PHYS 446 Fall 2007 Homework Assignment 5 Nov 9th

(COVERS ALL METALS: Lectures 7 and 8)

1. Copper has a mass density ρ = 8.95 g/cm^3, and electrical resistivity 1.55×10^{-8} ohm·m at room temperature. Assuming that the effective mass of electron in Cu $m^* = m_0$, (m_0 – free electron mass) calculate:
   a) The concentration of the conduction electrons
   b) The mean relaxation time τ
   c) The Fermi energy E_F and the Fermi velocity v_F
   d) The mean free path at the Fermi level l_F

2. The He^3 atom has spin ½ and is a fermion. The density of liquid He^3 is 0.081 g/cm^3 near absolute zero. Calculate the Fermi energy E_F and the Fermi temperature T_F.

3. Estimate the fraction of electrons excited above the Fermi level at room temperature for Cu and Na. The electron effective masses in Cu and Na are $1.0\times m_0$ and $1.2\times m_0$, respectively, where m_0 – free electron mass.

4. At what temperature T_0 does the specific heat of the free electrons become larger than the specific heat of the lattice? Express T_0 in terms of the Debye temperature and the electron concentration. Calculate T_0 for copper. (The Debye temperature is 343 K).

5. The residual resistivity for 1 atomic percent of As impurities in Cu is 6.8×10^{-8} ohm·m. Calculate the cross section for the electron scattering by one As impurity in Cu.

6. Cyclotron resonance has been observed in copper at a frequency of 24 GHz. Assuming that the effective mass of electron in Cu $m^* = m_0$, (m_0 – free electron mass) determine the value of applied magnetic field.

7. Using the free-electron model we obtained the Fermi wave vector – the radius of the Fermi sphere on **k**-space: $k_F = \left(3\pi^2 n\right)^{1/3}$, where n is the electron concentration. As n increases, the Fermi sphere expands.

a) Show that this sphere begins to touch the faces of the first Brillouin zone in a fcc lattice when the electron-to-atom ratio $n/n_a = 1.36$, where n_a is the concentration of atoms.

b) Suppose that some atoms in Cu crystal (Cu has a fcc lattice) are replaced by Zn atoms. Taking into account that Zn is bivalent, while Cu is monovalent, calculate the atomic ratio of Zn to Cu in ZnCu alloy (brass) at which the Fermi surface touches the first Brillouin zone faces. (Use free-electron model.)

8. a) Using the expression for electron velocity for a one-dimensional crystal in the tight-binding model, show that the velocity is zero at the Brillouin zone edge and zone center.

   b) Repeat the same for a three-dimensional simple cubic lattice, and show that the electron velocity at a Brillouin zone face is parallel to that face.

$$v = \frac{1}{\hbar}\nabla_{\mathbf{k}} E(\mathbf{k}) \qquad E(k) = E_0 + 4\gamma\left[\sin^2\left(\frac{k_x a}{2}\right) + \sin^2\left(\frac{k_y a}{2}\right) + \sin^2\left(\frac{k_z a}{2}\right)\right]$$

9. Using the tight-binding model, calculate the effective mass m* as a function of k in a one-dimensional lattice. Plot the m*(k) and show that the mass is independent on k only near the center and the edge of the Brillouin zone.

$$m^* = \hbar^2\left(\frac{d^2E}{dk^2}\right)^{-1}$$

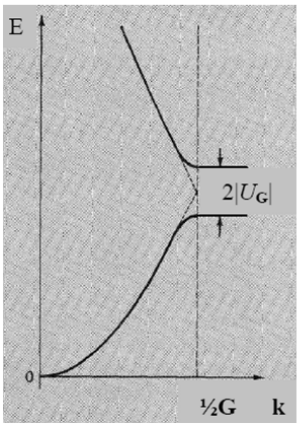
EXTRA CREDIT: Suppose that the crystal potential in one-dimensional lattice with lattice constant a is composed of a series of rectangular wells surrounding the atoms. Let the depth of each well is V_0 and its width is $a/5$.

a) Using the nearly-free electron model, calculate the values of the first three energy gaps and compare their magnitudes.

b) Evaluate these gaps for $V_0 = 5$ eV and $a = 4$ Å.

$$\left\langle \psi^0_{\mathbf{k}} \middle| U \middle| \psi^0_{\mathbf{k}-\mathbf{G}} \right\rangle = \frac{1}{V_c}\int_{cell} e^{-i\mathbf{k}\mathbf{r}} U(\mathbf{r}) e^{i(\mathbf{k}-\mathbf{G})\mathbf{r}} dr = \frac{1}{V_c}\int_{cell} U(\mathbf{r}) e^{-i\mathbf{G}\mathbf{r}} dr = U_{\mathbf{G}}$$

$$E = \frac{1}{2}\left(E^0(\mathbf{k}) + E^0(\mathbf{k}-\mathbf{G})\right) \pm \left[\frac{1}{4}\left(E^0(\mathbf{k}) - E^0(\mathbf{k}-\mathbf{G})\right)^2 + \left|U_{\mathbf{G}}\right|^2\right]^{1/2}$$



This results is particularly simple for point lying on the Bragg plane: $E^0(\mathbf{k}) = E^0(\mathbf{k}-\mathbf{G})$

Obtain $E = E^0(\mathbf{k}) \pm \left|U_{\mathbf{G}}\right|$

The magnitude of the band gap is equal to twice the Fourier component of the crystal potential.