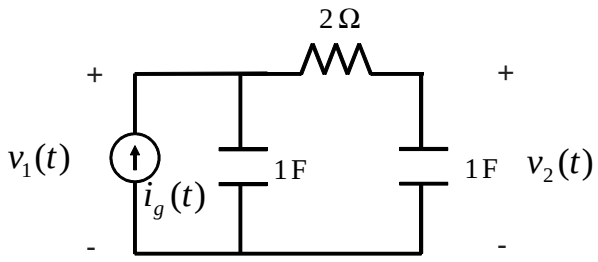
ECE 232 - Circuits and Systems II

Test 2

Please provide clear and complete answers. Don't forget to specify the units of measure!

Consider the circuit in the figure below. The initial energy in the two capacitors is zero at time $t = 0$ and the current source is $i_g(t) = e^{-t/2}u(t)$.



1. Draw the circuit in the Laplace domain.
2. Calculate $V_1(s)$.
3. Calculate $V_2(s)$.
4. Calculate $v_1(t)$ for $t \geq 0$.
5. Calculate $v_1(t)$ and $v_2(t)$ for $t \to \infty$ using partial fraction expansion.

Sol.: 1. Please see class notes or textbook.
2. We can calcualte the equivalent impedance seen by the source as

$$\begin{aligned} Z_{eq}(s) &= \frac{1}{s}||(2+\frac{1}{s}) \\ &= \frac{\frac{1}{s}(2+\frac{1}{s})}{2(1+\frac{1}{s})} = \frac{2s+1}{2s(s+1)} \\ &= \frac{s+1/2}{s(s+1)}. \end{aligned}$$

It follows that

$$\begin{aligned} V_1(s) &= \frac{1}{s+1/2} Z_{eq}(s) \\ &= \frac{1}{s(s+1)}. \end{aligned}$$

3. By the voltage division rule, we have

$$\begin{aligned} V_2(s) &= \frac{1/s}{2+1/s} V_1(s) \\ &= \frac{1/2}{s+1/2} V_1(s) \\ &= \frac{1/2}{s(s+1)(s+1/2)}. \end{aligned}$$

4. Using partial fraction expansion, we calculate

$$K_1 = V_1(s)s|_{s=0} = 1$$
$$K_2 = V_1(s)(s+1)|_{s=-1} = -1$$

from which we can conclude that

$$V_1(s) = \frac{1}{s} - \frac{1}{(s+1)}$$

and

$$v_1(t) = 1 - e^{-t} \text{ V}$$

for $t \geq 0$.
5. For $v_1(t)$, from the results at point 4., we have

$$v_1(t) = K_1 = 1 \text{ V}$$

for $t \to \infty$. Similarly, for $v_2(t)$, we can calculate

$$v_2(t) = K_1 = V_2(s)s|_{s=0} = 1 \text{ V}$$

for $t \to \infty$.