ECE 232 - Circuits and Systems II
Test 3

Please provide clear and complete answers. Don't forget to specify the units of measure!

Consider the circuit in the figure below (part (a)). The input is $v_s(t)$ and the output $v_o(t)$.

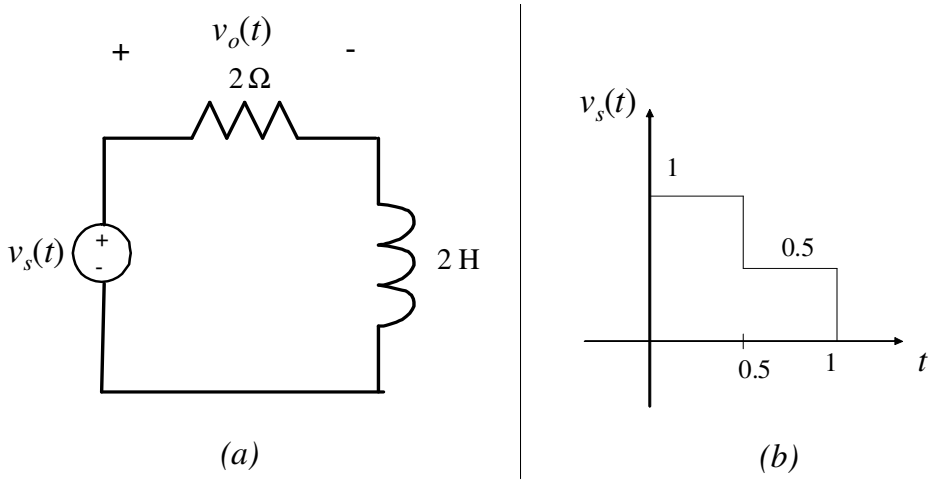

Figure 1:

1. (2 points) Calculate the impulse response $h(t)$.
2. (4 points) Calculate the output $v_o(t)$ when the input $v_s(t)$ is as shown in the figure, part (b).
3. (2 points) Plot $v_o(t)$.
4. (2 points) Calculate the output $v_o(t)$ in steady-state if the input is $v_s(t) = 3\cos(2t)u(t)$.

Sol.: **1.** The transfer function is

$$H(s) = \frac{2}{2+2s} = \frac{1}{s+1},$$

and the impulse response is then

$$h(t) = e^{-t}u(t).$$

2. Following the usual procedure, we get:

- For $t \leq 0$, $v_o(t) = 0$.
- For $0 \leq t \leq 0.5$

$$v_o(t) = \int_0^t e^{-\tau}d\tau = 1 - e^{-t}$$

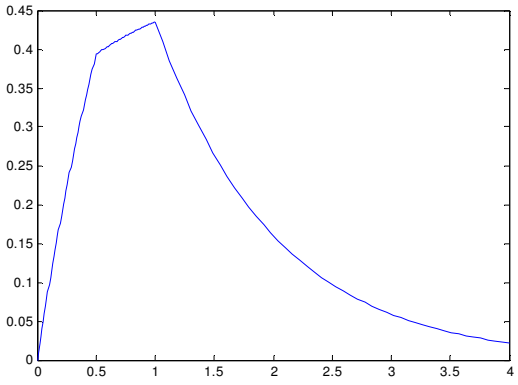

Figure 2:

- For $0.5 \leq t \leq 1$

$$\begin{aligned} v_o(t) &= \int_0^{t-1/2} \frac{1}{2} e^{-\tau} d\tau + \int_{t-1/2}^{t} e^{-\tau} d\tau \\ &= \frac{1}{2}(1 - e^{-(t-1/2)}) + (e^{-(t-1/2)} - e^{-t}) \\ &= \frac{1}{2} + \frac{1}{2} e^{-(t-1/2)} - e^{-t} \\ &= \frac{1}{2} + e^{-t}\left(\frac{1}{2} e^{1/2} - 1\right) \end{aligned}$$

- For $t \geq 1$

$$\begin{aligned} v_o(t) &= \int_{t-1}^{t-1/2} \frac{1}{2} e^{-\tau} d\tau + \int_{t-1/2}^{t} e^{-\tau} d\tau \\ &= \frac{1}{2}\left(e^{-(t-1)} - e^{-(t-1/2)}\right) + \left(e^{-(t-1/2)} - e^{-t}\right) \\ &= e^{-t}\left(\frac{1}{2} e - \frac{1}{2} e^{1/2} + e^{1/2} - 1\right) \\ &= e^{-t}\left(\frac{1}{2} e + \frac{1}{2} e^{1/2} - 1\right). \end{aligned}$$

3. The plot is as in the figure.

4. The frequency response at frequency 2 rad/s is

$$H(j2) = \frac{1}{j2 + 1} = 0.45 e^{-j1.1},$$

which leads to the steady-state output

$$v_o(t) = 1.35 \cos(2t - 1.1) u(t).$$