Math 611, Fall 2013
Show all your work. Due in class on September 10, 2013.

1. Problem 1 (25 points).

   For $f(x) = e^x$, construct a cubic polynomial $q(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3$ for which

   $$q(0) = f(0), \quad q(1) = f(1), \quad q'(0) = f'(0), \quad q'(1) = f'(1).$$

   Numerically compare it to e^x and the Taylor polynomial $T_3(x)$ for $0 \leq x \leq 1$. Plot $q(x)$, $f(x)$ and $T_3(x)$ versus x on the $x - y$ plane for $0 \leq x \leq 1$.

2. Problem 2 (25 points).

   First produce the Taylor polynomial of degrees 1, 2, 3, 4 for $f(x) = e^{x^2}$ with $a = 0$ the point of approximation. Then use the Taylor approximation for e^t and substitute $t = x^2$ to obtain Taylor polynomial approximations for e^{x^2}. Compare the two results.

3. Problem 3 (20 points).

   How large should n be chosen to have

   $$|e^x - T_n(x)| \leq 10^{-5}, \quad -1 \leq x \leq 1$$

   ?

4. Problem 4 (30 points).

   Evaluate

   $$I = \int_0^1 \frac{e^x - 1}{x} dx$$

   within an accuracy of 10^{-6}.