ECE 232 - Circuits and Systems II
Test 2, Fall 2011

Consider the circuit in the figure. We have the two sources $i_{s_1}(t) = 3u(t)$ [A] and $i_{s_2}(t) = 2\cos(t)u(t)$ [A]. At time $t = 0$, no energy is stored in the capacitor and in the inductor.

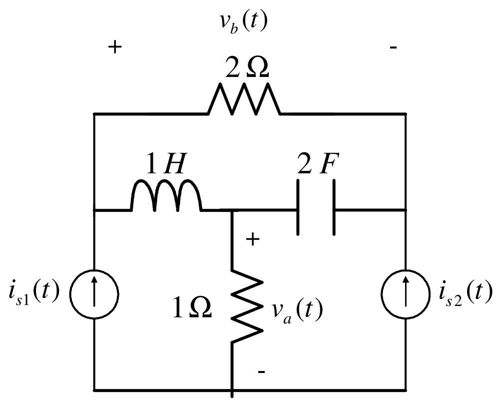

Figure 1:

a. Find the Laplace transform $V_a(s)$ of $v_a(t)$, for $t \geq 0$, and identify the transfer functions from the two sources $I_{s_1}(s)$ and $I_{s_2}(s)$ to $V_a(s)$.

Sol.: We use the superposition principle. Note that $I_{s_1}(s) = 3/s$ and $I_{s_2}(s) = 2s/(s^2+1)$. When considering source $I_{s_1}(s)$, we set $I_{s_2}(s) = 0$, and thus we get $V_a(s) = 3/s$. Instead, when considering source $I_{s2}(s)$, we set $I_{s1}(s) = 0$, and thus we get $V_a(s) = 2s/(s^2+1)$. Overall, we obtain

$$V_a(s) = \frac{3}{s} + \frac{2s}{s^2+1},$$

and the two transfer functions are both simply $H(s) = 1$.

b. Find poles and zeros of $V_a(s)$. Which poles are steady-state and which poles are transient? Finally, calculate $v_a(t)$ for $t \geq 0$.

Sol.: We obtain

$$V_a(s) = \frac{3(s^2+1)+2s^2}{s(s^2+1)} = \frac{5s^2+3}{s(s^2+1)},$$

so we have poles $s = \pm j$ (steady state), $s = 0$ (steady state) and we have zeros $s = \pm j\sqrt{3/5}$. Converting in time domain, we get

$$v_a(t) = (3 + 2\cos t)u(t).$$

c. Assume $i_{s2}(t) = 0$ for simplicity (but we still have the other source, namely $i_{s_1}(t) = 3u(t)$). Find the Laplace transform $V_b(s)$ of $v_b(t)$, for $t \geq 0$, and identify the transfer functions from the source $I_{s_1}(s)$ to $V_b(s)$.

Sol.: Using the current division rule, we get

$$\begin{aligned} V_b(s) &= 2 \cdot \frac{3}{s} \cdot \frac{s}{s + 2 + \frac{1}{2s}} \\ &= \frac{3}{s} \cdot \frac{2s^2}{s^2 + 2s + 1/2}, \end{aligned}$$

so that the transfer function from the source $I_{s_1}(s)$ to $V_b(s)$ is given by $H(s) = \frac{2s^2}{s^2+2s+1/2}$.

d. Find poles and zeros of $V_b(s)$. Which poles are steady-state and which poles are transient? Finally, calculate $v_b(t)$ for $t \geq 0$.

Sol.: We have

$$V_b(s) = \frac{6s}{s^2 + 2s + 1/2},$$

and hence the poles are $s = -1 + 1/\sqrt{2} = -0.29$ and $s = -1 - 1/\sqrt{2} = -1.71$, both transient, and the zero is $s = 0$.
To find $v_b(t)$ we use partial fraction expansion and write

$$\begin{aligned} V_b(s) &= \frac{6s}{s^2 + 2s + 1/2} \\ &= \frac{6s}{(s + 1 - 1/\sqrt{2})(s + 1 + 1/\sqrt{2})} \\ &= \frac{K_1}{s + 1 - 1/\sqrt{2}} + \frac{K_2}{s + 1 + 1/\sqrt{2}}, \end{aligned}$$

where

$$\begin{aligned} K_1 &= \frac{6s}{s + 1 + 1/\sqrt{2}}\Big|_{s=-1+1/\sqrt{2}} = \frac{6(-1 + 1/\sqrt{2})}{2/\sqrt{2}} \\ &= -1.24 \end{aligned}$$

and

$$\begin{aligned} K_2 &= \frac{6s}{s + 1 - 1/\sqrt{2}}\Big|_{s=-1-1/\sqrt{2}} = \frac{6(-1 - 1/\sqrt{2})}{-2/\sqrt{2}} \\ &= 7.24. \end{aligned}$$

It follows that

$$v_b(t) = -1.24e^{-(-1+1/\sqrt{2})t} + 7.24e^{-(-1-1/\sqrt{2})t}, \ t \geq 0.$$