ECE 232 - Circuits and Systems II
Test 2

Please provide clear and complete answers. Don't forget to specify the units of measure!

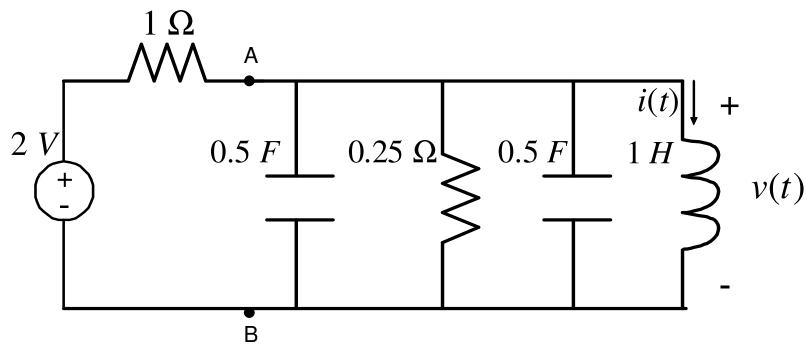

Figure 1:

Consider the circuit in the figure and assume at first that the initial energy is zero.
1. (1 point) Evaluate the impedance $Z(s)$ seen at the terminals A and B by the series of source and 1 Ω resistor.
2. (1 point) Calculate the Laplace transform $V(s)$ of the voltage $v(t)$.
3. (1 point) Calculate the poles and zeros of $V(s)$ and indicate which poles are steady-state and which poles are transient.
4. (1 point) Identify the transfer function between the voltage source and $V(s)$.
5. (2 points) Calculate $v(t)$.
6. (2 points) Calculate the Laplace transform $I(s)$ of the current $i(t)$. Evaluate the steady-state value of $i(t)$ *using partial fraction expansion.*
7. (2 points) Assume now that the initial current in the inductor is 1 A. Evaluate the Laplace transform $V(s)$.

Solution:
1. We have

$$Z(s) = \frac{2}{s}||\frac{2}{s}||0.25||s = \frac{1}{4+\frac{1}{s}+s} = \frac{s}{s^2+4s+1}.$$

2. By the voltage division rule, we can write

$$\begin{aligned} V(s) &= \frac{2}{s}\frac{Z(s)}{1+Z(s)} \\ &= \frac{2}{s^2+4s+1+s} \\ &= \frac{2}{s^2+5s+1}. \end{aligned}$$

3. No zeros. Poles: $s_1 = -2.5+\sqrt{(2.5)^2-1} = -0.2087$, $s_2 = -2.5-\sqrt{(2.5)^2-1} = -4.7913$. All poles are transient.

4. We have

$$H(s) = \frac{Z(s)}{1+Z(s)} = \frac{s}{s^2+5s+1}.$$

5. Using partial fraction expansion,

$$\begin{aligned} V(s) &= \frac{2}{s^2+5s+1} \\ &= \frac{2}{(s+0.2087)(s+4.7913)} \\ &= \frac{K_1}{s+0.2087} + \frac{K_2}{s+4.7913}, \end{aligned}$$

with

$$\begin{aligned} K_1 &= \frac{2}{-0.2087+4.7913} = 0.4364 \\ K_2 &= \frac{2}{-4.7913+0.2087} = -0.4364. \end{aligned}$$

It follows that

$$v(t) = 0.4364(e^{-0.2087t} - e^{-4.7913t}),\ t \geq 0.$$

6. We have

$$\begin{aligned} I(s) &= \frac{V(s)}{s} \\ &= \frac{2}{s(s^2+5s+1)}. \end{aligned}$$

The steady-state pole is s=0 and thus the steady-state value of $i(t)$ is obtained as

$$i(\infty) = \frac{2s}{s(s^2+5s+1)}|_{s=0} = 2.$$

7. Using the parallel representation of an inductor in the frequency domain, along with the superposition principle, we get

$$\begin{aligned} V(s) &= \frac{2}{s}\frac{Z(s)}{1+Z(s)} - \frac{1}{s}(Z(s)||1) \\ &= \frac{2}{s}\frac{Z(s)}{1+Z(s)} - \frac{1}{s}\frac{Z(s)}{1+Z(s)} \\ &= \frac{1}{s}\frac{Z(s)}{1+Z(s)} \\ &= \frac{1}{s^2+5s+1}. \end{aligned}$$