Carrier Frequency Offset Estimation in qHLRT Modulation Classifier with Antenna Arrays

Hong Li*, Ali Abdi*, Yeheskel Bar-Ness*, and Wei Su†

*Center for Wireless Communication and Signal Processing Research
ECE Department, New Jersey Institute of Technology, Newark, NJ 07102 USA
Email: {hl8, ali.abdi, yeheskel.barness}@njit.edu

†U.S. Army RDECOM CERDEC, Fort Monmouth, NJ 07703 USA

***Abstract*— A likelihood ratio test (LRT) -based modulation classifier is sensitive to unknown parameters, such as carrier frequency offset (CFO), symbol rate, etc. To deal with the limited knowledge of CFO, in this paper, a quasi-hybrid likelihood ratio test (qHLRT) -based approach is proposed for linear modulation classification. In the qHLRT algorithm, a non-maximum likelihood (ML) estimator is used to reduce the computational burden of multivariate maximization. Several of blind, non-ML CFO estimators are studied and their performance are compared with both single and multiple receiving antennas systems. It is shown that the nonlinear least-squares (NLS) CFO estimator is the best choice for the qHLRT algorithm, particularly with antenna arrays, which are introduced to combat the effect of channel fading on modulation classification.**

I. Introduction

Modulation classification (MC) is important in both military and commercial communication applications. It is a challenging problem, especially in non-cooperative environments, where no prior knowledge on the incoming signal is available. A likelihood ratio test (LRT) -based MC method relies on the likelihood function (LF) of the received signal and views the MC problem as a multiple-hypothesis testing problem. It has been shown that the LRT -based MC approaches are sensitive to unknown parameters, such as CFO, phase offset, amplitude, etc [1] [2]. In computing the LF, the unknown parameters can be treated either as random variables (RV) or unknown deterministics, which results in three likelihood techniques to solve the multiple-hypothesis testing problem: (a) the average likelihood ratio test (ALRT), where the unknown quantities are treated as RVs with probability density functions (pdf) known and the LF is computed by averaging over them; (b) the generalized likelihood ratio test (GLRT), where the unknown parameters are treated as unknown deterministics, because a pdf cannot be assigned to these unknowns. In this case, a logical procedure is to estimate the unknown parameters assuming certain hypothesis is true and use these estimates in the LRT as if they were correct. If maximum likelihood (ML) estimates are used, the test is called GLRT; (c) the hybrid likelihood ratio test (HLRT), where only the pdfs of several parameters are known, and ML estimates are used for the rest.

Although ALRT provides an optimal solution, it suffers from high computational complexity and even mathematical intractability. Therefore, GLRT and HLRT were investigated as possible alternatives [3] [4]. Nevertheless, the former fails in identifying nested constellations, e.g., 16QAM and 64QAM. With the increasing number of unknown parameters, however, both GLRT and HLRT experience again high computational complexity, as the LF requires ML estimations, and thus, a multivariate maximization. Therefore, a quasi-HLRT (qHLRT) classifier, which relies on simple yet accurate parameter estimators, can be used instead [5].

Since modulation recognition is a non-cooperative communication practice, a blind, nondata-aided open-loop algorithm is needed for estimating the unknown signal parameters, such as the CFO. Several blind, non-ML CFO estimation schemes have been proposed in the literature [6]–[10]. In order to find the proper one for our proposed qHLRT method, those estimation schemes are compared in this paper.

It is well known that antenna array is of great benefit to combat fading. A likelihood-based multi-antenna modulation classifier was shown [5] to be able to achieve significant performance improvement. It is worthwhile to investigate how much a CFO estimator can benefit from antenna arrays, and to determine appropriate CFO estimators for the array -based qHLRT classifier.

II. Signal Model

Consider the baseband representation for a noise-free signal, wherein the complex envelope of the signal for the i-th modulation format is expressed as

$$\begin{aligned} s(t;\mathbf{v}_i) &= \alpha_0 e^{j2\pi f_e t + j\varphi_0} \sum_{k=1}^{N} s_k^{(i)}(t) g(t-(k-1)T), \\ 0 &\leq t \leq NT \end{aligned} \tag{1}$$

where $\mathbf{v}_i$ denotes the vector of unknown quantities for the i-th modulation type, $i = 1, 2, \ldots, N_{\text{mod}}$, and N_{mod} is the number of candidates modulations what we are considering. Channel amplitude and channel phase shift are indicated by α_0 and φ_0, respectively, which are assumed known in this paper[1],

[1]In [5], the modulation classification problem with unknown channel amplitude and phase shift was investigated. Here, we focus on the CFO estimation issue.

and f_e is the unknown CFO. $\{s_k^{(i)}\}_{k=1}^N$ indicates a sequence of N complex transmitted data symbols, taken from the i-th finite-alphabet modulation format. $g(t)$ is a raised cosine pulse shaping filter and T is the symbol duration. Eq. (1) is applicable to M-ary ASK, PSK, and QAM modulations.

For the quasi hybrid likelihood ratio (qHLR) based classifier presented in this paper, f_e is considered as unknown deterministic parameter to be estimated, whose estimate is denoted by $\hat{f}_e$. The data symbols $\{s_k^{(i)}\}_{k=1}^N$ are considered as independent and identically distributed (iid) random variables. Without loss of generality, α_0 is set to 1 and φ_0 is set to 0. Therefore, the vector of unknown quantities of the i-th modulation $\mathbf{v}_i$ bears the form $\mathbf{v}_i = \left[f_e \ \mathbf{s}_i^T\right]^T$, where $\mathbf{s}_i = [s_1^{(i)} \ s_2^{(i)} \ \ldots s_N^{(i)} \ ]^T$ and $(\cdot)^T$ is the transpose operator.

The modulation classification problem hereby is to identify the transmitted constellation based on the following noise-corrupted received complex envelope:

$$r(t) = s(t;\mathbf{v}_i) + w(t), \qquad 0 \le t \le NT \tag{2}$$

where $w(t)$ is the complex additive white Gaussian noise (AWGN) with two-sided power spectral density N_0, and correlation $E[w(t)w^*(t+\tau)] = N_0\delta(\tau)$, wherein $(\cdot)^*$ is the complex conjugate and $\delta(\cdot)$ is Dirac delta.

III. Array -Based Likelihood Ratio Test

The likelihood-based approach for modulation classification requires the computation of the likelihood or log-likelihood function of $r(t)$ over the interval $0 \le t \le NT$. It chooses the i-th hypothesis H_i (the i-th modulation candidate) for which the likelihood function is maxmized, assuming that the a priori probabilities of all hypotheses are equal. It is well known that with complex Gaussian distribution of $w(t)$ and for the i-th hypothesis, the likelihood function of $r(t)$, conditioned on the unknown vector $\mathbf{v}_i$, is given by

$$\Lambda\left[r(t)|\mathbf{v}_i;H_i\right] = \exp\left\{\frac{2}{N_0}\mathrm{Re}\left[\int_0^{NT} r(t)s^*(t;\mathbf{v}_i)dt\right] - \frac{1}{N_0}\int_0^{NT}|s(t;\mathbf{v}_i)|^2 dt\right\}, \tag{3}$$

where Re$[\cdot]$ stands for the real part of complex quantities.

Intuitively, signal classification may perform better under a higher SNR, provided that all the other parameters are fixed. Therefore, when properly combining the signals received by different antenna array elements, the received SNR improves due to the array gain, thereby increasing the classification accuracy. The diversity gain of an array in fading channels is likely to improve the classification performance, as discussed in [5] and for unknown CFO at the end of this paper.

Consider a receiver consisting of L array antenna branches, where the complex envelope of received signal at each branch bears the same form as in Eq. (2). Using vector notation we have

$$\mathbf{r}(t) = \mathbf{s}(t;\mathbf{v}_i) + \mathbf{w}(t), \tag{4}$$

where $\mathbf{r}(t) = [r_1(t), r_2(t), \ldots, r_L(t)]^T$, $\mathbf{w}(t) = [w_1(t), w_2(t), \ldots, w_L(t)]^T$, $\mathbf{v}_i = \left[f_e \ \mathbf{s}_i^T\right]^T$ and $\mathbf{s}(t;\mathbf{v}_i) = s(t;\mathbf{v}_i)\mathbf{1}$, with $\mathbf{1} = [1,\ldots,1]^T$. We assume the same CFO for all the branches. This is a reasonable assumption because all the antenna branches may share the same oscillator. Furthermore, $w_l(t)$'s are independent complex AWGNs, with the same two-sided power spectral density N_0. Then the LF of the array is the product of L LF given in (3)

$$\Lambda\left[\mathbf{r}(t)|\mathbf{v}_i;H_i\right] = \prod_{l=1}^{L}\exp\left\{\frac{2}{N_0}\mathrm{Re}\left[\int_0^{NT} r_l(t)s_l^*(t;\mathbf{v}_{i,l})dt\right] - \frac{1}{N_0}\int_0^{NT}\left|s_l(t;\mathbf{v}_{i,l})\right|^2 dt\right\}. \tag{5}$$

Since the likelihood function contains the unknown parameter f_e, as well as the unknown data symbols $\{s_k^{(i)}\}_{k=1}^N$, the modulation classification becomes a multiple composite hypothesis testing problem. In qHLRT, the unknown parameters are treated as deterministic and replaced by their non-ML yet accurate enough estimates, whereas the unknown data symbols are considered as random variables and are averaged over the conditional pdf of the i-th hypothesis $p(\mathbf{s}_i|H_i)$. Therefore, three steps are involved in calculating the qHLRT likelihood function for the i-th modulation. First, the likelihood function, conditioned on the unknown $\mathbf{v}_i$ is averaged over $p(\mathbf{s}_i|H_i)$

$$\Lambda\left[\mathbf{r}(t)|f_e;H_i\right] = \int \Lambda\left[\mathbf{r}(t)|\mathbf{v}_i;H_i\right]p(\mathbf{s}_i|H_i)d\mathbf{s}_i. \tag{6}$$

Then a blind algorithm is applied to estimate the unknown parameter f_e. Finally, by substituting the estimate $\hat{f}_e$ into (6) we obtain

$$\Lambda\left[\mathbf{r}(t)|H_i\right] = \Lambda\left[\mathbf{r}(t)|\hat{f}_e;H_i\right]. \tag{7}$$

The decision is made according to the following criterion, to choose $\bar{\imath}$ as the modulation type

$$\bar{\imath} = \arg\max_{i=1,2,\cdots,N_{\mathrm{mod}}} \Lambda\left[\mathbf{r}(t)|H_i\right]. \tag{8}$$

In this paper, the unknown parameter f_e will be estimated via two different approaches. One is the cyclostationary approach, which uses oversample data symbols. The other is the nonlinear least-squares approach, which only requires symbol rate sampling. Both approaches are elaborated in the following section, and the estimation performance are compared thereby.

IV. Estimation of Carrier Frequency Offset

Since modulation recognition is a non-cooperative communication practice, only a nondata-aided open-loop algorithm is applicable for estimating the unknown CFO. In qHLRT scheme, we use non-ML algorithms for CFO estimation, to avoid the complexity of ML estimation.

Based on the sampling rate of the data sequence, CFO estimation algorithms can be divided into two categories: over-sampled and symbol rate-sampled. An over-sampled CFO estimator was proposed in [6] and [7], which relies explicitly on the cyclostationarity of the over-sampled data. This is modualtion-independent, i.e., the processing procedure is same no matter what kind of modulation is involved. Sometimes,

however, a processing with symbol-rate sampling is preferable. A class of nonlinear least-squares (NLS) estimators with symbol-rate-sampling are discussed in [8]–[10]. Optimal NLS estimators are also designed such that their asymptotic (large sample) variance is minimized [10]. These optimal NLS estimators are modulation dependent and are more complex to calculate.

A. Cyclic Correlation Approach

For fully digital nondata-aided CFO estimation of a linearly modulated wavefom, an approach was proposed by Gini and Giannakis [6], that exploits the second-order cyclostationarity of the over-sampled received sequence.

After the receiver matched filter, the signal is (over)sampled at a rate P/T, where P is an integer, to obtain the discrete-time data as follows[2]

$$x(n) = e^{j(2\pi/P)f_e nT} \sum_k s_k^{(i)} g(n-kP) + w(n). \quad (9)$$

It is shown that for raised cosine pulse shaping, the unknown CFO can be estimated from $x(n)$ as

$$\hat{f}_e = -\frac{P}{4\pi T L_g} \sum_{\tau=1}^{L_g} \frac{1}{\tau} \arg\left\{\hat{\mathcal{M}}_{2x}(1;\tau)\hat{\mathcal{M}}_{2x}(-1;\tau)\right\} \quad (10)$$

where L_g is the window length of delay τ, $\hat{\mathcal{M}}_{2x}(k;\tau)$ is the estimate of the cyclic correlation $\mathcal{M}_{2x}(k;\tau)$, defined as $\mathcal{M}_{2x}(k;\tau) = (1/P)\sum_{n=0}^{P-1} E\left\{x(n)x^*(n+\tau)\right\}\exp(-j(2\pi/P)kn)$. Note that $\mathcal{M}_{2x}(k;\tau)$ is periodic with respect to k with period P, and $\{2\pi k/P,\ k=-P/2,\cdots,P/2-1\}$ are called cyclic frequencies or cycles. The estimate of $\hat{\mathcal{M}}_{2x}(k;\tau)$ is obtained from $\{x(n)\}_{n=0}^{K-1}$, $K=PN$, according to

$$\hat{\mathcal{M}}_{2x}(k;\tau) = \frac{1}{K}\sum_{n=0}^{K-\tau-1} x(n)x^*(n+\tau)e^{-j(2\pi/P)kn}, \qquad \tau > 0. \quad (11)$$

Estimates of negative lags are obtained from Hermitian symmetry as $\mathcal{M}_{2x}(k;-\tau) = \exp(-j2\pi k\tau/P)\mathcal{M}_{2x}^*(k;\tau)$.

B. Nonlinear Least-Squares Approaches

The NLS estimator was originally proposed by Viterbi and Viterbi (V&V) [8] as a blind carrier phase estimator for fully modulated phase-shift keying (M-PSK) transmission. Based on the V&V algorithm, Efstathiou and Aghvami introduced blind carrier phase and frequency offset estimators for 16-QAM [9] [11]. Wang *et al.* extended this algorithm to general QAM modualtions [10].

At the output of the receiver matched filter, with symbol rate sampling, we obtain the discrete-time data as

$$r_n = s_n^{(i)} T e^{j2\pi f_e nT} + w_n, \quad (12)$$

[2]To distinguish over-sampled discrete-time data from the symbol rate sampled data, we hereby consider x as the over-sampled sequence, whereas r is the symbol-rate sampled sequence.

where $w_n = \int_{(n-1)T}^{nT} w(t)dt$ with variance σ_w^2. r_n can be represented in its polar form as $r_n = \rho_n e^{j\phi_n}$. By applying a nonlinear transformation one obtains the sequence y_n as

$$y_n = F(\rho_n)\, e^{jD\phi_n} \quad (13)$$

where $F(\cdot)$ is a real-valued non-negative nonlinear function and D is an integer which depends on the modulation format. For CFO estimation in QAM modulation, D is set to 4, whereas in M-PSK modulation, we have $D = M$.

1) Monomial Nonlinear Estimators: The conventional V&V-like nonlinearities rely on the monomial transformations $F(\rho_n) = \rho_n^k$, $k = 0,\ldots,4$, which are simpler to compute than the optimal matched nonlinearities presented in the next subsection. Define the class of processes $y_n^{(k)}$, obtained via the monomial transformations

$$y_n^{(k)} = \rho_n^k e^{jD\phi_n}, \quad k = 0,1,\ldots,4, \quad (14)$$

and the zero-mean processes $u_n^{(k)} = y_n^{(k)} - E\{y_n^{(k)}\}$. It turns out that $E\{y_n^{(k)}\}$ is a constant amplitude chirp signal, and hence, $y_n^{(k)} = E\{y_n^{(k)}\} + u_n^{(k)}$ can be interpreted as a constant amplitude harmonic embedded in additive noise. The class of monomial NLS estimators are given by

$$\hat{f}_e^{(k)} = \frac{1}{D}\arg\max_{|f_0|<\frac{1}{2}} \frac{1}{N}\left|\sum_{n=0}^{N-1} y_n^{(k)} e^{-j2\pi f_0 n}\right|. \quad (15)$$

2) Optimal (Matched) Nonlinear Estimators: An optimal or "matched" nonlinear CFO estimator for linear modulations is modulation/constellation-dependent and achieves the smallest asymptotic (large sample) variance, within the family of blind NLS estimators.

The optimal NLS estimators for CFO has the same form as the monomial NLS estimators in (15), except that the function $F(\cdot)$ in (13) is optimally derived as $F_{\min}(\cdot)$, to minimize the asymptotic variances for a specific modulation. For M-PSK, $F_{\min}(\cdot)$ is given by

$$F_{\min}^{(\text{PSK})}(\rho_n) = \frac{I_M\left(\frac{2\rho_n}{\sigma_w^2}\right)}{I_0\left(\frac{2\rho_n}{\sigma_w^2}\right) - I_{2M}\left(\frac{2\rho_n}{\sigma_w^2}\right)}, \quad (16)$$

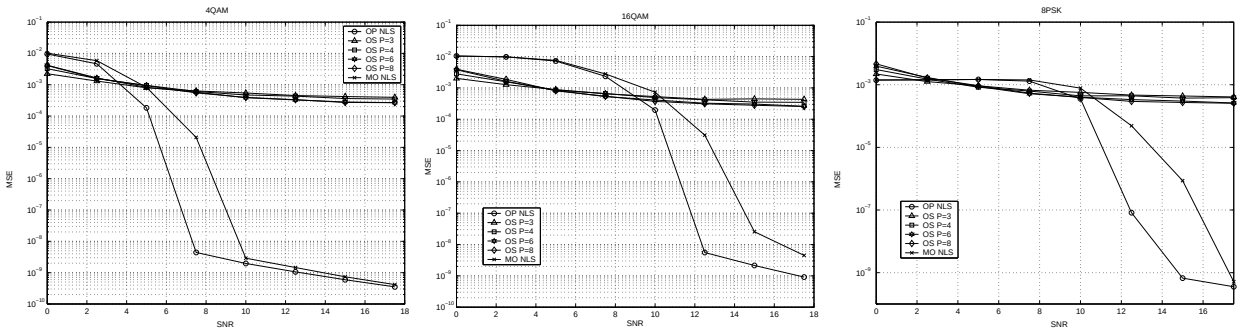
where $I_M(\cdot)$ denotes the Mth-order modified Bessel function of the first kind. For M-QAM, the $F_{\min}(\cdot)$ is a little

$$F_{\min}^{(\text{QAM})}(\rho_n) = \frac{\xi_2(\rho_n)}{\xi_1(\rho_n) - \xi_3(\rho_n)}, \quad (17)$$

where

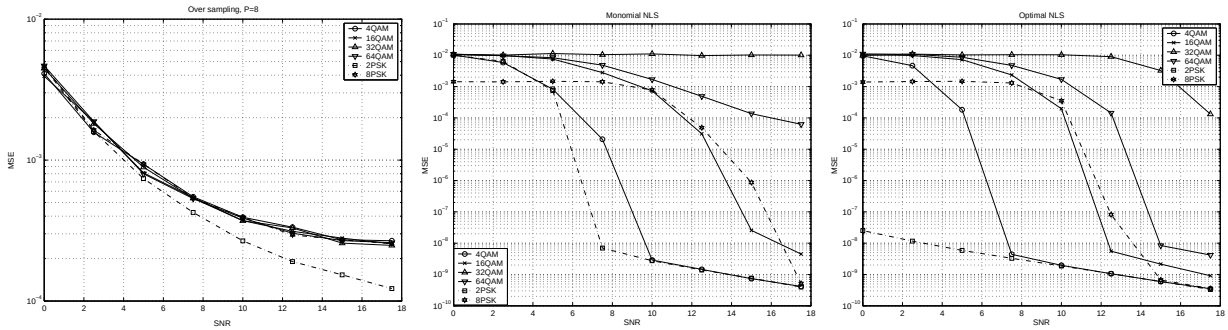
$$\xi_i(\rho_n) = (-1)^{i-1}\frac{8\rho_n}{M\sigma_w^2} e^{-(\rho_n^2/\sigma_w^2)} \sum_{l,k\in\mathcal{A}_M} \cos(4(i-1)\varphi_{l,k}) e^{-(\varrho_{l,k}^2/\sigma_w^2)} I_{4(i-1)}\left(\frac{2\rho_n\varrho_{l,k}}{\sigma_w^2}\right),$$

$\varphi_{l,k} = \psi_{\max\{l,k\},\min\{l,k\}}$, $\varrho_{l,k}$ and $\psi_{l,k}$ are the amplitudes and angles of the normalized QAM constellation points, respectively, and $\mathcal{A}_M$ is the set of constellation points of M-QAM. Note that optimal NLS CFO estimators need perfect



(a) 4QAM (b) 16QAM (c) 8PSK

Fig. 1. CFO estimation performance for different modulations, $f_eT = 0.05$.



(a) over-sampled cyclic correlation estimator (b) monomial NLS estimator (c) optimal NLS estimator

Fig. 2. CFO estimation performance w.r.t. modulation type, $f_eT = 0.05$.

knowledge of the noise power σ_w^2.

C. Estimators Performance Comparisons

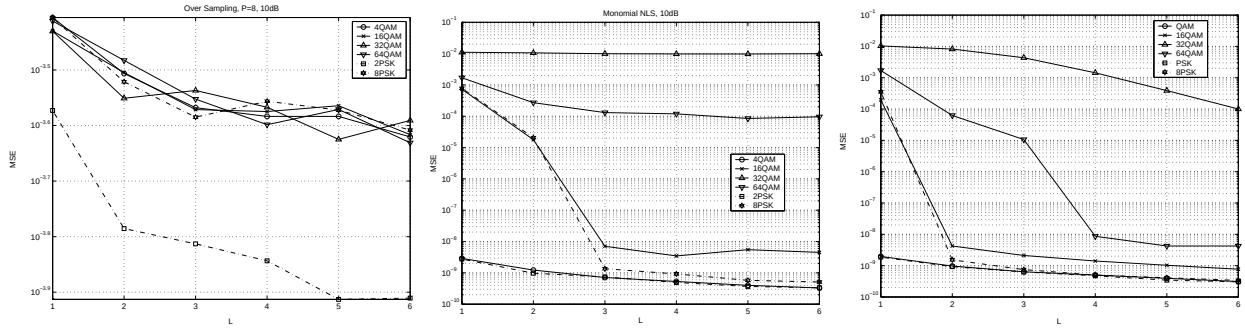
As mentioned in the first section, the likelihood ratio test -based modulation classifier is very sensitive to unknown parameters. Therefore, the performance study of aforementioned CFO estimators is of great importance. In this subsection, we first investigate the accuracy of those estimators in terms of the mean square error (MSE) versus SNR, when the modulation type is fixed. Then the applicability of each CFO estimator to different modulation types is studied. Finally we examine array-based CFO estimators.

In the simulations, a pool of candidate modulations to be classified includes 4QAM, 16QAM, 32QAM, 64QAM, 2PSK, and 8PSK. Raised cosine pulse shaping filter is used with a roll-off factor of 0.35. Normalized constellations are generated to assure fair comparison, i.e., $E[|s_k^{(i)}|^2] = 1$. The received SNR per symbol per branch is defined as $\gamma = S_0T/N_0$, where S_0 is the power of the signal, which due to the normalization is independent of the modulation type. In all the experiments, we set $T = 1$ and $S_0 = 1$, and change SNR by varying N_0. The length of data symbol sequence is $N = 200$, and every simulation result is obtained via performing 1000 Monte Carlo trials.

Three CFO estimators are examined in this paper. The first is the optimal NLS estimator (OP NLS), the second is the 4-th order monomial NLS estimator (MO NLS), i.e., $k = 4$ in (14), and the third is the cyclic correlation estimator with over sampled sequence (OSCC). The over sampling rate P varies from 3 to 8.

1) Estimators with a Single Antenna: As shown in the Fig. 1, we observe that in the low SNR region, OSCC estimators perform better than NLS estimators. However, as SNR increases, the MSEs drop dramatically for both NLS estimators, whereas for the OSCC estimators, the performance enhancement is very small. One reason for the poor performance of cyclic-based estimator might be the small number of symbols, which is 200. As expected, the OP NLS estimation always function better than the conventional MO NLS estimator, but at the cost of computational complexity. Note that the over sampling rate does not affect the CFO estimation performance. Therefore, in a practical application, one may save the processing effort by using a lower sampling rate when OSCC estimator is applied.

The performance of each CFO estimator for different modulation types is depicted in Fig. 2. For the OSCC estimator, the MSE performance does not change a lot, when modulation is changed. As for NLS estimators, the CFO estimation results heavily depend on the modulation type, as well as the modulation order. With those MSE vs. SNR plots, we can determine which CFO estimation approach is suitable for the given scenario. For instance, let the involved modulation



(a) over-sampled cyclic correlation estimator (b) monomial NLS estimator (c) optimal NLS estimator

Fig. 3. CFO estimation performance as a function of number of antenna, $f_eT = 0.05, SNR = 10dB$.

types be 4QAM, 16QAM, 2PSK, 4PSK, and 8PSK, and the operational SNR is higher that 14dB. Then in order to keep the system complexity low, a monomial NLS CFO estimator, rather than a complicated OP NLS one, is accurate enough to provide MSE$< 10^{-5}$. However, if 64QAM is included, to keep MSE below 10^{-5}, the OP NLS has to be used.

2) Estimators with an Antenna Array: To improve the performance of CFO estimators, here we use an antenna array. Fig. 3 shows the results, where the SNR is fixed at 10dB. In general, as expected, MSE of each CFO estimation scheme decreases as the number of elements increases. In most cases, significant improvement appears only when the second and third elements are added. Also, only OP NLS estimator shows consistent performance improvement with respect to the number of antenna, for all types of modulation.

In fact, in AWGN environments, by adding another receiving branch to a single antenna, the system obtains the same performance enhancement as would be obtained via a 3dB SNR increase in a single antenna system. For example, we observe that for OP NLS estimator and 16QAM modulation, the MSE decreases from 2×10^{-4} to 4×10^{-9} by increasing SNR from 10dB to 13dB in a single antenna system, as depicted in Fig. 1(b). Similarly, Fig. 3(c) illustrates the same MSE improvement by adding one more antenna to a 10dB SNR single antenna receiver.

V. Simulation of the Proposed qHLRT and Discussion

In this section, we study the array-based qHLRT classifier using computer simulations. The performance improvement of the proposed classifier will be compared to the classifier with error free estimate of the unknown CFO. The impact of the number of antennas is also addressed. Further, the impact of the time invariant Rayleigh fading channel is examined.

Simulation set-up is the same as the previous section. We define the probability of correct classification as $P_{cc} = N_{\text{mod}}^{-1} \sum_{i=1}^{N_{\text{mod}}} P_c^{(i|i)}$, to evaluate the performance of classifiers. $P_c^{(i|i')}$ is the probability to claim that the i-th modulation is received, where in fact the i'-th modulation has been originally transmitted. To estimate the unknown CFO, we employ the monomial and optimal NLS estimators.

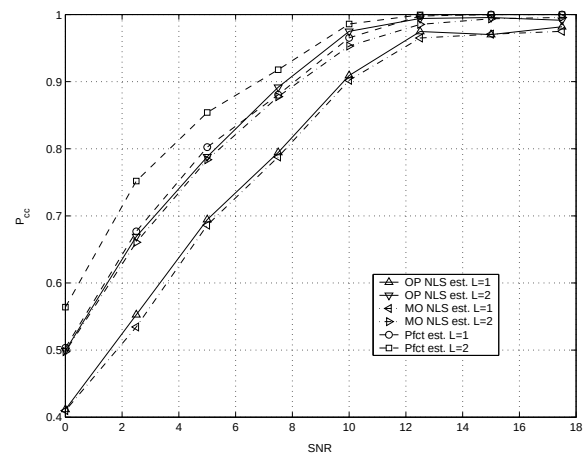


Fig. 4. Performance comparison versus SNR. $f_eT = 0.05$.

A. Performance of the Array-Based qHLRT Classifier in AWGN

Fig. 4 illustrates the system performance of the array-based qHLRT classifier. The solid lines indicate the performance of the classifier with OP NLS estimator, whereas the dash-dot lines correspond to MO NLS estimator. As a benchmark, the performance of the ALRT classifier with perfect CFO estimate (ideal case) is plotted in the same figure with dash lines. Note that by adding only one additional antenna, we obtain a large performance improvement, in comparison with the single antenna qHLRT case.

We observe that the performance with MO NLS estimator is very close to the one with OP NLS estimator, in AWGN channels. This means that one may use less complex MO NLS estimator for CFO, with accurate enough classification results.

B. Performance of the Array-Base qHLRT Classifier in Fading

In a wireless multipath channel, transmitted signals face the fading. In this subsection, the time invariant Rayleigh fading

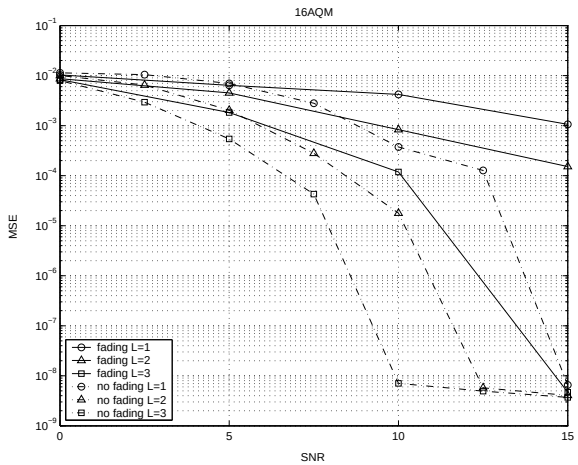


Fig. 5. Array -based monomial NLS estimator performance in Rayleigh fading. $f_eT = 0.05$.

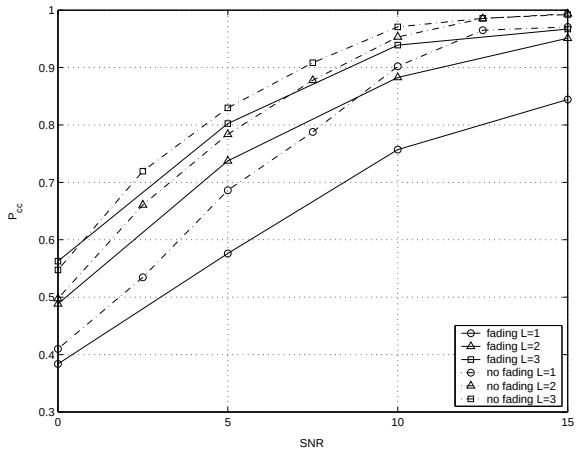


Fig. 6. The proposed classifier performance in Rayleigh fading. $f_eT = 0.05$.

effect on the proposed qHLRT classifier using the MO NLS estimator is investigated.

The solid lines in Fig. 5 show the MSE of the monomial NLS estimator in Rayleigh fading for 16QAM, whereas the dash-dot lines illustrate the MSE without fading (AWGN only). Obviously, the accuracy of the CFO estimator is reduced due to fading. However, when the number of receiver antennas increases, the estimator degrades less. For example, for $L = 2$ in Rayleigh fading, one needs 7dB more to get a MSE at 10^{-4}, when compared to the AWGN case. However, for $L = 3$, the penalty is only 3dB. It means that the NLS method, coupled with an antenna array, offers an effective way to estimate CFO, when fading is present.

Fig. 6 depicts the fading effect on the system's overall performance. The 3-receiver antenna system working under fading channel performs close to the one functioning under AWGN. Therefore, antenna array is confirmed to combat fading in signal classification, even when CFO is estimated by a simple scheme.

Note that in the simulation, the fading channel parameters (amplitudes and phases) are assumed known, since our focus is on the performance of CFO estimators and their impacts on modulation classification. The classifier with estimated fading parameters is one of our ongoing research topics and the results will be presented in [12], in which we have developed, based on the comparison results addressed in the previous section, a new CFO estimation technique that takes advantage of antenna arrays for modulation classification.

VI. Conclusion

In this paper, a quasi-hybrid likelihood ratio test approach is employed to classify signals with unknown carrier frequency offset. A number of blind, non-ML CFO estimators are studied. Their performance is compared with both single and multiple receiving antennas. The symbol-rate-sampling nonlinear least-squares estimator with a monomial nonlinearity shows a good compromise between complexity and performance. The effect of channel fading on the proposed classifier is examined, and antenna array is considered as an effective tool, especially when fading is present.

References

[1] O. Dobre, A. Abdi, Y. Bar-Ness, and W. Su;, "Blind modulation classification: a concept whose time has come," in *Advances in Wired and Wireless Communication, 2005 IEEE/Sarnoff Symposium*, Apr. 2005, pp. 223–228.

[2] H. Li, O. A. Dobre, Y. Bar-Ness, and W. Su, "Quasi-hybrid likelihood modulation classification with nonlinear carrier frequency offsets estimation using antenna arrays," in *Proc. IEEE MILCOM*, 2005, to appear.

[3] D. Boiteau and C. L. Martret, "A generalized maximum likelihood framework for modualtion classification," in *Proc. IEEE ICASSP*, 1998, pp. 2165–2168.

[4] P. Panagiotou, A. Anastasopoulos, and A. Polydoros, "Likelihood ratio test for modulation classification," in *Proc. IEEE MILCOM*, 2000, pp. 670–674.

[5] A. Abdi, O. A. Dobre, R. Choudhry, Y. Bar-Ness, and W. Su, "Modulation classification in fading channels using antenna arrays," in *Proc. IEEE MILCOM*, 2004, pp. 211–277.

[6] F. Gini and G. B. Giannakis, "Frequency offset and symbol timing recovery in flat-fading channels: A cyclostationary approach," *IEEE Trans. Commun.*, vol. 46, pp. 400–411, 1998.

[7] M. Ghogho, A. Swami, and T. Durrani, "On blind carrier recovery in time-selective fading channels," in *Proc. Asilomar Conf. Signals, Syst., Comput.*, 1999, pp. 243–247.

[8] A. J. Viterbi and A. M. Viterbi, "Nonlinear estimation of PSK-modulated carrier phase with application to burst digital transmissions," *IEEE Trans. Inform. Theory*, vol. 29, pp. 543–551, 1983.

[9] D. Efstathiou and A. H. Aghvami, "A comparison study of the estimation period of carrier phase and amplitude gain error for 16-QAM Rayleigh faded burst transmission," in *Proc. IEEE GLOBECOM*, 1994, pp. 1904–1908.

[10] Y. Wang, E. Serpedin, and P. Ciblat, "Optimal blind nonlinear least-squares carrier phase and frequency offset estimation for general QAM modulations," *IEEE Trans. Wireless Commun.*, vol. 2, pp. 1040–1054, 2003.

[11] D. Efstathiou and A. H. Aghvami, "Feedforward synchronization techniques for 16-QAM TDMA demodulations," in *Proc. IEEE ICC*, 1996, pp. 1432–1436.

[12] H. Li, A. Abdi, Y. Bar-Ness, and W. Su, "A qHLRT modulation classifier in fading channels with antenna arrays," in *Proc. IEEE Sarnoff Conference*, 2006, to appear.