

Optimal Decision Making with Rational Inattention Using Noisy Data

Yuan Zhao
Electrical and Computer
Engineering Department
New Jersey Institute of
Technology
Newark, NJ
yz847@njit.edu

Atah Abdi
New York, NY
atah.abdi@gmail.com

Mark Dean
Department of Economics
Columbia University
New York, NY
mark.dean@columbia.edu

Ali Abdi
Electrical and Computer
Engineering Department
New Jersey Institute of
Technology
Newark, NJ
ali.abdi@njit.edu

Abstract — Rational inattention of decision makers to costly data and information and resources affects their optimal decision making strategies. The theory of rational inattention has found applications in several areas such as economics, finance and psychology. In this paper, we study scenarios where the available data is noisy. The noise may have been generated because of inaccuracies or errors in data collection methods, or the data may have been intentionally distorted to protect private or secret information. Here we introduce a formulation for rationally inattentive decision making when the data is noisy, and derive its optimal decision making strategy. Using a stock trading problem as an example, we demonstrate that as the noise level in the data increases, probability of correct decision decreases. This results in less payoff for the decision maker, when using noisy data. We also show how the noise level and information cost parameters can be estimated using the developed formulation. The results are useful for developing decision making strategies, when using noisy data.

Keywords — Decision making, rational inattention, noisy signals.

I. INTRODUCTION

Understanding how decision makers utilize scarce resources to make choices is of interest in various disciplines, including financial market analysts, economists and psychologists. In this regard and within the rational inattention (RI) framework [1], a decision maker tries to use a limited amount of attention to make an optimal decision, by optimizing the utility net of information cost. To quantify RI, Sims [2] modeled the process of acquiring information as a channel, where input is the underlying state and output is the observed information. He then used mutual information to measure the amount of attention. Matejka and McKay derived a general solution for the discrete choice RI problem [1]. Essentially, they showed that the decision maker's optimal strategy follows a multinomial logit model. From a practical perspective, recently Dean and Neligh [3] conducted experiments and collected experimental data, to study the RI model using human subjects, when they were in the process of making decisions. Several applications of the RI model to finance and financial decision making are discussed in [4].

In prior studies, the presence of noise in the data, as well as in acquiring and processing information, was not considered. To address this important problem, here we introduce a formulation for RI decision making when the data is noisy and derive its optimal decision making strategy. We also apply the new model to some synthetic data, to show how to estimate the model parameters. The rest of this paper is organized as follows. The basic noiseless RI model and its optimal solution are presented in Section II. A financial market exemplary problem is formulated in Section III, and its optimal RI decision making strategy is derived as well. The new RI decision making framework where the available data is noisy is introduced in Section IV, and its corresponding optimal strategy is derived as well. Section V is devoted to the RI model parameter estimation from data. The concluding remarks are provided in Section VI.

II. THE RATIONAL INATTENTION MODEL

In the RI model, a decision maker tries to use a limited amount of attention to optimize the utility net of information cost (to be defined later). To do so, he needs to use the information that he finds to be useful and discards the information that he considers not to be helpful. Rational inattention can be modeled as an information channel that has some input and output data, as proposed by Sims [2]. Using this model, one can measure the amount of attention or information cost using the mutual information between the input and output of the channel.

Following the formulation for rational inattention to discrete choices given in [1], a decision maker (DM) takes these two steps (Fig. 1): The DM selects an information strategy to refine his/her belief about the state, and then, the DM decides based on the belief generated in the first step. The underlying state V , the received signal (or the observed information) S and the action A in Fig. 1 are considered to be discrete random variables. The V and S random variables represent the input and output of the said information channel, respectively. A decision strategy is

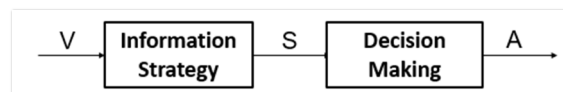


Fig. 1. Rationally inattentive decision making steps.

represented by the probability of S conditioned on V , $P(S|V)$. In the second step in Fig. 1, the DM chooses an action from the set Ξ , i.e., $A \in \Xi$, based on the received signal S .

We define $U(A,V)$ as the payoff of taking the action A based on the underlying state V (also called the utility function). We also define utility net of information cost [1] as follows:

$$\Lambda = E[U(A,V)] - \lambda I(V;S), \quad (1)$$

where E is the mathematical expectation, $\lambda > 0$ is the unit cost of information and $I(V;S)$ is the mutual information [5] between V and S :

$$\begin{aligned} I(V;S) &= H(S) - H(S|V), \\ &= -\sum_s P(s) \log P(s) + \sum_{s,v} P(s,v) \log P(s|v). \end{aligned} \quad (2)$$

In Equation (2), $H(\cdot)$ and $H(\cdot|V)$ are entropy and conditional entropy, respectively, and \log is the base e natural logarithm. Additionally, $P(s)$, $P(s,v)$ and $P(s|v)$ are shorthand notations for the individual, joint and conditional probabilities $P(S=s)$, $P(S=s, V=v)$ and $P(S=s|V=v)$, respectively.

To simplify the notation, here we consider a two state framework where there are two underlying states and two actions, i.e., $V=1,2$ and $A=a,b$. To find the best decision strategy, $P(A|V)$, the DM needs to solve the following constrained optimization problem for Λ in (1):

$$\begin{aligned} \max_{P(A|V)} \Lambda &= \max_{P_{1a}, P_{1b}, P_{2a}, P_{2b}} E[U(A,V)] - \lambda I(V;A), \\ \text{subject to } &P_{1a}, P_{1b}, P_{2a}, P_{2b} \geq 0, \\ &P_{1a} + P_{1b} = 1, P_{2a} + P_{2b} = 1, \end{aligned} \quad (3)$$

where S in (1) is replaced with A , since each action corresponds to one specific signal. Additionally, $P_{1a} = P(a|1)$, $P_{1b} = P(b|1)$, $P_{2a} = P(a|2)$, and $P_{2b} = P(b|2)$ are the probabilities of the actions conditioned on the states. All these conditional probabilities are depicted in a state-action trellis diagram in Fig. 2.

According to [1], the following action probability distributions - given the states - constitute the optimal strategy that maximizes the utility net of information cost Λ in (3):

$$P_{1a} = \frac{P_a e^{\frac{U(a,1)}{\lambda}}}{P_a e^{\frac{U(a,1)}{\lambda}} + P_b e^{\frac{U(b,1)}{\lambda}}}, P_{2b} = \frac{P_b e^{\frac{U(b,2)}{\lambda}}}{P_a e^{\frac{U(a,2)}{\lambda}} + P_b e^{\frac{U(b,2)}{\lambda}}}, \lambda > 0, \quad (4)$$

where $P_a = P(a)$ and $P_b = P(b)$. If $\lambda = 0$, the DM simply selects the action with the highest payoff with probability 1, since there is no information cost.

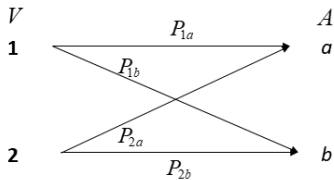


Fig. 2. State-action trellis diagram. The variables V and A represent the state and action, respectively.

TABLE I. Reward Matrix for the Financial Market Problem

Action	State 1: Stock Appreciates	State 2: Stock Depreciates
Bet on Appreciation	r	$-r$
Bet on Depreciation	$-r$	r

III. A FINANCIAL MARKET EXEMPLARY PROBLEM AND ITS OPTIMAL RI DECISION STRATEGY

In this section, we consider a mathematically-tractable financial market problem, to be able to derive a closed-form optimal RI decision strategy. This allows us to later revisit the optimal RI decision making problem when the data, i.e., the underlying states, are noisy – as discussed in the next section. Consider a stock trading problem, where the DM is supposed to choose between betting on appreciation or depreciation. If his/her bet is correct, will get a reward r , and if incorrect, will have a $-r$ loss, as specified in the reward matrix in TABLE I.

In the state-action trellis diagram (Fig. 2) for this stock trading problem, the underlying state $V=1,2$ represents stock appreciation and depreciation, respectively, whereas the action $A=a,b$ refers to betting on appreciation and depreciation, respectively. The DM does not have any information about the underlying states, so, needs to observe and acquire some information. For mathematical tractability, let us assume $P_{2b} = 1$ and $P_{2a} = 0$, which indicate that DM's preferred choice is the action b . This simplifies the constrained optimization problem for Λ in (3) to:

$$\begin{aligned} \max_{P(A|V)} \Lambda &= \max_{P_{1a}, P_{1b}} E[U(A,V)] - \lambda I(V;A) \\ \text{subject to } &P_{1a} \geq 0, P_{1b} \geq 0, P_{1a} + P_{1b} = 1, \end{aligned} \quad (5)$$

where the payoff U is defined as follows, based on the reward matrix given in TABLE I:

$$U(A,V) = \begin{cases} r, & A=a, V=1, \text{ or } A=b, V=2, \\ -r, & A=b, V=1, \text{ or } A=a, V=2. \end{cases} \quad (6)$$

Upon solving the optimization problem in (5) (see Appendix A), we obtain the following optimal strategy for the DM in the considered stock trading problem:

$$P_{1a} = \max \left(0, \frac{e^{\frac{2r}{\lambda}} - g^{-1}}{e^{\frac{2r}{\lambda}} - 1} \right), \lambda > 0, \quad (7)$$

where $g = P(V=1)$ and also $P_{1b} = 1 - P_{1a}$. Probability of correct decision by the DM can be computed according to $P_{correct}(r; \lambda) = gP_{1a} + (1-g)P_{2b}$. For $g = 1/2$, equi-probable states and upon substituting P_{1a} from (7) and $P_{2b} = 1$ coming from our problem formulation, the correct decision probability for the optimal strategy simplifies to $0.5P_{1a} + 0.5$, which gives:

$$P_{correct}(r; \lambda) = \max \left(0.5, \frac{e^{\frac{2r}{\lambda}} - 1.5}{e^{\frac{2r}{\lambda}} - 1} \right), \lambda > 0. \quad (8)$$

Equation (8) is plotted in Fig. 3a for $\lambda = 1, 5, 20$ and 50. We observe that as the reward r increases, the probability of correct

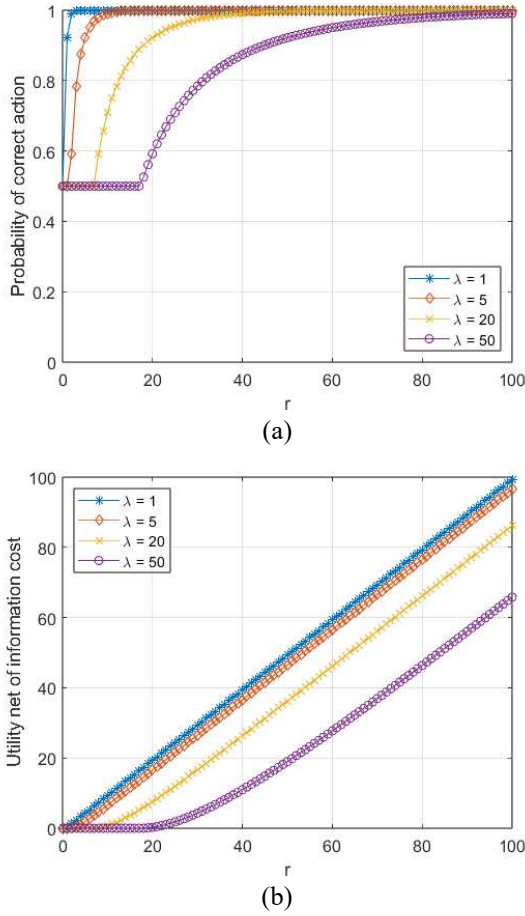


Fig. 3. Optimal strategy of a rationally inattentive decision maker for different costs of information of 1, 5, 20 and 50. (a) Probability of correct decision, (b) Maximized utility net of information cost.

decision increases, because the DM becomes more attentive to the information and makes more accurate and better decisions. However, as the information cost λ increases, the probability of correct decision decreases.

Using (6), the average payoff can be computed according to $E[U(A, V)] = rgP_{1a} + r(1-g)P_{2b} - rgP_{1b} - r(1-g)P_{2a}$. Upon substituting $P_{2b} = 1$, $P_{2a} = 0$, $P_{1b} = 1 - P_{1a}$ and P_{1a} of the optimal strategy derived in (7), together with $g = 1/2$, we obtain $E[U(A, V)] = rP_{1a}$. After its substitution in (5), along with the second expression for $I(V; A)$ in Equation (A3) of Appendix A, the maximized Λ is computed and plotted in Fig. 3b. We note that for any given reward r , Λ decreases as the information cost λ increases. This is because the usage of more expensive information further reduces the average payoff of the DM.

IV. RATIONAL INATTENTION MODEL FOR NOISY DATA AND ITS OPTIMAL STRATEGY

In previous studies, the effect of using noisy data on RI decision making has not been considered, to the best of our knowledge. This is while the data can be noisy for various reasons, such as inaccuracies or errors in data collection

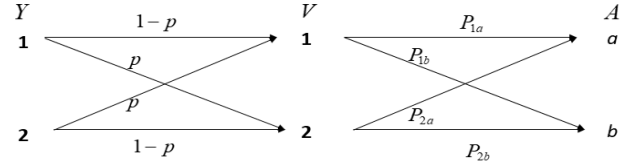


Fig. 4. State-action trellis diagram with noisy data. The variables Y and V represent the noiseless and noisy states, respectively, whereas A is the action.

methods, or the data being intentionally distorted to protect private or secret information [6].

To incorporate the noisy data into the RI decision making framework, we expand the state-action trellis diagram of the basic - noiseless - RI model of Fig. 2 to Fig. 4, where V now represents the noisy state, whereas Y stands for the noiseless state. The relation between the noisy and noiseless states is specified by the noise parameter p , such that $p = P(V = 2 | Y = 1) = P(V = 1 | Y = 2)$. This means that noise can flip a Y state to its opposite value, with probability of p .

To formulate the noisy RI constrained optimization problem, we replace $U(A, V)$ in (3) and subsequently (5) with $U(A, Y)$, which results in:

$$\begin{aligned} \max_{P(A|V)} \Lambda &= \max_{P_{1a}, P_{1b}} E[U(A, Y)] - \lambda I(V; A) \\ \text{subject to } &P_{1a} \geq 0, P_{1b} \geq 0, P_{1a} + P_{1b} = 1. \end{aligned} \quad (9)$$

By solving the above optimization problem for equi-probable states $P(Y = 1) = P(Y = 2) = 1/2$ (see Appendix B), we obtain the following optimal strategy for the DM in the considered stock trading problem using noisy data:

$$P_{1a} = \max \left(0, \frac{e^{\frac{2(1-2p)r}{\lambda}} - 2}{e^{\frac{2(1-2p)r}{\lambda}} - 1} \right), \lambda > 0. \quad (10)$$

The DM's probability of correct decision $P_{correct}(r; \lambda, p)$ equals $P(Y = 1)P(A = a | Y = 1) + P(Y = 2)P(A = b | Y = 2)$. Note that $P(A = a | Y = 1) = \sum_v P(V = v | Y = 1)P(A = a | V = v) = (1-p)P_{1a} + pP_{2a}$. Also $P(A = b | Y = 2) = pP_{1b} + (1-p)P_{2b}$. By substituting $P_{2b} = 1$, $P_{2a} = 0$, $P_{1b} = 1 - P_{1a}$ and P_{1a} of the optimal strategy derived in (10), we finally obtain $P_{correct}(r; \lambda, p) = 0.5(1-2p)P_{1a} + 0.5$, which can be written as:

$$P_{correct}(r; \lambda, p) = \max \left(0.5, \frac{(1-p)e^{\frac{2(1-2p)r}{\lambda}} - 1.5 + 2p}{e^{\frac{2(1-2p)r}{\lambda}} - 1} \right), \lambda > 0. \quad (11)$$

When there is no noise, $p = 0$, the above equation reduces to (8), as expected. Equation (11) is plotted in Fig. 5a for $\lambda = 1$ and $p = 0, 0.1, 0.3$ and 0.5 . We observe that as the noise level increases, the probability of correct decision decreases and the DM makes more incorrect decisions.

Using (6), the average payoff $E[U(A, Y)]$ can be shown to be $rP(A = a, Y = 1) + rP(A = b, Y = 2) - rP(A = a, Y = 2) - rP(A = b, Y = 1)$. The first two probabilities can be computed using the formulas derived immediately after (10). The last two probabilities can be similarly computed using

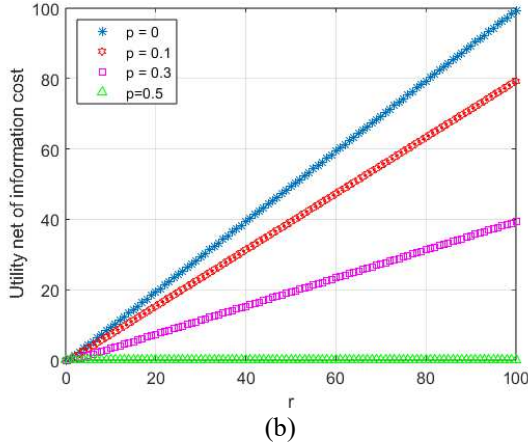
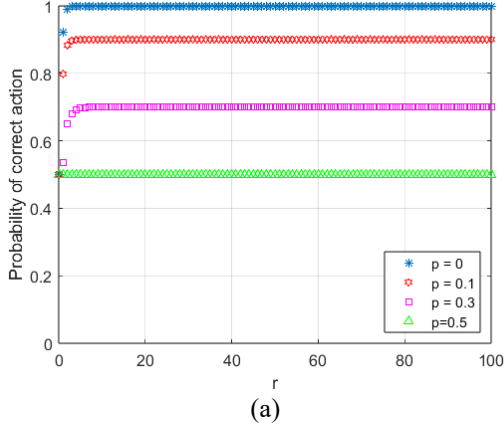


Fig. 5. Optimal strategy of a rationally inattentive decision maker using noisy data with different noise levels of 0, 0.1, 0.3 and 0.5. (a) Probability of correct decision, (b) Maximized utility net of information cost.

$P(A=b | Y=1) = \sum_v P(V=v | Y=1)P(A=b | V=v) = (1-p)P_{1b} + pP_{2b}$ and $P(A=a | Y=2) = pP_{1a} + (1-p)P_{2a}$. The average payoff then becomes $0.5(1-2p)r(P_{1a} + P_{2b} - P_{1b} - P_{2a})$. Substitution of $P_{2b} = 1$, $P_{2a} = 0$, $P_{1b} = 1 - P_{1a}$ and P_{1a} of the optimal strategy derived in (10) finally results in $E[U(A, Y)] = (1-2p)rP_{1a}$. By substituting this result in (9), together with $I(V; A)$ in Equation (A8) of Appendix B, the maximized Λ is computed and plotted in Fig. 5b, for $\lambda = 1$ and $p = 0, 0.1, 0.3$ and 0.5 . We observe its decrease as the noise level increases, i.e., less payoff for the DM when using noisy data.

V. RATIONAL INATTENTION NOISY MODEL PARAMETER ESTIMATION

In this section, we use some data to demonstrate how the noisy RI model parameters, i.e., the information cost λ and the noise level p , can be estimated. Motivated by the experimental data collection method of [3], we consider a hypothetical market study with 16 participants, where each participant is facing this decision problem: According to TABLE I, there are two underlying states 1 and 2, representing stock appreciation and depreciation, respectively, and two actions a and b , referring to betting on appreciation and depreciation, respectively. There is a reward r for correctly taking the action a when the underlying

state is 1, or correctly taking the action b , when the underlying state is 2, and the reward for incorrect actions is $-r$. Note that all these are the same as the payoff U defined in Equation (6).

Each participant faces four decision problems that correspond to four different reward levels, and decides a or b . The participants are aware that the payoff of each action is state dependent, i.e., r or $-r$, as shown in TABLE I. The results of this survey are shown in TABLE II (see the last page of the paper), where the decision of each participant is shown by a “state,action” pair. The pairs $(1,a)$ and $(2,b)$ represent winning decisions, whereas the pair $(1,b)$ indicates a losing decision. There is no pair $(2,a)$ in TABLE II, since in Section III and to reach the simplified optimization problem in (5), we considered that for the state 2, the decision maker never takes the action a . Also note that each row of TABLE II corresponds to a specific trade size, and the associated trade reward r is assumed to be 10% of the trade size, i.e., $r = 0.1, 10, 100$ and 1000 .

To estimate the probability of correct decision using the data of TABLE II for each r , we note that $\hat{P}_{correct}(r) = 0.5\hat{P}(a|1) + 0.5\hat{P}(b|2)$, where $\hat{\cdot}$ stands for estimation. For $r = 0.1$, as an example and using the first row of TABLE II, it can be shown that $\hat{P}(a|1) = 2/8$ and $\hat{P}(b|2) = 8/8$, which result in $\hat{P}_{correct}(0.1) = 0.625$. Using the rest of the rows we obtain $\hat{P}_{correct}(10) = 0.75$, $\hat{P}_{correct}(100) = 0.81$ and $\hat{P}_{correct}(1000) = 0.88$. Using these probability estimates and the least squares FindFit command in the Mathematica software, the parameters λ and p are estimated using (11). The estimates are $\hat{\lambda} = 0.13$ and $\hat{p} = 0.186$.

The above numerical example demonstrates how the information cost λ and the noise level p can be jointly estimated from experimental data.

VI. CONCLUSION

In this paper, we have studied optimal decision making of a rationally inattentive individual who relies on noisy data and resources. In our new formulation and in addition to the information cost that appears in the context of decisions with rational inattention, there is a noise parameter. This parameter specifies how much the available - noisy - data differ from the true - noiseless - underlying states. The introduced formulation is applied to a financial market exemplary problem in stock trading, and the optimal decision strategy is derived. We have shown that as the noise level in the data increases, the payoff for the decision maker decreases because of making more incorrect decisions, due to relying on the noisy data. The results indicate how important it is to have access to reliable and trustworthy data, and also quantify the detrimental effect of noisy or distorted data on decision strategies.

APPENDIX

A. Derivation of the Optimal RI Decision Strategy for the Financial Market Exemplary Problem

We use the Lagrange multiplier method, to solve the constrained optimization problem in (5). The Lagrangian is given by:

$$L = E[U(A, V)] - \lambda I(V; A) + gP_{1a}\xi_a(1) + gP_{1b}\xi_b(1) - g\mu(1)(P_{1a} + P_{1b} - 1), \quad (A1)$$

where $g = P(V=1)$, $\xi_a(1) \geq 0$ and $\xi_b(1) \geq 0$ are Lagrange multipliers on $P_{1a} \geq 0$ and $P_{1b} \geq 0$, respectively, and $\mu(1)$ is the multiplier on $P_{1a} + P_{1b} = 1$. Furthermore, using (6) and (2), we respectively obtain:

$$E[U(A, V)] = r(gP_{1a} + (1-g)P_{2b}) - r(gP_{1b} + (1-g)P_{2a}), \quad (A2)$$

$$= r(gP_{1a} + (1-g) - gP_{1b}),$$

$$I(V; A) = -P_a \log P_a - P_b \log P_b + gP_{1a} \log P_{1a} + gP_{1b} \log P_{1b} \\ + (1-g)P_{2a} \log P_{2a} + (1-g)P_{2b} \log P_{2b}, \quad (A3)$$

$$= -P_a \log P_a - P_b \log P_b + gP_{1a} \log P_{1a} + gP_{1b} \log P_{1b}.$$

The last equations in (A2) and (A3) are obtained by substituting $P_{2b} = 1$ and $P_{2a} = 0$, coming from our problem formulation.

The first-order conditions with respect to P_a and P_b are obtained by taking derivatives of L in (A1), using the last equations of (A2) and (A3), and also by noting that $P_a = gP_{1a}$ and $P_b = gP_{1b} + 1 - g$:

$$\partial L / \partial P_a = r + \lambda(\log P_a - \log P_{1a}) + \xi_a(1) - \mu(1) = 0, \quad (A4)$$

$$\partial L / \partial P_b = -r + \lambda(\log P_b - \log P_{1b}) + \xi_b(1) - \mu(1) = 0.$$

Similarly to [1] and when $P_a > 0$, $P_b > 0$ and $-\infty < r < \infty$, we have $\xi_a(1) = \xi_b(1) = 0$. Therefore, solving for P_a and P_b using (A4) results in:

$$P_a = P_a e^{\frac{r-\mu(1)}{\lambda}}, \quad P_b = P_b e^{\frac{-r-\mu(1)}{\lambda}}. \quad (A5)$$

By solving for $\mu(1)$ using the second equation in (A5), together with $P_b = 1 - P_a$, $P_b = gP_{1b} + 1 - g = 1 - gP_{1a}$ and $P_a = gP_{1a}$, and plugging in the result in the first equation of (A5), we finally obtain the DM's optimal strategy given in (7).

B. Derivation of the Optimal RI Decision Strategy Using Noisy Data for the Financial Market Exemplary Problem

To solve the constrained optimization problem in (9), together with $P(Y=1) = P(Y=2) = 0.5$ that results in $g = 0.5$, we form the following Lagrangian:

$$L = E[U(A, Y)] - \lambda I(V; A) \\ + 0.5P_{1a}\xi_a(1) + 0.5P_{1b}\xi_b(1) - 0.5\mu(1)(P_{1a} + P_{1b} - 1), \quad (A6)$$

where $\xi_a(1) \geq 0$ and $\xi_b(1) \geq 0$ are Lagrange multipliers on $P_{1a} \geq 0$ and $P_{1b} \geq 0$, respectively, and $\mu(1)$ is the multiplier on

$P_{1a} + P_{1b} = 1$. Using (6) and as shown at the end of Section IV, we have:

$$E[U(A, Y)] = 0.5(1-2p)r(P_{1a} + P_{2b} - P_{1b} - P_{2a}), \quad (A7)$$

$$= 0.5(1-2p)r(P_{1a} + 1 - P_{1b}).$$

The last equation in (A7) is obtained by substituting $P_{2b} = 1$ and $P_{2a} = 0$, coming from our problem formulation. Additionally, by replacing g in the last equation of (A3) with 0.5, we obtain:

$$I(V; A) = -P_a \log P_a - P_b \log P_b \\ + 0.5P_{1a} \log P_{1a} + 0.5P_{1b} \log P_{1b}. \quad (A8)$$

Differentiation of L in (A6), along with (A7) and (A8), with respect to P_{1a} and P_{1b} , and upon noticing that $P_a = 0.5P_{1a}$ and $P_b = 0.5P_{1b} + 0.5$, results in:

$$\partial L / \partial P_{1a} = (1-2p)r + \lambda(\log P_a - \log P_{1a}) - \mu(1) = 0, \quad (A9)$$

$$\partial L / \partial P_{1b} = -(1-2p)r + \lambda(\log P_b - \log P_{1b}) - \mu(1) = 0.$$

Note that similarly to Appendix A we have $\xi_a(1) = \xi_b(1) = 0$. Solving for P_{1a} and P_{1b} using (A9) results in:

$$P_a = P_a e^{\frac{(1-2p)r-\mu(1)}{\lambda}}, \quad P_b = P_b e^{\frac{-(1-2p)r-\mu(1)}{\lambda}}. \quad (A10)$$

For $p = 0$, no noise in data, (A10) reduces to (A5). By solving for $\mu(1)$ using the second equation in (A10), together with $P_{1b} = 1 - P_{1a}$, $P_b = 0.5P_{1b} + 0.5 = 1 - 0.5P_{1a}$ and $P_a = 0.5P_{1a}$, and substituting the result in the first equation of (A10), at the end we obtain the DM's optimal strategy provided in (10).

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TABLE II. Decision Data of 16 Participants of a Hypothetical Market Study where Green and Red Entries Denote Correct and Incorrect Decisions, Respectively

Trade Size (in \$1000)	Trade Reward r (in \$1000)	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	#13	#14	#15	#16
1	0.1	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(1,a)	(1,b)	(1,b)	(1,b)	(1,b)	(1,b)	(1,a)	(1,b)
100	10	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(1,b)	(1,a)	(1,b)	(1,b)	(1,a)	(1,a)	(1,b)	(1,a)
1000	100	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(1,a)	(1,a)	(1,a)	(1,a)	(1,b)	(1,b)	(1,a)	(1,b)
10000	1000	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(2,b)	(1,a)	(1,a)	(1,a)	(1,b)	(1,a)	(1,a)	(1,b)	(1,a)