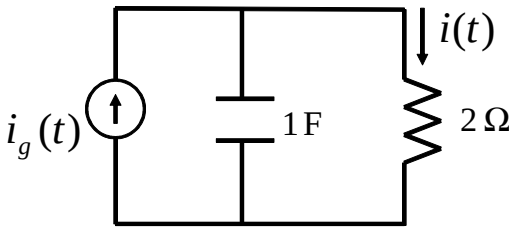
ECE 232 - Circuits and Systems II

Test 3

Please provide clear and complete answers. Don't forget to specify the units of measure!

Consider the circuit in the figure below.



1. (2 points) Calculate the transfer function between $i_g(t)$ and $i(t)$ and the corresponding impulse response.
2. (2 points) Assume that the input $i_g(t)$ is a rectangle of height 1/2 and duration 1/4 seconds. Calculate the current $i(t)$ for all $0 \leq t \leq 1/4$ using the convolution integral.
3. (2 points) Continue the point above by calculating $i(t)$ for all $t \geq 1/4$.
4. (2 point) Plot the resulting $i(t)$ for all $t \geq 0$.
5. (2 points) If the input is $i_g(t) = (2 + 2\cos(0.5t))u(t)$ A, what is the output $i(t)$ in steady state?

Sol.: 1. By the current division rule, we get

$$H(s) = \frac{1/s}{2 + 1/s} = \frac{1}{2s+1} = \frac{1/2}{s + 1/2}.$$

The impulse response is then

$$h(t) = \frac{1}{2} e^{-\frac{t}{2}} u(t)$$

.

2. For $0 \leq t \leq 1/4$, we have

$$\begin{aligned} i(t) &= i_g(t) * h(t) \\ &= \int_0^t \frac{1}{2}\frac{1}{2} e^{-\frac{\tau}{2}} d\tau \\ &= \frac{1}{4}\,(-2)\, e^{-\frac{\tau}{2}}|_0^t \\ &= -\frac{1}{2}(e^{-t/2} - 1) \\ &= \frac{1}{2} - \frac{1}{2} e^{-t/2}. \end{aligned}$$

3. For $t \geq 1/4$, we have

$$\begin{aligned} i(t) &= i_g(t) * h(t) \\ &= \int_{t-1/4}^{t} \frac{1}{2}\frac{1}{2}e^{-\frac{\tau}{2}}d\tau \\ &= \frac{1}{4}\left(-2\right)e^{-\frac{\tau}{2}}|_{t-1/4}^{t} \\ &= \frac{1}{2}e^{-(t-1/4)/2} - \frac{1}{2}e^{-t/2} \\ &= \frac{1}{2}e^{-t/2}(e^{1/8} - 1). \end{aligned}$$

5. We have

$$H(0) = 1$$

and

$$H(j0.5) = \frac{1}{\sqrt{2}}e^{-j\pi/4},$$

and so, in the steady-state regime, we can write

$$i(t) = (2 + \sqrt{2}\cos(0.5t - \frac{\pi}{4}))u(t).$$