ECE 232 - Circuits and Systems II
Test 3

For a given circuit, we know that, if the input is $x(t) = 4\delta(t-3)$, then the output is given by $y(t) = 8e^{-2(t-3)}u(t-3)$.

a. Find the impulse response and the transfer function of the circuit.

b. Draw an RC circuit that has the obtained impulse response and transfer function (specify the values of R and C).

c. If the input is $x(t) = u(t) - u(t-2)$ (i.e., a rectangle of unit amplitude and duration 2 s), find the output using the convolution integral.

d. If the input is $x(t) = (\cos(2t) + 2)u(t)$, find the output in steady state.

Sol.: a. The impulse response is

$$h(t) = 2e^{-2t}u(t),$$

and the transfer function is

$$H(s) = \frac{2}{s+2}.$$

b. It is a low-pass filter, so the output is the voltage on the capacitor. We need to have $1/\tau = 2$, and thus $RC = 0.5$. A possible choice is $R = 0.5\ \Omega$ and $C = 1\ F$.

c. Please see notes and textbook for the procedure. We get the output

$$y(t) = \begin{cases} 1 - e^{-2t} & \text{for } 0 \le t \le 2 \\ e^{-2t}(e^4 - 1) & \text{for } t \ge 2 \\ 0 & \text{otherwise} \end{cases}$$

d. We need to calculate

$$H(j2) = \frac{1}{j+1} = \sqrt{2}e^{-j\frac{\pi}{4}}$$

and

$$H(j0) = 1.$$

From this, we get that the output in steady state is

$$y(t) = (\sqrt{2}\cos\left(2t - \frac{\pi}{4}\right) + 2)u(t).$$