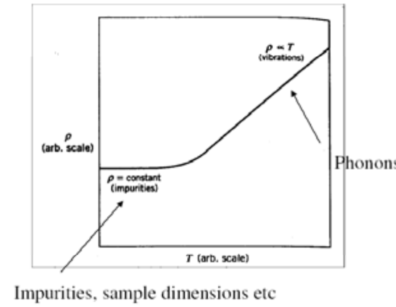
Phys 446 Solid State Physics
Lecture 11 Nov 25
(Ch. 10)

SUPERCONDUCTIVITY

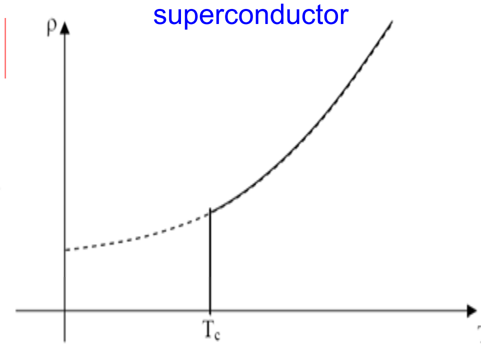
Superconductivity is a phenomenon occurring in certain materials at low temperatures, characterized by exactly zero electrical resistance

Discovered in 1911 by Heike Kamerlingh Onnes - the resistance of solid mercury abruptly disappeared at the temperature of 4.2 K

normal metal



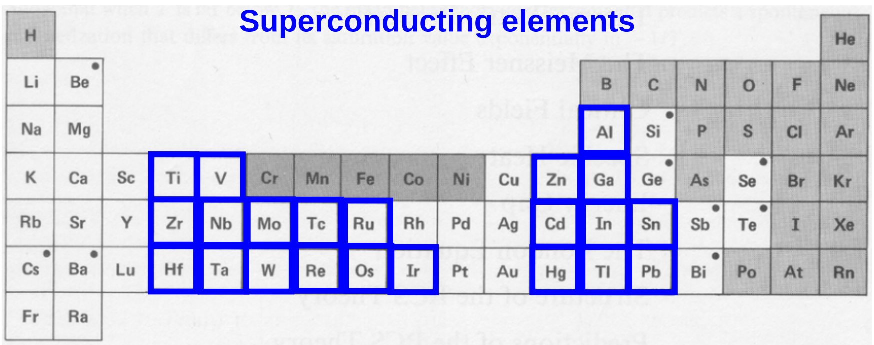
superconductor



$$\rho = \frac{m}{ne^2\tau}$$

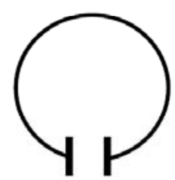
zero ρ means there are some electrons with infinite τ – *no scattering*

Superconducting elements



- More than 20 metallic elements are superconductors
- Cu, Au, Ag, Na, K and magnetically ordered metals (Fe, Ni, Co) are *not superconductors*
- Certain elements are superconducting at high pressures or as thin films
- Highest T_c of an element is 9.3 K for Nb
- There are thousands of alloys and compounds that exhibit superconductivity
- The highest T_c superconductors tend to be poor conductors in the normal state
- Record T_c is currently ~ 138 K (a ceramic consisting of Tl, Hg, Cu, Ba, Ca, Sr, and O)

Persistent current in a superconductor



Most sensitive method of measuring a small resistance;

→ Look for the decay of current around a closed superconducting loop

R = Resistance of the loop
L = Self-inductance

$$V = -L\frac{dI}{dt} = IR$$

Current should decay with time constant τ = L/R

$$I(t) = I_0 e^{-\frac{R}{L}t}$$

No decay observed!

This gives an upper value on the value of R:

Resistivity $\rho < 10^{-26}$ Ω-cm for a superconductor

Compare with $\rho < 10^{-8}$ Ω-cm for Cu

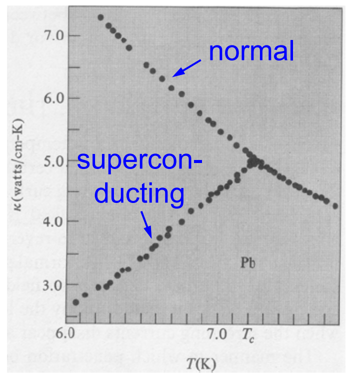
18 orders of magnitude difference!

Limitations of persistent current flow

Persistent currents will flow in a superconductor unless:

1) A sufficiently large magnetic field is applied
2) The current exceeds a certain critical current, I_c (the Silsbee effect)
3) An AC electric field above a certain frequency is applied
   The transition from dissipationless to normal response occurs at $\omega \sim \Delta/\hbar$, where Δ is the energy gap

Superconductors are poor thermal conductors

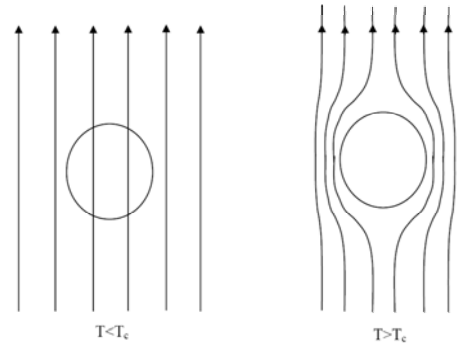


Thermal conductivity of lead
From J.H.P. Watson and G. M Graham, Can. J. Phys. **41***, 1738 (1963)*

Perfect diamagnetism

Second basic property of superconductors – they expel magnetic field completely when in superconducting phase ($T<T_c$).

This phenomenon is called ***the Meissner effect***.



$$\boldsymbol{B} = \mu_o(\boldsymbol{H} + \boldsymbol{M}) = 0$$

$$\chi = \frac{M}{H} = -1$$

Distinguishes the superconductor from an ideal but normal conductor, for which $dB/dt = 0$

Could explain the expulsion of the flux if a S/C is moved into a magnetic field → motion of metal produces currents, which do not decay because $R = 0$

However, $R = 0$ does not explain the flux expulsion when a S/C is already in a magnetic field and is cooled from its normal to S/C state (through T_c)

Magnetic properties: type I superconductors

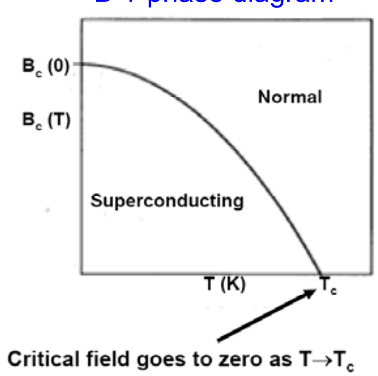
Superconductors are divided into 2 types, depending on their behavior in a magnetic field.

All superconducting elements are type I, except Nb

Superconductivity is destroyed by the presence of some magnetic field, ***the critical field***, B_c. Typically B_c ~ tens of mT for type I

$$B_c(T) = B_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

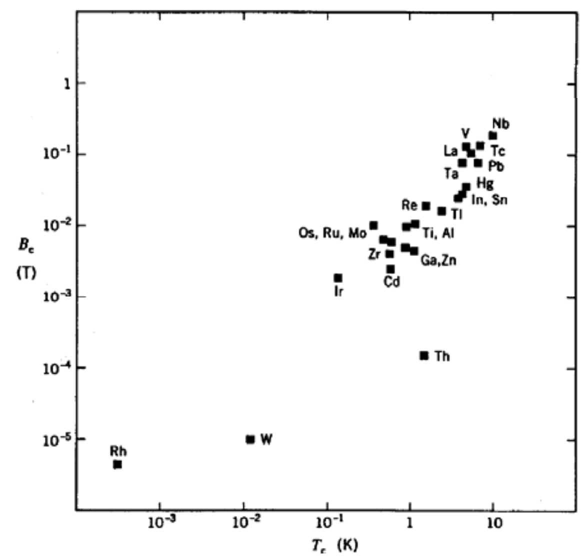
B-T phase diagram



Field B_c does not need to be external:

Critical current I_c causes a magnetic field B_c, which destroys superconductivity

Rough correlation between critical field and T_C



Penetration depth

Surface currents expel magnetic flux from type I superconductors

Currents actually penetrate the sample slightly (~ 100 nm)

Magnetic field decreases exponentially inside sample:

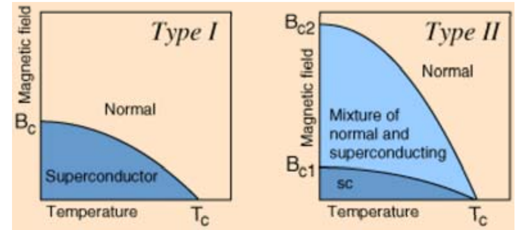
$$B(x) = B_o e^{-x/\lambda} \qquad \lambda(T) = \frac{\lambda(0)}{\sqrt{1-\left(\frac{T}{T_c}\right)^4}}$$

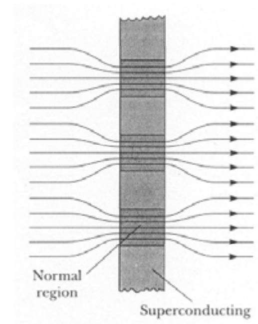
As $T \to T_c$, λ increases and B penetrates deeper into the sample.

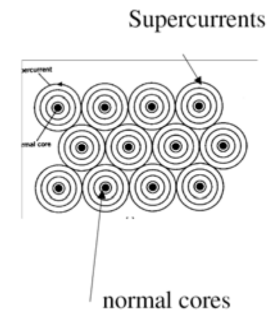
At T_c, $\lambda \to \infty$, and the sample goes normal

Type II superconductors

- Alloys (and Nb and V)
- Higher T_C, B_C, I_C than type I
- More applications e.g.Nb_3Sn magnets
- 2 critical fields:
  $B < B_{C1}$ - just like a type I
  $B > B_{C2}$ - normal
  Intermediate fields – sample has both superconducting and normal regions



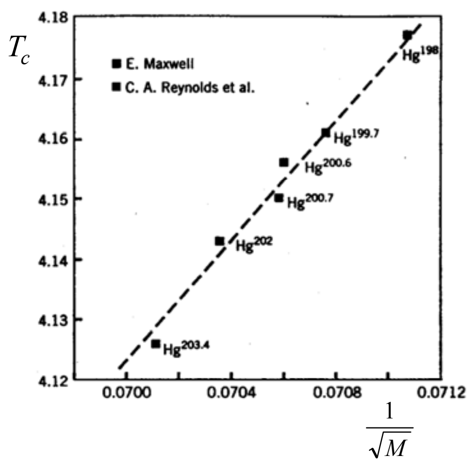




Isotope effect

- T_C depends on atomic mass: $T_c \propto \frac{1}{\sqrt{M}}$
- More general: $T_c M^a = constant$
- Suggests superconductivity is not just an electronic effect

Hg - Maxwell, Reynolds et al. (1950)



Isotope effect: 1st direct evidence of interaction of electrons and the lattice

Suggests that lattice vibrations play a part in the superconducting process

Experimental evidence for an energy gap – heat capacity

Specific heat of normal metal has form:

AT + BT³

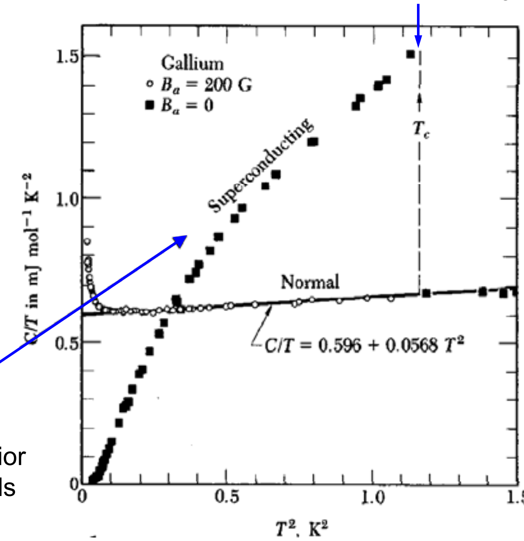
Electronic vibrations ***Phonons***

Exponential T-dependence

$$C \propto e^{-b/kT}$$

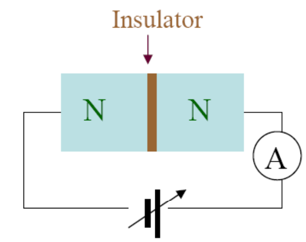
Characteristic of thermal behavior of a system whose excited levels are separated from the ground state by an energy 2Δ

Discontinuity at T_C

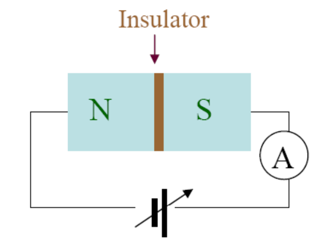


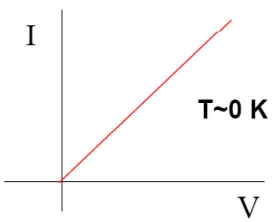
From Kittel-Phillips

Experimental evidence for an energy gap – tunneling



Thin insulator → electrons tunnel

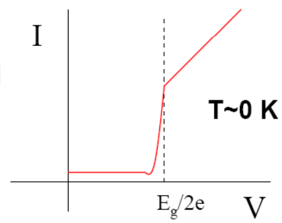




Ohm's law obeyed

As T increases towards T_c, threshold voltage decreases

⇒ *energy gap decreases with increasing T*



No current flows below eV=Δ

Theory

<u>*Phenomenological:*</u>

- F & H. London (1935)
- Ginzburg& Landau (1950)

<u>*Quantum:*</u>

- Fröhlich (1950)
- Bardeen, Cooper & Schrieffer, BCS (1957)

London model

- Using two fluid model of Gorter and Casimir: Assume only a fraction of electrons $n_s(T)/n$ participate in supercurrent
- $n_s(T)$ is the 'density of superconducting electrons: $n_s \sim n$ at $T<<Tc$, $n_s \to 0$ at $T \to T_c$
- $n - n_s$ electrons exhibit normal dissipation
- Current and supercurrent flow in parallel ⇒ superconducting electrons carry all current, normal current is inert and can be ignored

London equations

In an electric field $\mathbf{E}$, S/C electrons will accelerate without dissipation, so we can relate the mean velocity v_s to the current density $\mathbf{j}$:

$$m\frac{d\mathbf{v_s}}{dt} = -e\mathbf{E} \quad \text{using} \quad \mathbf{j} = -e\mathbf{v_s}n_s \quad \text{get} \quad \boxed{\frac{d}{dt}\mathbf{j} = \frac{n_s e^2}{m}\mathbf{E}}$$

1st London equation

In a steady state, $\mathbf{j} = \text{const} \Rightarrow \mathbf{E} = 0$ Electric field inside a S/C vanishes

Maxwell's equation: $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \quad \Rightarrow \quad \frac{\partial\mathbf{B}}{\partial t} = 0 \Rightarrow \mathbf{B} = \text{const}$

These equations describe the magnetic fields and current densities within a perfect conductor, but they are incompatible with the Meissner effect.

From the above, have

$$\frac{\partial\mathbf{B}}{\partial t} = -\frac{m}{n_s e^2}\nabla\times\frac{\partial\mathbf{j}}{\partial t}$$

London assumed that

$$\boxed{\mathbf{B} = -\frac{m}{n_s e^2}\nabla\times\mathbf{j}}$$

(2nd London equation)

i.e. to successfully predict the Meissner effect the constant of integration must be chosen to be zero

Combining equation $\mathbf{B} = -\frac{m}{n_s e^2}\nabla\times\mathbf{j}$ and $\nabla\times\mathbf{B} = \mu_0\mathbf{J}$

and using $\nabla\times\nabla\times\mathbf{B} = \nabla(\nabla\cdot\mathbf{B}) - (\nabla\cdot\nabla)\mathbf{B} = -\nabla^2\mathbf{B}$

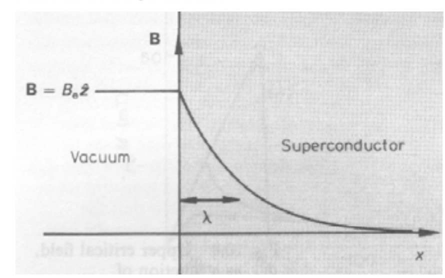
get $\nabla^2\mathbf{B} = \frac{\mu_0 n_s e^2}{m}\mathbf{B}$ and $\nabla^2\mathbf{j} = \frac{\mu_0 n_s e^2}{m}\mathbf{j}$

Solution (one-dimensional case): $B(x) = B_o\, e^{-x/\lambda}$

$$\lambda = \left(\frac{m}{\mu_0 n_s e^2}\right)^{1/2}$$

- the London penetration depth

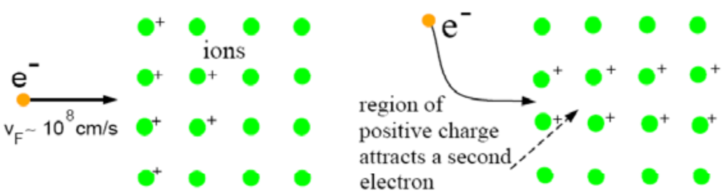
i.e. the Meissner effect is predicted



Solution for $\mathbf{j}$ gives a *surface current* – exponentially decaying into a S/C

BCS theory

- Fröhlich (1950): e-e attraction via phonons (…. Isotope effect)
- Cooper (1956): electrons just above the Fermi surface form bound pairs
- Most stable when center of mass is at rest and total spin = 0,
  So, $+\boldsymbol{k}\uparrow$ and $-\boldsymbol{k}\downarrow$
- Attractive interaction is provided by lattice vibrations – phonons
- First electron deforms the lattice and second electron is then attracted by the deformation (i.e. the changed positive charge distribution)



- Time-scales: electron motion ~ 10^{-16} s; lattice deformed for ~10^{-13} s
  In this time, first electron has traveled $\sim v_F \tau \sim 10^6 ms^{-1} \times 10^{-13} s \sim 1000$ Å

Lattice deformation attracts 2nd electron without it feeling the Coulomb repulsion of the 1st

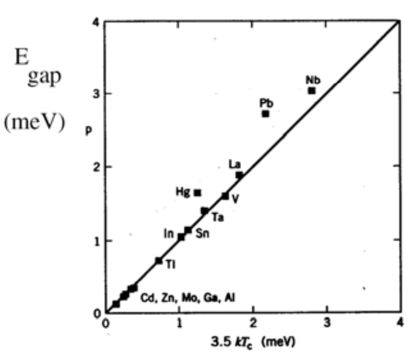
- Cooper calculation: solve Schrödinger eq.for 2 interacting electrons in the presence of a Fermi sphere of non-interacting electrons. Only effect of *N-2* electrons - restrict $\boldsymbol{k}$ values of e-e pair to be > $\boldsymbol{k}_F$, i.e. outside the Fermi sphere
- Cooper pair – boson.
- A single, coherent wave function extending over entire system. Can't change momentum of a pair without changing all pairs
- Bardeen, Cooper and Schrieffer (BCS) → extend Cooper's theory, construct a ground state where all electrons form bound pairs
- Each electron now has 2 roles:
  - Provide restriction on allowed wavevectors via Pauli principle
  - Participate in bound pair (called a Cooper pair)
- Electron-phonon interactions: responsible for resistance of metals *and* superconductivity
- Superconductors are generally poor conductors in normal state

Energy gap: a Cooper pair has a lower energy than 2 individual electrons. BCS gives

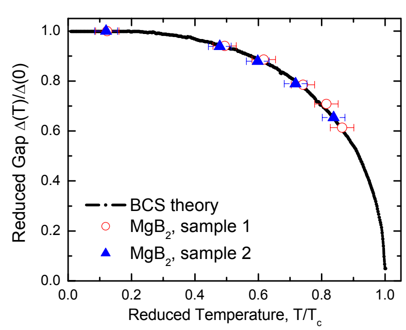
$$E_{gap}(T=0) = 3.53 k_B T_c$$

$$E_{gap} \sim 10^{-4} E_F$$

BCS calculated gap



T-dependence of energy gap



Summary

- When a superconductor is cooled below the critical temperature (T_C), it enters a new state, in which its resistance vanishes.
- Superconductors expel magnetic field completely when in superconducting phase – the Meissner effect
- When a magnetic field higher than a certain value called *the critical field* (B_c) is applied to a superconductor, it reverts to a normal state
- Type I and type II superconductors are distinguished by their behavior in a magnetic field. In a type II S/C there are 2 critical fields. At intermediate fields, the material has both superconducting and normal regions
- Electrodynamics of superconductors is described by phenomenological London equations
- BCS theory – microscopic mechanism for superconductivity through the formation of *e-e* Cooper pairs via electron-phonon interaction.

  A Cooper pair has a lower energy than 2 individual electrons. The energy difference is 2Δ - energy gap.