ECE 232 - Circuits and Systems II
Test 3

The frequency response of a circuit is given by

$$H(j\omega) = \frac{0.5}{j\omega + 1}.$$

a. Write and sketch the amplitude response. What type of filter is it? What is the maximum value $H_{\max}$ of $|H(j\omega)|$? What is the $3dB$ cut-off frequency?

b. If the input is $x(t) = \delta(t)$, find the output.

c. If the input is $x(t) = u(t) - u(t-1)$ (i.e., a rectangle of unit ampliture and duration 1 s), find the output using the convolution integral.

d. If the input is $x(t) = 3\cos(2\pi t)$, find the output in steady-state.

Sol.:
a. The amplitude response is

$$|H(j\omega)| = \frac{0.5}{\sqrt{1+\omega^2}}.$$

The maximum value of $|H(j\omega)|$ is obtained for $\omega = 0$ and is given by $H_{\max} = 0.5$. It is a low-pass filter with cut-off frequency ω_c given by

$$|H(j\omega_c)| = \frac{0.5}{\sqrt{1+\omega_c^2}} = \frac{H_{\max}}{\sqrt{2}} \rightarrow \omega_c = 1 \text{ rad/s}.$$

b. The impulse response is

$$h(t) = \mathcal{L}^{-1}\left(\frac{0.5}{s+1}\right) = 0.5e^{-t}u(t).$$

c. We have for $0 \leq t \leq 1$

$$y(t) = 0.5\int_0^t e^{-\tau}d\tau = 0.5(1-e^{-t}),$$

and for $t \geq 1$

$$\begin{aligned} y(t) &= 0.5\int_{t-1}^{t} e^{-\tau}d\tau \\ &= 0.5e^{-t}(e-1). \end{aligned}$$

d. We have

$$H(j2\pi) = \frac{0.5}{j2\pi+1} = 0.0786\angle -80.95^\circ.$$

so that the output in steady-state is given by

$$y(t) = 0.2358\cos(2\pi t - 80.95^\circ).$$