ECE 232 - Circuits and Systems II
Test 3

Please provide clear and complete answers. Don't forget to specify the units of measure!

For the circuit in the figure below, where input and output are denoted by $v_I(t)$ and $v_O(t)$, respectively, answer the following questions. Assume that the initial conditions are zero.

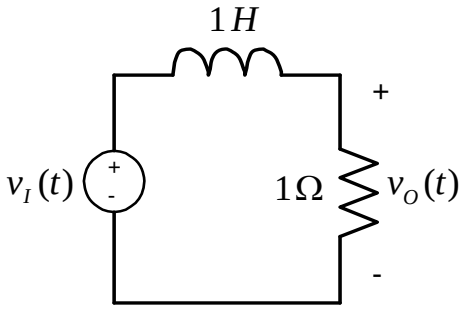

Figure 1:

a. What type of filter is it? Justify your answer by considering the behavior of the circuit for high and low frequencies. What is the cut-off frequency?

b. Find the transfer function $H(s)$ and the impulse response $h(t)$.

c. If the input is $v_I(t) = \cos(10t)u(t)$, what is the output in steady-state? (Hint: Recall the definition of frequency response).

d. Use the convolution integral to find the output if the input is $v_I(t) = \exp(-t)u(t)$. (Hint: Write down the integral between $v_I(t)$ and $h(t)$ using the definition!)

Sol.:
a. It is a low-pass filter as it can be justified by following the same reasoning used in class (please see notes). The cut-off frequency is $\omega_c = 1/\tau = R/L = 1$ rad/s.

b. We have

$$\begin{aligned} H(s) &= \frac{1}{1+s} \\ h(t) &= e^{-t}u(t) \end{aligned}$$

c. Evaluating the frequency response for $s = j10$, we get

$$H(j10) = \frac{1}{1+j10} = 0.0995\angle -84.29^\circ$$

In steady-state, we thus have

$$\begin{aligned} v_O(t) &= |H(j10)|\cos(10t + \theta(j10))u(t) \\ &= 0.0995\cos(10t - 84.29^\circ). \end{aligned}$$

d.

$$\begin{aligned} v_I(t) * h(t) &= \int_0^t v_I(\tau)v_I(t-\tau)d\tau \\ &= \int_0^t e^{-\tau}e^{-(t-\tau)}d\tau \\ &= \int_0^t e^{-\tau-t+\tau}d\tau = \int_0^t e^{-t}d\tau = te^{-t}. \end{aligned}$$