ECE 232 - Circuits and Systems II
Test 2

Consider the circuit in the figure. At time $t = 0$, the energy in the two inductors is zero. The source is $v_g(t) = 6\cos(10t)u(t)$.

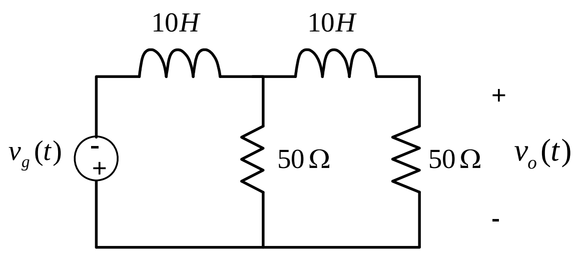

Figure 1:

a. Find the equivalent circuit in the s-domain. Find the transfer function between $v_g(t)$ and $v_o(t)$.

Sol.: The equivalent circuit is obtained following the usual rules. To find the transfer function, we can for instance calculate the equivalent impedance

$$Z_{eq}(s) = 50||(10s + 50) = 50\frac{(s+5)}{(s+10)}.$$

We then calculate $V_O(s)$ by applying the voltage division rule twice

$$\begin{aligned} V_o(s) &= -V_g(s) \cdot \frac{Z_{eq}(s)}{10s + Z_{eq}(s)} \cdot \frac{50}{10s+50} \\ &= -V_g(s) \cdot \frac{50\frac{(s+5)}{(s+10)}}{10s + 50\frac{(s+5)}{(s+10)}} \cdot \frac{50}{10s+50} \\ &= -V_g(s) \cdot \frac{250}{10s(s+10) + 50(s+5)} \\ &= -V_g(s) \cdot \frac{250}{10s^2 + 100s + 50s + 250} \\ &= -V_g(s) \cdot \frac{25}{s^2 + 15s + 25} \end{aligned}$$

so that the desired transfer function is

$$H(s) = -\frac{25}{s^2 + 15s + 25}.$$

b. Find poles and zeros of $V_o(s)$. Which poles correspond to transient and which to steady-state components of the solution?

Sol.: We have

$$\begin{aligned} V_o(s) &= -V_g(s) \cdot \frac{25}{s^2+15s+25} \\ &= -\frac{150s}{(s^2+100)(s^2+15s+25)} \\ &= -\frac{150s}{(s^2+100)(s+13.09)(s+1.91)}. \end{aligned}$$

The zero is $s = 0$. The poles are:

$$\begin{aligned} s &= \pm j10 \text{ steady-state} \\ s &= -13.09 \text{ transient} \\ & \quad -1.91 \text{ transient.} \end{aligned}$$

c. Evaluate $v_o(t)$ for $t \geq 0$. Identify the transient and steady-state components.

Sol.: Using partial fraction expansion we can write

$$-\frac{150s}{(s^2+100)(s^2+15s+25)} = \frac{K_1}{s-j10} + \frac{K_1^*}{s+j10} + \frac{K_2}{s+13.09} + \frac{K_3}{s+1.91},$$

where we can calculate

$$\begin{aligned} K_1 &= -(s-j10)\frac{150s}{(s^2+100)(s^2+15s+25)}\bigg|_{s=j10} \\ &= -\frac{150s}{(s+j10)(s^2+15s+25)}\bigg|_{s=j10} \\ &= -\frac{1500}{20(-100+j150+25)} \\ &= 0.44\angle 63.43^\circ, \end{aligned}$$

and

$$\begin{aligned} K_2 &= -\frac{150s}{(s^2+100)(s+1.91)}\bigg|_{s=-13.09} \\ &= -0.64, \end{aligned}$$

and

$$\begin{aligned} K_3 &= -\frac{150s}{(s^2+100)(s+13.09)}\bigg|_{s=-1.91} \\ &= 0.24, \end{aligned}$$

We thus have

$$v_o(t) = \left(\begin{array}{c} 0.88\cos(10t+63.43^\circ) - 0.64\exp(-13.09t) \\ +0.24\exp(-1.91t) \end{array} \right) u(t)\ V,$$

where the first term ($0.88\cos(10t+63.43°)$) is steady-state and the second ($-0.64\exp(-13.09t)+0.24\exp(-1.91t)$) is transient.

d. Assume now that the inductor close to the source has initial energy 20 J and the initial current in the inductor flows towards the left (i.e., towards the source). Obtain an equivalent circuit in the s-domain and find $V_o(s)$.

Sol.: We have $\frac{1}{2}Li_L^2(0) = 5i_L^2(0) = 20$, so that $i_L(0) = 2\ A$. An equivalent circuit can be found by using the usual rules.
To evaluate $V_o(s)$, it is convenient to use the series equivalent of the inductor, where we need to add a voltage source of $Li_L(0) = 20\ V$. Given the direction of the current, the polarity of the source is the the same as that of source $v_g(t)$. We now realize that the transfer function from the source due to the initial conditions is the same as the one from the source $v_g(t)$. Therefore, using the superposition principle, we get

$$V_o(s) = -(V_g(s) + 20)\frac{25}{s^2 + 15s + 25}.$$