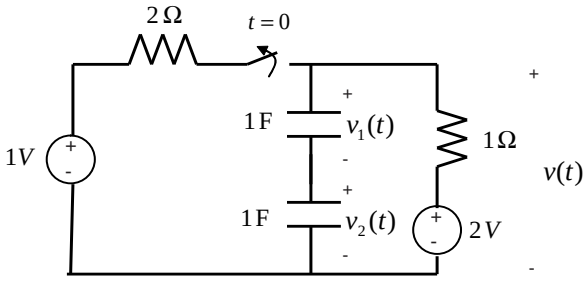
ECE 232 - Circuits and Systems II

Test 1

Please provide clear and complete answers. Don't forget to specify the units of measure!

Consider the circuit in the figure below. The switch has been closed for a long time before it opens at time $t = 0$. It is known that $v_1(0) = \frac{5}{6}$ V.



1. Calculate $v(t)$ for $t \geq 0$ (Hint: The equivalent capacitor for a series is $\frac{C_1 C_2}{C+C_2}$).

2. Plot $v(t)$ for $t \geq 0$.

3. Calculate the current flowing in the two capacitors, and also $v_1(t)$ and $v_2(t)$ for $t \geq 0$.

4. Find the total energy dissipated by the 1 Ω resistor. Calculate the initial and final energy stored in the capacitors. How much energy is provided in total by the 2 V source for $t \geq 0$?

5. Assume now that the 2 V voltage source turns off at time $t = 1$ s (i.e., it becomes a short circuit). Calculate $v(t)$ for $t \geq 1$ s.

Sol.:
1. We have

$$v(0) = 2 - \frac{1}{3} = \frac{5}{3} \text{ V},$$
$$C_{eq} = 0.5 \text{ F},$$
$$v(\infty) = 2 \text{ V}$$

and

$$\tau = RC_{eq} = 0.5 \text{ s}.$$

It follows that

$$v(t) = 2 + (5/3 - 2)e^{-2t}$$
$$= 2 - \frac{1}{3}e^{-2t} \text{ V}$$

for $t \geq 0$.
2. Please see class notes on how to plot.

3. We can calculate the current flowing in the capacitors

$$i(t) = C_{eq}\frac{dv(t)}{dt} = 0.5(\frac{2}{3}e^{-2t}) = \frac{1}{3}e^{-2t} \text{ A}$$

for $t \geq 0$. We can then obtain

$$\begin{aligned} v_1(t) &= \frac{5}{6} + \int_0^t i(x)dx \\ &= 1 - \frac{1}{6}e^{-2t} \text{ V}, \end{aligned}$$

and

$$v_2(t) = v_1(t)$$

for $t \geq 0$.
4. The total energy dissipated by the 1 Ω resistor is

$$\begin{aligned} \int_0^\infty i(t)^2 dt &= \int_0^\infty \frac{1}{9}e^{-4t}dt \\ &= \frac{1}{36} = 0.027 \text{ J}. \end{aligned}$$

The initial energy in the capacitors is $C_{eq}v(0)^2/2 = 0.694$ J and the final energy is $C_{eq}v(\infty)^2/2 = 1$ J. The energy has increased by $1 - 0.694 = 0.306$ J. The overall energy provided by the 2 V source is thus $0.027 + 0.306 = 0.333$ J.
5. We have

$$v(1) = 2 - \frac{1}{3}e^{-2} = 1.955 \text{ V},$$

and

$$v(\infty) = 0 \text{ V}$$

so that

$$v(t) = 1.955e^{-2(t-1)} \text{ V}.$$