Statistical Land Clutter Modeling For Airborne Radar

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Abstract

This paper presents a new approach for modeling the radar land clutter, by considering broad regions of ground and all types of roughness. In this way, many kinds of terrains with and without mountains and their composite forms are taken into account. Finally a generalized PDF is introduced for the envelope of the clutter.

1 Introduction

Modern radars require accurate models of clutter to reject it efficiently. The performance of conventional radar signal processing methods usually degrades specially when the clutter envelope has a non-Rayleigh statistics. So it is vital to develop appropriate non-Rayleigh probability density functions (PDF's) for the clutter envelope.

There are two common approaches to find a PDF for clutter envelope. In the first approach, one should obtain enough data by a large number of measurements, and then find a PDF that fits best to the data. In the second one, some presumptions which coincide with the real world conditions should be considered to construct a statistical model. Then the PDF of clutter envelope can be derived by analyzing the model. In this contribution, we consider the second approach. So it is neccssary to introduce a reasonable statistical model.

To introduce a general model, we have considered broad regions of ground, with taking into account all types of roughness and their composite forms. The structure of these kinds of ground roughness can be extracted from various topographical maps. The topographical maps also show the random nature of those parameters like large dimension roughness height, and so on.

In what follows, and according to the above discussion, we show how various PDF's can be obtained for the clutter envelope, by considering different ground structures. Finally, a generalized PDF is introduced by taking a new shadowing effect into account, called the large dimension shadowing effect.

2 Basic Ideas

As mentioned earlier, we want to find a PDF for the clutter envelope at each time between two radar pulses. In this paper, we consider that the radar antenna pattern has a main lobe that looks at the ground through a small grazing angle. Moreover, the side lobes are assumed to be so weak that can be ignored. The radar beam can be divided into a large number of rays. By ray we mean a very narrow beam which illuminates a very small region of the ground, having only one reflection. It should be noticed that according to [1], the reflection coefficient of a rough surface can be defined as:

$$R_f = \rho_d R_{fo} \tag{1}$$

in which ρ_d stands for the effect of surface roughness, and R_{fo} represents the effect of polarization. Considering A as the ray amplitude at the radar antenna output, the following two types of reflected rays can be defined:

a) ***BackScattered (BS) ray :*** This kind of ray is backscattered toward the radar, with the amplitude A_{BS} and the phase θ_{BS}. For the amplitude one can take $A_{BS} = P_{BS} A R_f$, in which P_{BS} is a constant determined by the propagation characteristics of the free space. This amplitude has a random nature, due to the unknown ground structure and related ρ_d coefficient [1]. So in general, A_{BS} must be taken as a random variable which we show its PDF by $f_{A_{BS}}(a)$. As a rough approximation and based on the experimental data in [1], someone may consider ρ_d approximately equal to 0.3, making A_{BS} a deterministic value.

To understand the statistics of θ_{BS}, it should be noticed that the distance between radar and the ground surface can be determined with some tolerance. However, due to the high frequency of radar, this tolerance introduces an uncertainty in phase measurement, making it a random variable with uniform PDF on $[0,2\pi[$, i.e. $f_{\theta_{BS}}(\theta) = 1/2\pi$. It should be noted that A_{BS} and θ_{BS} are independent random variables.

b) ***Multiple Scattered (MS) ray :*** Some rays may travel toward the radar after multiple scattering on the ground. The resultant ray will have the amplitude A_{MS} and the phase θ_{MS}. In this case, and after $\aleph$ times of scattering, we have:

$$A_{MS} = P_{MS} A \prod_{i=1}^{\aleph} R_{f,i} \tag{2}$$

In the above formula, P_{MS} is a constant, determined by the propagation characteristics of the free space, and $R_{f,i}$ is the random reflection coefficient of the ith scatterer. Assuming that $\aleph$ is large enough, $R_{f,i}$'s are independent random variables, and the Lindenberg conditions [2] hold for $R_{f,i}$'s (roughly speaking, there is no dominant $R_{f,i}$), it can be concluded that central limit theorem (CLT) holds, and A_{MS} becomes a lognormal random variable:

$$f_{A_{MS}}(a) = \frac{1}{a\sqrt{2\pi\sigma^2}} \exp\left(- \frac{(\log_e a - \eta)^2}{2\sigma^2} \right) \tag{3}$$

where η and σ are the parameters of the lognormal PDF. The moment of order 2γ for this random variable, which will be used later, is:

$$E\left[A_{MS}{}^{2\gamma}\right] = \exp\left(2\gamma\eta + 2\gamma^2\sigma^2\right) \tag{4}$$

in which E denotes mathematical expectation.

Due to multiple scattering, it is evident that θ_{MS} has a uniform PDF on $[0,2\pi[$, i.e. $f_{\theta_{MS}}(\theta) = 1/2\pi$. It should be noted that A_{MS} and θ_{MS} are independent random variables.

In general, the clutter envelope variable, R, is a combination of the above two kinds of rays. Thus, if N_{BS} and N_{MS} represent the number of BS and MS rays respectively, with $j=\sqrt{-1}$, then R can be calculated according to the following random phasor sum:

$$R = \left| \sum_{p=1}^{N_{BS}} A_{BS,p} \exp(j\theta_{BS,p}) \sum_{q=1}^{N_{MS}} A_{MS,q} \exp(j\theta_{MS,q}) \right| \quad (5)$$

3 The Proposed Ground Structure Based PDF's

Now we introduce a general model for the ground structure. This model includes some degrees of ground roughness according to the Rayleigh criterion, their composite forms, and the terrain slope according to the existence of the projections like hills and mountains. In what follows, various ground structures, along with their associated clutter envelope PDF, $f_R(r)$, are discussed:

1- A terrain without any mountain

(i) smooth surface: In this case, there is no reflection according to the small grazing angle of the airborne main lobe. This means that $N_{BS}=N_{MS}=0$, which gives $f_{R_{1,i}}(r) = \delta(r)$, and $\delta(r)$ is the Dirac delta function.

(ii) very rough surface : For this structure, reception of BS rays is not probable, and there are only a large number of MS rays, which result in $N_{BS}=0$ and $N_{MS} >> 1$. Since $A_{MS,q}$'s are independent and the Lindenberg conditions are satisfied, CLT makes R a Rayleigh random variable:

$$f_{R_{1,ii}}(r) = \frac{r}{\alpha_{1,ii}^2} \exp\left(-\frac{r^2}{2\alpha_{1,ii}^2}\right),$$

$$\alpha_{1,ii}^2 = \frac{1}{2} \sum_{q=1}^{N_{MS}} E[A_{MS,q}^2] \quad (6)$$

where $E[A_{MS,q}^2]$ can be calculated using (4), by taking $\gamma=1$.

(iii) composite rough surface : In this situation, we have $N_{BS}=0$ and $N_{MS} >> 1$. So, similar to the previous case, R becomes a Rayleigh random variable, i.e. $f_{R_{1,iii}}(r) = f_{R_{1,ii}}(r)$.

2- A terrain having mountains

(i) smooth surface : In this case, and according to the slope of the ground, there may be either several independent BS rays or no BS ray. Anyway, there is no MS ray. So we can take $N_{BS}=0$ or $>> 1$, and $N_{MS}=0$, leading us to the fact that $f_{R_{2,i}}(r) = \delta(r)$, or:

$$f_{R_{2,i}}(r) = \frac{r}{\alpha_{2,i}^2} \exp\left(-\frac{r^2}{2\alpha_{2,i}^2}\right),$$

$$\alpha_{2,i}^2 = \frac{1}{2} \sum_{p=1}^{N_{BS}} E[A_{BS,p}^2], \quad (7)$$

where for each p, $E[A_{BS,p}^2]$ must be calculated according to the given $f_{A_{BS,p}}(a)$.

(ii) very rough surface : Based on the same reasoning in the part 1-ii, we have $f_{R_{2,ii}}(r) = f_{R_{1,ii}}(r)$.

(iii) composite rough surface : For this situation, we expect to have a limited number of BS and MS rays. So to obtain the PDF of R, the following random vectors problem must be solved:

There are $N = N_{BS} + N_{MS}$ random vectors $\{A_{BS,p} \exp(j\theta_{BS,p})\}_{p=1,\dots,N_{BS}}$ and $\{A_{MS,q} \exp(j\theta_{MS,q})\}_{q=1,\dots,N_{MS}}$. All of these $2N$ random variables are independent. $\theta_{BS,p}$'s and $\theta_{MS,q}$'s have uniform PDF's on $[0,2\pi[$. $A_{BS,p}$'s are assumed to be arbitary positive random variables; while $A_{MS,q}$'s are lognormal random variables with different parameters, as defined in (3). What is the PDF of R in (5)?

Using the results reported in [3], the following solution for the above problem can be derived:

$$f_{R_{2,iii}}(r) = 2\beta r \exp(-\beta r^2) \sum_{m=0}^{\infty} C_m L_m(\beta r^2), \quad (8)$$

in which β is a positive constant that controls the convergence rate of (8) [4], $L_m(.)$ is the mth order Languerre polynomial, and C_m is given by:

$$C_m = \sum_{k=0}^{m} \frac{(-\beta)^k m!}{(m-k)!(k!)^2} \mu_N^{(2k)} \quad (9)$$

In the above formula, $\mu_N^{(2k)}$ can be computed recursively:

$$\mu_l^{(2k)} = \begin{cases} \nu_l^{(2k)}, & l=1 \quad k=0,1,... \\ \sum_{i=0}^{k} \left(\frac{k!}{i!(k-i)!}\right)^2 \mu_{l-1}^{(2i)} \nu_l^{(2k-2i)}, & l=2,...,N \quad k=0,1,... \end{cases}$$

(10)

where:

$$\nu_l^{(2k)} = \begin{cases} E[A_{BS,l}^{2k}] & l=1,...,N_{BS} \quad k=0,1,... \\ E[A_{MS,l}^{2k}] & l=N_{BS}+1,...,N \quad k=0,1,... \end{cases}$$

(11)

and $E[A_{MS,l}^{2k}]$ can be calculated using (4), by taking $\gamma=k$.

To obtain more accurate PDF's that fit better to the terrain structure, it is necessary to take $A_{BS,p}$'s and $A_{MS,q}$'s as dependent random variables. For this general case, the $\mu_N^{(2k)}$ in (9) must be computed according to [3]:

$$\mu_N^{(2k)} = (-4)^k \frac{(k!)^2}{(2k)!} \left. \frac{\partial^{2k} \Lambda(\rho)}{\partial \rho^{2k}} \right|_{\rho=0} \quad (12)$$

where:

$$\Lambda(\rho) = E\left[\prod_{p=1}^{N_{BS}} J_0(A_{BS,p}\,\rho) \prod_{q=1}^{N_{MS}} J_0(A_{MS,q}\,\rho) \right]$$

(13)

and $J_0(.)$ is the zero order Bessel function.

4 The Generalized PDF

In the previous section, we studied two composite ground structures 1-iii and 2-iii separately. Now we combine them by introducing the probability of observing a mountain according to [5] and [6]. If $M_{\Re_1}$ is the event of observing a mountain in the horizontal direction between 0 and $\Re_1$, then the probability of this event is:

$$P(M_{\Re_1}) = \lambda \Re_1 \exp(-\lambda \Re_1) \times S_L(\theta_i) \quad (14)$$

where λ is the poisson parameter in 1/km, and its value can be found using topographical maps. In addition, $\Re_1 = \Re \cos\theta_g + x$, in which $\Re$ is the distance between radar and the ground surface in the direction of the main lobe axis, θ_g is the grazing angle, and x is a parameter which can be determined using topographical maps.

In formula (14), $S_L(\theta_i)$ represents a new shadowing function, called the large dimension shadowing function:

$$S_L(\theta_i) = \frac{1}{2} - \frac{1}{\pi} \tan^{-1}\left(\frac{\cos\theta_i}{K}\right) \quad . \quad (15)$$

which shouldn't be confused with the function defined in [7]. In (15), θ_i is the incident angle and:

$$K = \sqrt{b'} \,/\, I_L \quad (16)$$

where b' and I_L are the variance and the correlation distance of the large dimension roughness, respectively.

Now by introducing $P(\overline{M}_{\Re_1}) = 1 - P(M_{\Re_1})$, and using the total limit theorem, the generalized PDF will be obtained:

$$f_{R_{ge}}(r) = f_{R_{ge}}(r|M_{\Re_1})P(M_{\Re_1}) + f_{R_{ge}}(r|\overline{M}_{\Re_1})P(\overline{M}_{\Re_1}) \tag{17}$$

where: $f_{R_{ge}}(r|M_{\Re_1}) = f_{R_{2,iii}}(r)$ and

$f_{R_{ge}}(r|\overline{M}_{\Re_1}) = f_{R_{1,iii}}(r)$.

It should be noted that the above result can be extended to the case in which $\Re$ in (2) is small, specially for a terrain without any mountain. In this case, CLT does not hold and consequently A_{MS} can not be a lognormal random variable. So it is necessary to substitute (3) and (4) by suitable formulas.

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