On the Average Crossing Rates in Selection Diversity

Hong Zhang, *Student Member, IEEE,* and Ali Abdi, *Member, IEEE*

Abstract

This letter presents new results on the average crossing rate of the combined signal of a selection diversity in Rayleigh fading channels. Exact closed-form expressions are derived for the inphase zero crossing rate, inphase rate of maxima, phase zero crossing rate, and the instantaneous frequency zero crossing rate of the output of the selection combiner. The utility of the new theoretical formulas, validated by Monte Carlo simulations, are briefly discussed as well.

Index Terms

Zero crossing rate, selection combining, Rayleigh channels, fading channels, instantaneous frequency.

I. Introduction

Diversity combining techniques are often used to combat the effect of fading and selection combining (SC) is one of the simplest diversity methods [1]. It is well known that the average crossing rate represents the fading rate and effectively quantifies the impact of the diversity combiner on channel fluctuations [2]. Another application is speed/Doppler estimation which is important for many applications such as handoff, adaptive modulation and equalization, power control, etc. [1] [7] [11] [12]. However, to the best of our knowledge, only the envelope level crossing rate (ELCR) in diversity systems has been considered so far [2] [3] [13] [14]. In this

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H. Zhang and A. Abdi are with Center for Wireless Communications and Signal Processing Research (CWCSPR), Department of Electrical and Computer Engineering, New Jersey Institute of Technology, Newark, NJ 07012 USA (emails: hz7@njit.edu, ali.abdi@njit.edu)

letter, closed-form expressions are derived which demonstrate the impact of SC on the fluctuation rates of the inphase component, the phase, and the instantaneous frequency in Rayleigh fading channels. Monte Carlo simulation results are provided to verify the theoretical expressions as well. Finally, application of these results to speed estimation is briefly discussed.

II. Signal and Channel Models

Consider a noisy Rayleigh frequency-flat fading channel and a combiner with L branches. The received lowpass complex envelope at the i-th branch is

$$z_i(t) = h_i(t) + n_i(t) \ , \quad i = 1, 2, ..., L, \tag{1}$$

where the independent zero-mean complex Gaussian processes $h_i(t)$ and $n_i(t)$ represent the channel gain (assuming a pilot has been transmitted) and the additive noise, respectively. In the Cartesian coordinates we have

$$z_i(t) = x_i(t) + jy_i(t), \tag{2}$$

where $j^2 = -1$, and $x_i(t)$ and $y_i(t)$ are the inphase and quadrature components, respectively. Using the polar representation we obtain

$$z_i(t) = r_i(t) \exp\{-j\theta_i(t)\}, \tag{3}$$

where $r_i(t)$ and $\theta_i(t)$ are the envelope and phase of $z_i(t)$, defined by

$$r_i(t) = \sqrt{x_i^2(t) + y_i^2(t)}, \ \ \tan\theta_i(t) = y_i(t)/x_i(t). \tag{4}$$

The autocorrelation function of $z_i(t)$ is defined by $C_{z_i(t)} = E[z_i(t)z_i^*(t+\tau)]/2$ with $*$ as the complex conjugate. We also need the n-th spectral moment of $z_i(t)$, $b_{i,n}$, $n = 0, 1, 2, ...,$ given by [4]

$$b_{i,n} = (2\pi)^n \int_{-\infty}^{\infty} f^n S_{z_i}(f) df = \left. \frac{d^n C_{z_i}(\tau)}{j^n d\tau^n} \right|_{\tau=0}, \tag{5}$$

in which $S_{z_i}(f)$ is the power spectral density of $z_i(t)$. As an example, assume the bandlimiting receive filter on each branch, with B_{RX} as the bandwidth, has a non-zero response only over the range $-f_D < f < f_D$, where f_D is the maximum Doppler frequency. Then for Clarke's isotropic scattering model [1] with signal power P_0 and white noise with $N_0/2$ as the two-sided power spectral density on each branch, respectively, we obtain $C_{z_i}(\tau) = P_0 J_0(2\pi f_D \tau)/2 + N_0 B_{RX} \operatorname{sinc}(2B_{RX}\tau)/2$, where $J_0(.)$ is the *zero*-th order Bessel function of the first kind, $B_{RX} =$

f_D , and $\mathrm{sinc}(\lambda) = \sin(\pi\lambda)/(\pi\lambda)$. This results in $b_0 = P_0/2 + N_0 B_{RX}/2$, $b_1 = b_3 = 0$, $b_2 = \pi^2 P_0 f_D^2 + 2\pi^2 N_0 B_{RX}^3/3$, and $b_4 = 3\pi^4 P_0 f_D^4 + 8\pi^4 N_0 B_{RX}^5/5$, where the index i on $b_{i,n}$ is omitted, due to assumption of identically distributed branches.

Let $z(t)$, $x(t)$, $y(t)$, $r(t)$, and $\theta(t)$ denote the complex envelope, inphase part, quadrature part, envelope, and the phase at the output of the SC, respectively. Then, based on the definition of an L-branch SC we have [1]

$$\gamma(t) = \gamma_i(t) \text{ if } r_i(t) = \max(\{r_k(t)\}_{k=1}^{L}), \tag{6}$$

where $\gamma(t) \in \{z(t), x(t), y(t), \theta(t)\}$ represents random process at the SC output and $\gamma_i(t) \in \{z_i(t), x_i(t), y_i(t), \theta_i(t)\}$ corresponds to the i-th branch. In the next section we derive closed-form expressions for the zero crossing rates (ZCRs) of $x(t)$, $\theta(t)$, $d\theta(t)/dt$ (the instantaneous frequency), and the rate of maxima (ROM) of $x(t)$. The results hold for a large class of fading correlations which have even-symmetric power spectra.

III. New Results on the Average Crossing Rates

Given a stationary real random process $\chi(t)$, $N_\chi(\chi_{th}, T)$ represents the number of times that the process crosses the threshold level χ_{th} with positive (or negative) slope, over the time interval T. Also let $M_\chi(T)$ denote the number of maxima of the process $\chi(t)$, over the time interval T. Based on [5], the expected values of $N_\chi(\chi_{th}, T)$ and $M_\chi(T)$ can be calculated according to

$$E[N_\chi(\chi_{th}, T)] = T \int_0^\infty \dot{\chi} f_{\chi\dot{\chi}}(\chi_{th}, \dot{\chi}) d\dot{\chi}, \tag{7}$$

$$E[M_\chi(T)] = T \int_0^\infty \ddot{\chi} f_{\dot{\chi}\ddot{\chi}}(0, \ddot{\chi}) d\ddot{\chi}, \tag{8}$$

where $f_{\chi\dot{\chi}}(\chi, \dot{\chi})$ is the joint probability density function (PDF) of $\chi(t)$ and $\dot{\chi}(t)$, $f_{\dot{\chi}\ddot{\chi}}(\dot{\chi}, \ddot{\chi})$ is the joint PDF of $\dot{\chi}(t)$ and $\ddot{\chi}(t)$, and dot denotes differentiation with respect to t. In what follows, we derive closed-form expressions for four crossing rates in SC diversity systems: the inphase zero crossing rate (IZCR) $E[N_x(0, T)]/T$, the inphase rate of maxima (IROM) $E[M_x(T)]/T$, the phase zero crossing rate (PZCR) $E[N_\theta(0, T)]/T$, and the instantaneous frequency zero crossing rate (FZCR) $E[N_{\dot{\theta}}(0, T)]/T$. The branches are independent and identically distributed (iid) and odd-numbered spectral moments are zero due to the even symmetry of the power spectrum in each branch.

A. IZCR

The covariance matrix of the three dimensional Gaussian random vector $\overrightarrow{V}_{IZCR} = [x_i \; y_i \; \dot{x}_i]^{tr}$, with tr as the transpose, at the i-th branch, is given by [4]

$$A_{IZCR} = \begin{bmatrix} b_0 & 0 & 0 \\ 0 & b_0 & 0 \\ 0 & 0 & b_2 \end{bmatrix}, \tag{9}$$

where the subscript i of $b_{i,n}$ is dropped, since we have identically distributed branches. Clearly, the PDF of $\overrightarrow{V}_{IZCR}$ can be written as

$$f_{\vec{V}_{IZCR}}(x_i, y_i, \dot{x}_i) = \frac{\exp\left\{-\frac{1}{2}\left(\frac{x_i^2 + y_i^2}{b_0} + \frac{\dot{x}_i^2}{b_2}\right)\right\}}{(2\pi)^{3/2} b_0 b_2^{1/2}}. \tag{10}$$

Another random vector of interest is $\overrightarrow{W}_{IZCR} = [r_i \; x_i \; \dot{x}_i]^{tr}$, whose PDF can be determined from the PDF of $\overrightarrow{V}_{IZCR}$ in (10) as

$$f_{\vec{W}_{IZCR}}(r_i, x_i, \dot{x}_i) = \sum \frac{f_{\vec{V}_{IZCR}}(x_i, y_i, \dot{x}_i)}{|J_{IZCR}(x_i, y_i, \dot{x}_i)|}, \tag{11}$$

where $J_{IZCR}(x_i, y_i, \dot{x}_i) = -y_i/\sqrt{x_i^2 + y_i^2}$ is the Jacobian [5] of the transformation $\overrightarrow{V}_{IZCR} \rightarrow \overrightarrow{W}_{IZCR}$. Based on (4), we then obtain the PDF of $\overrightarrow{W}_{IZCR}$

$$f_{\vec{W}_{IZCR}}(r_i, x_i, \dot{x}_i) = \frac{r_i \exp\left\{-\frac{1}{2}\left(\frac{r_i^2}{b_0} + \frac{\dot{x}_i^2}{b_2}\right)\right\}}{\sqrt{2\pi b_2}\pi b_0 \sqrt{r_i^2 - x_i^2}}. \tag{12}$$

According to (34) in the Appendix, the joint PDF of $x(t)$ and $\dot{x}(t)$ at the output of the L-branch SC can be shown to be

$$f_{x\dot{x}}(x, \dot{x}) = \frac{L}{\sqrt{2\pi b_2}\pi b_0} \exp\left\{-\frac{\dot{x}^2}{2b_2}\right\} \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \int_{|x|}^{\infty} \frac{r_1 \exp\left\{-\frac{(k+1)r_1^2}{2b_0}\right\}}{\sqrt{r_1^2 - x^2}} dr_1, \tag{13}$$

where $\binom{m}{n} = m!/[(m-n)!n!]$. Equation (13) can be simplified to

$$f_{x\dot{x}}(x, \dot{x}) = \frac{1}{\sqrt{2\pi b_2}} \exp\left(-\frac{\dot{x}^2}{2b_2}\right) \frac{L}{\sqrt{2\pi b_0}} \sum_{k=0}^{L-1} \binom{L-1}{k} \frac{(-1)^k \exp\left(-\frac{k+1}{2b_0}x^2\right)}{\sqrt{k+1}}. \tag{14}$$

It is interesting to note that the inphase component of the SC complex envelope and its derivative at any time instant t are independent. Also, the distribution of $\dot{x}(t)$ does not depend on L , and still is Gaussian. By substituting (14) into (7) with $x_{th} = 0$, we obtain the average zero crossing rate of the SC inphase component as

$$\frac{E[N_x(0,T)]}{T} = \frac{L}{2\pi}\sqrt{\frac{b_2}{b_0}}\sum_{k=0}^{L-1}\binom{L-1}{k}\frac{(-1)^k}{\sqrt{k+1}}. \tag{15}$$

In the absence of noise, with isotropic scattering and $L = 1$, (15) reduces to the well-known inphase zero crossing rate $\sqrt{2}f_D/2$ [1].

B. IROM

Here we define two random vectors $\overrightarrow{V}_{IROM} = [x_i\ \ddot{x}_i\ y_i\ \dot{x}_i]^{tr}$ and $\overrightarrow{W}_{IROM} = [r_i\ \dot{x}_i\ \ddot{x}_i\ \theta_i]^{tr}$. The covariance matrix of the Gaussian vector $\overrightarrow{V}_{IROM}$ is [4]

$$A_{IROM} = \begin{bmatrix} b_0 & -b_2 & 0 & 0 \\ -b_2 & b_4 & 0 & 0 \\ 0 & 0 & b_0 & 0 \\ 0 & 0 & 0 & b_2 \end{bmatrix}. \tag{16}$$

Then the PDF of $\overrightarrow{V}_{IROM}$ can be written as

$$f_{\vec{V}_{IROM}}(x_i, \ddot{x}_i, y_i, \dot{x}_i) = \frac{\exp\left(-\frac{1}{2}\vec{V}_{IROM}^{tr}A_{IROM}^{-1}\vec{V}_{IROM}\right)}{4\pi^2\sqrt{\det(A_{IROM})}}, \tag{17}$$

where $\det(.)$ is the determinant. The Jacobian of the transformation $\overrightarrow{V}_{IROM} \to \overrightarrow{W}_{IROM}$ is $J_{IROM}(x_i, \ddot{x}_i, y_i, \dot{x}_i) = \sqrt{x_i^2 + y_i^2} = r_i$. So, we obtain the PDF of $\overrightarrow{W}_{IROM}$ as

$$\begin{aligned} f_{\vec{W}_{IROM}}(r_i, \dot{x}_i, \ddot{x}_i, \theta_i) =& \frac{r_i}{4\pi^2\sqrt{\det(A_{IROM})}} \exp\left\{\frac{b_2^3 r_i^2 \sin^2\theta_i - b_0^2 b_2 \ddot{x}_i^2 - 2b_0 b_2^2 r_i \ddot{x}_i \cos\theta_i}{2\det(A_{IROM})}\right\} \\ &\times \exp\left\{-\frac{(b_0 b_2 b_4 r_i^2 - b_0 b_2^2 \dot{x}_i^2 + b_0^2 b_4 \dot{x}_i^2)}{2\det(A_{IROM})}\right\}. \end{aligned} \tag{18}$$

Based on (34) in the Appendix, the joint PDF of $\dot{x}(t)$ and $\ddot{x}(t)$ at the output of the L-branch SC can be expressed as

$$\begin{aligned} f_{\dot{x}\ddot{x}}(\dot{x}, \ddot{x}) =& L\sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k \int_0^\infty \int_0^{2\pi} f_{\vec{W}_{IROM}}(r_1, \dot{x}, \ddot{x}, \theta_1) \exp\left(-\frac{k r_1^2}{2b_0}\right) d\theta_1 dr_1 \\ =& \frac{1}{\sqrt{2\pi b_2}} \exp\left(-\frac{\dot{x}^2}{2b_2}\right) \frac{L}{\pi\sqrt{2\pi b_0 B}} \exp\left(-\frac{b_0}{2B}\ddot{x}^2\right) \sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k \\ &\times \int_0^{2\pi}\int_0^\infty \exp\left\{-\frac{b_2 \ddot{x}\cos\theta_1}{B} r_1 - \left(\frac{k+1}{2b_0} + \frac{b_2^2\cos^2\theta_1}{2b_0 B}\right) r_1^2\right\} r_1 dr_1 d\theta_1, \end{aligned} \tag{19}$$

where $B = b_0 b_4 - b_2^2$. Note that at any given time t, random variables $\dot{x}$ and $\ddot{x}$ are independent, and $\dot{x}$ is Gaussian, as observed in (14), with zero mean and variance b_2. After substituting (19) into (8) and some lengthy calculations, finally we obtain the average rate of maxima of the SC inphase component

$$\frac{E[M_x(T)]}{T} = \frac{L}{2\pi} \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \left\{ \frac{B}{\sqrt{(k+1)^2 b_0 b_2 B + (k+1) b_0 b_2^3}} + \frac{b_2^{1/2}}{b_0^{1/2}(k+1)^{3/2}} \right. \left. - \frac{4B^{3/2}}{3\pi b_0^{1/2} b_2^{5/2}} \int_0^{\frac{\pi}{2}} \frac{{}_2F_1\left(\frac{3}{2}, 2; \frac{5}{2}; -\frac{(k+1)B}{b_2^2 \cos^2\alpha}\right)}{\cos^2\alpha} d\alpha \right\}, \quad (20)$$

where ${}_2F_1(.,.;.;.)$ is the hypergeometric function [6]. For $L = 1$, with noise-free isotropic scattering, (20) reduces to $\sqrt{3} f_D/2$, the IROM derived in [7].

C. PZCR

The joint PDF of the vector $\overrightarrow{W}_{PZCR} = [r_i \ \theta_i \ \dot{\theta}_i]^{tr}$ is given in [8]

$$f_{\vec{W}_{PZCR}}(r_i, \theta_i, \dot{\theta}_i) = \frac{r_i^2 \exp\left\{ -\frac{1}{2b_0}\left(r_i^2 + \frac{b_0}{b_2} r_i^2 \dot{\theta}_i^2 \right) \right\}}{(2\pi)^{3/2} (b_0^2 b_2)^{1/2}}. \quad (21)$$

Substitution of (21) into (34) from the Appendix leads to

$$f_{\theta\dot{\theta}}(\theta, \dot{\theta}) = \frac{L}{4\pi} \sqrt{\frac{b_0}{b_2}} \sum_{k=0}^{L-1} \binom{L-1}{k} \frac{(-1)^k}{\left(k + 1 + \frac{b_0}{b_2}\dot{\theta}^2\right)^{3/2}}. \quad (22)$$

Interestingly, θ and $\dot{\theta}$ are independent, θ is uniformly distributed over $(-\pi, \pi]$, and the PDF of $\dot{\theta}$ is consistent with the result given in [9]. By substituting (22) into (7),[1] we obtain the average zero crossing rate of the SC phase

$$\frac{E[N_\theta(0,T)]}{T} = \frac{L}{4\pi} \sqrt{\frac{b_2}{b_0}} \sum_{k=0}^{L-1} \binom{L-1}{k} \frac{(-1)^k}{\sqrt{k+1}}.^{2} \quad (23)$$

Note that for $L = 1$ and the no-noise isotropic scattering scenario, (23) simplifies to $\sqrt{2} f_D/4$, which is consistent with the result that one can derive from [8].

[1]Since (22) does not depend on θ, (23) is valid for any $\dot{\theta}_{th}$. To simplify the notation, $\dot{\theta}_{th} = 0$ is chosen in (23).

[2]For $\theta = 0$ and π, we have $y = r\sin\theta = 0$. So, $E[N_y(0,T)] = E[N_\theta(0,T)] + E[N_\theta(\pi,T)]$, as we always have $r > 0$. On the other hand, since (22) does not depend on θ, we get $E[N_\theta(0,T)] = E[N_\theta(\pi,T)]$. Therefore $E[N_\theta(0,T)] = E[N_y(0,T)]/2$, which agrees with the fact that (23) is half of (15).

D. FZCR

We define the random vector $\overrightarrow{W}_{FZCR} = [r_i \; \theta_i \; \dot{\theta}_i \; \ddot{\theta}_i]^{tr}$, whose PDF is given by [8]

$$f_{\vec{W}_{FZCR}}(r_i, \theta_i, \dot{\theta}_i, \ddot{\theta}_i) = \frac{r_i^3}{(2\pi)^2 (b_0 b_2 B + 4b_0^2 b_2^2 \dot{\theta}_i^2)^{\frac{1}{2}}} \exp\left\{ -\frac{r_i^2}{2b_0} \left(1 + \frac{b_0}{b_2} \dot{\theta}_i^2 + \frac{b_0^2 \ddot{\theta}_i^2}{B + 4b_0 b_2 \dot{\theta}_i^2} \right) \right\}. \tag{24}$$

As before, by substituting (24) into (34) in the Appendix and some simplifications, we obtain the joint PDF of $\dot{\theta}(t)$ and $\ddot{\theta}(t)$ at the output of the L-branch SC

$$f_{\dot{\theta}\ddot{\theta}}(\dot{\theta}, \ddot{\theta}) = \frac{L}{4\pi \left(b_0 b_2 B + 4 b_0^2 b_2^2 \dot{\theta}^2 \right)^{\frac{1}{2}}} \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \times \left\{ \frac{1}{2b_0} \left(k + 1 + \frac{b_0}{b_2} \dot{\theta}^2 + \frac{b_0^2 \ddot{\theta}^2}{B + 4b_0 b_2 \dot{\theta}^2} \right) \right\}^{-2}. \tag{25}$$

[3]

After substituting (25) into (7) with $\dot{\theta}_{th} = 0$, we obtain the average zero crossing rate of the SC instantaneous frequency

$$\frac{E[N_{\dot{\theta}}(0, T)]}{T} = \frac{1}{2\pi} \sqrt{\frac{b_4}{b_2} - \frac{b_2}{b_0}}, \tag{26}$$

which interestingly, does not depend on L. For the noise-free and isotropic scattering case, (26) reduces to $f_D/2$, which for a single branch, can be derived from [8].

IV. Simulation Results and Conclusion

To verify the four new crossing rate expressions, Monte Carlo simulations using the spectral method [10] have been conducted. In each simulation, we have generated 100 independent realizations of L iid zero-mean complex Gaussian processes, with 10000 complex samples per realization, over $T = 1$ second. The simulated autocorrelation function at each branch is $P_0 J_0(2\pi f_D \tau)/2$ with $P_0 = 1$. As illustrated in Fig. 1, there is a perfect agreement between the theoretical and simulation results. Note that the crossing rates are normalized by f_D in Fig. 1.

When there is no noise, it is easy to show that all these closed-form expressions are linear functions of f_D, so that they can be used for speed estimation [12]. One also observes that some crossing rates of the combined signal do not necessarily decrease,[4] as the diversity order

[3]By integrating (25) with respect to $\ddot{\theta}$, the PDF of $\dot{\theta}$ can be obtained, which agrees with the expression given in [9].

[4]The ELCR goes to zero as $L \to \infty$ [3].

increases, as shown by IROM and FZCR in Fig. 1. Furthermore, for a given Doppler, IROM, IZCR, and PZCR can be utilized to measure the fluctuation rate of the combined signal versus the number of branches, whereas FZCR can not.

APPENDIX

DERIVATION OF THE JOINT PDF OF THE COMBINER OUTPUT AND ITS DERIVATIVE

According to (6), the SC diversity system produces the signal $\gamma(t)$ and its derivative $\dot{\gamma}(t)$ such that

$$\gamma(t) = \gamma_i(t) \text{ and } \dot{\gamma}(t) = \dot{\gamma}_i(t) \text{ if } r_i(t) = \max(\{r_k(t)\}_{k=1}^{L}), \tag{27}$$

which means the output at time instant t is coming from the branch with the largest instantaneous envelope. Of course i may change as t changes.

We define the event $e_i = \{\Omega : r_i(t) = \max(\{r_k(t)\}_{k=1}^{L})\}$, $i = 1, 2, ..., L$, with the probability $P(e_i)$, where Ω represents a point of the sample space. Based on the total probability theorem [5], the joint distribution function of $\gamma(t)$ and $\dot{\gamma}(t)$ can be written as

$$F(\gamma, \dot{\gamma}) = \sum_{i=1}^{L} F(\gamma, \dot{\gamma}|e_i)P(e_i), \tag{28}$$

where $F(\gamma, \dot{\gamma}|e_i) = P(\gamma_i \leq \gamma,\, \dot{\gamma}_i \leq \dot{\gamma}|e_i)$. This results in

$$F(\gamma, \dot{\gamma}) = \sum_{i=1}^{L} P(\gamma_i \leq \gamma, \dot{\gamma}_i \leq \dot{\gamma}, e_i). \tag{29}$$

With iid branches, (29) simplifies to

$$\begin{aligned} F(\gamma, \dot{\gamma}) &= LP(\gamma_1 \leq \gamma, \dot{\gamma}_1 \leq \dot{\gamma}, e_1) \\ &= LP(\gamma_1 \leq \gamma, \dot{\gamma}_1 \leq \dot{\gamma}, r_2 < r_1, r_3 < r_1, ..., r_L < r_1) \\ &= L\int_0^{\infty} P(\gamma_1 \leq \gamma, \dot{\gamma}_1 \leq \dot{\gamma}, r_2 < r_1, r_3 < r_1, ..., r_L < r_1|r_1) f_{r_1}(r_1) dr_1 \\ &= L\int_0^{\infty} P(\gamma_1 \leq \gamma, \dot{\gamma}_1 \leq \dot{\gamma}|r_1) P^{L-1}(r_2 < r_1|r_1) f_{r_1}(r_1) dr_1. \end{aligned} \tag{30}$$

Since the envelope of each branch is Rayleigh distributed, we obtain

$$P(r_2 < r_1|r_1) = \int_0^{r_1} \frac{r_2}{b_0} \exp\left(-\frac{r_2^2}{2b_0}\right) dr_2 = 1 - \exp\left(-\frac{r_1^2}{2b_0}\right), \tag{31}$$

which after substitution in (30) and using the binomial expansion, simplifies (30) to

$$F(\gamma, \dot{\gamma}) = L \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \int_0^\infty \exp\left(-\frac{k}{2b_0} r_1^2\right) P(\gamma_1 \le \gamma, \dot{\gamma}_1 \le \dot{\gamma} | r_1) f_{r_1}(r_1) dr_1. \quad (32)$$

By taking the derivative of (32) with respect to γ and $\dot{\gamma}$ we obtain the joint PDF of $\gamma(t)$ and $\dot{\gamma}(t)$

$$f_{\gamma\dot{\gamma}}(\gamma, \dot{\gamma}) = L \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \int_0^\infty \exp\left(-\frac{k}{2b_0} r_1^2\right) f_{\gamma_1\dot{\gamma}_1}(\gamma, \dot{\gamma} | r_1) f_{r_1}(r_1) dr_1, \quad (33)$$

where $f_{\gamma_1\dot{\gamma}_1}(.,.|r_1)$ is the joint PDF of $\gamma(t)$ and $\dot{\gamma}(t)$, conditioned on $r_1(t)$. Equation (33) can be written in the following more compact form

$$f_{\gamma\dot{\gamma}}(\gamma, \dot{\gamma}) = L \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \int_0^\infty f_{r_1\gamma_1\dot{\gamma}_1}(r_1, \gamma, \dot{\gamma}) \exp\left(-\frac{k}{2b_0} r_1^2\right) dr_1, \quad (34)$$

where $f_{r_1\gamma_1\dot{\gamma}_1}(.,.,.)$ is the joint PDF of $r_1(t)$, $\gamma_1(t)$, and $\dot{\gamma}_1(t)$.

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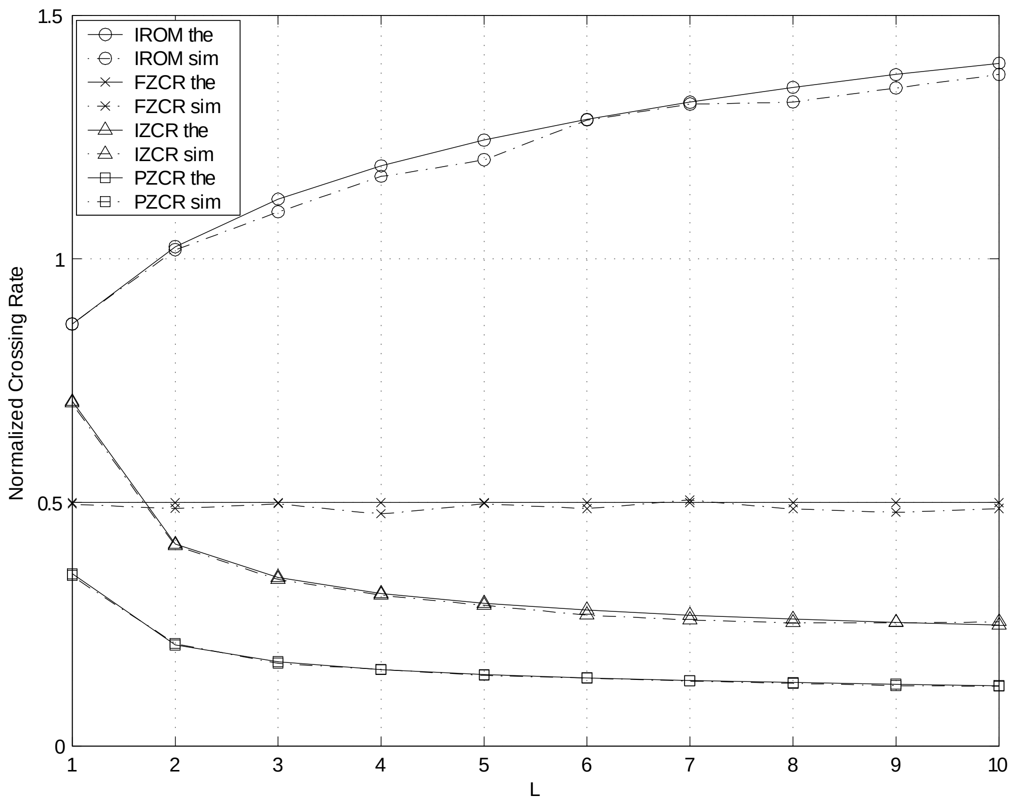

Fig. 1. Normalized average zero crossing rates versus the number of branches L.

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