Sum of Gamma Variates and Performance of Wireless Communication Systems over Nakagami Fading Channels

Mohamed-Slim Alouini, *Member, IEEE,*
Ali Abdi, *Student Member, IEEE,*
and Mostafa Kaveh, *Fellow, IEEE.*

This is an expanded version of work which is going to be presented at the IEEE International Symposium on Information Theory (ISIT'2001), Washington, DC, USA, June 2001. This work was supported by the National Science Foundation under the Wireless Initiative Grant Number 9979443. M.-S. Alouini, A. Abdi, and M. Kaveh are with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455, USA. M.-S. Alouini can be reached at Tel: (612) 625-9055, Fax: (612) 625-4583, and E-mail: <alouini@ece.umn.edu>. A. Abdi can be reached at Tel: (612) 625-6831, Fax: (612) 625-4583, and E-mail: <abdi@ece.umn.edu>. M. Kaveh can be reached at Tel: (612) 625-0720, Fax: (612) 625-4583, and E-mail: <kaveh@ece.umn.edu>.

Abstract

Capitalyzing on the Moschopoulos single gamma series representation of the probability density function (PDF) of the sum of gamma variates, we provide a PDF-based approach for the performance analysis of maximal-ratio combining and postdetection equal-gain combining diversity techniques as well as co-channel interference of cellular mobile radio systems over Nakagami fading channels with arbitrary parameters. Aside from putting under the same umbrella many of the past results obtained via characteritic function (CF) or moment generating function (MGF)-based approaches, the proposed approach also allows the derivation of additional performance measures which are harder to analyze via CF or MGF-based approaches.

Keywords

(1) Nakagami Fading Channels, (2) Correlated Fading, (3) Diversity Systems, (4) Cellular Mobile Radio Systems, (5) Co-Channel Interference, (6) Outage Probability, (7) BER Performance, and (8) Shannon Capacity over Fading Channels.

I. Introduction

The wide versatility, experimental validity, and analytical tractability of the Nakagami distribution [1] has made it a very popular fading model for performance analysis investigations in two important topics of wireless communications, namely, (i) diversity schemes (e.g., [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]) and (ii) co-channel interference (CCI) in cellular mobile radio systems (e.g. [16, 17, 18, 19, 20, 21, 22, 23, 24, 25]). While initial investigations have dealt mainly with independent identically distributed (i.i.d) diversity paths or cochannel interferers, more recent work has focused on the performance analysis for the non-i.i.d. (arbitrary channel parameters and arbitray correlation) case.

Many of these performance analysis problems require determination of the statistics of the sum (over the L diversity paths or the N_I cochannel interferers) of the squared envelopes of Nakagami faded signals, or equivalently the sum of gamma variates since the square of a Nakagami variate follows a gamma distribution [1]. Expressions have been derived for the probability density function (PDF) of the sum of gamma variates by Mathai [26], Moschopoulos [27], and Sim [28, Appendix] for queuing type of problems. These findings have been reported in the statistics literature but, apparently, have not attracted the attention of researchers working on wireless communication theory. Accordingly, the reported results on communication over Nakagami fading channels that were mentioned earlier, have depended on the use of variants of the characteristic function (CF) or the moment generating function (MGF) of the sum of gamma variates. In this paper, we show that by starting with the Moschopoulos result [27] on the sum of independent gamma variates (which is easily extendable to the sum of correlated gamma variates, as we show later), we are able to provide a PDF-based approach for the performance analysis of (i) maximal-ratio combining (MRC) [29, Sect. 5.5.3] and postdetection equal-gain combining (EGC) [29, Sect. 5.5.6] with noncoherent detection over non-identically distributed and arbitrarily correlated Nakagami diversity paths, and (ii) CCI in cellular mobile radio systems with non-identically distributed and arbitrarily correlated Nakagami interferers. The resulting easy-to-evaluate expressions give (i) alternative formulas for previously known/published results obtained via CF or MGF-based approaches and (ii) new formulas for additional performance measures. These measures, which are harder to analyze via CF or MGF-based approaches, include the Shannon capacity of diversity systems over non-i.i.d. Nakagami diversity paths, the PDF and moments of the carrier-to-interference ratio (CIR) and the average bit-error-rate (BER) in presence of non-identically distributed and arbitrarily correlated Nakagami interferers. Due to space limitations, the derivations of these expressions (which can be obtained with a good table of integrals such as [30]) are omitted here, but all these results have been checked and validated by numerical integration and/or Monte-Carlo simulations.

The remainder of this paper is organized as follows. The next section presents the Moschopoulos result on the sum of independent gamma variates as well as its extension to the sum of correlated gamma variates. Section III applies these results to derive formulas for the outage probability, average BER, and Shannon capacity of MRC and postdetection EGC over non-i.i.d. Nakagami diversity paths. Section IV uses these results to study the PDF and moments of the CIR, outage probability, and average BER of cellular mobile radio systems subject to arbitrary correlated, not necessarily identically distributed Nakagami faded cochannel interferers.

II. Sum of Gamma Variates

In this section, we first recall the definition of a gamma variate and then present two key results on the sums of independent and correlated gamma variates.

Definition 1. *X follows a gamma distribution with parameters $\alpha > 0$ and $\beta > 0$ if the PDF of X is given by*

$$p_X(x) = \frac{x^{\alpha-1}\ e^{-x/\beta}}{\beta^\alpha\ \Gamma(\alpha)}\ U(x), \tag{1}$$

where $\Gamma(\cdot)$ is the gamma function [30] and $U(\cdot)$ is the unit step function. In what follows, we will use the shorthand notation $X \sim \mathcal{G}(\alpha, \beta)$ to denote that X is gamma distributed with parameters α and β.

A. Sum of Independent Gamma Variates

Theorem 1 (Moschopoulos, 1985 [27]). *Let $\{X_n\}_{n=1}^N$ be a set of N independent not necessarily identically distributed gamma variates with parameters α_n and β_n, respectively, (i.e., $X_n \sim \mathcal{G}(\alpha_n, \beta_n)$), then the PDF of $Y = \sum_{n=1}^N X_n$ can be expressed as*

$$p_Y(y) = \prod_{n=1}^{N} \left(\frac{\beta_1}{\beta_n}\right)^{\alpha_n} \sum_{k=0}^{\infty} \frac{\delta_k \; y^{\sum_{n=1}^N \alpha_n + k - 1} \; e^{-y/\beta_1}}{\beta_1^{\sum_{n=1}^N \alpha_n + k} \; \Gamma\left(\sum_{n=1}^N \alpha_n + k\right)} \; U(y), \tag{2}$$

where $\beta_1 = \min_n\{\beta_n\}$ and the coefficients δ_k can be obtained recursively by the formula

$$\begin{cases} \delta_0 & = 1 \\ \delta_{k+1} & = \frac{1}{k+1} \sum_{i=1}^{k+1} \left[\sum_{j=1}^N \alpha_j \left(1 - \frac{\beta_1}{\beta_j}\right)^i\right] \; \delta_{k+1-i}, \qquad k = 0, 1, 2, \cdots . \end{cases} \tag{3}$$

Proof: See [27]. □

A MATHEMATICA program that implements the Moschopoulos representation for the PDF of the sum of gamma variates is given in Appendix A. Contrary to partial fraction-based techniques (e.g. [26]) that typically restrict the $\{\alpha_n\}_{n=1}^N$ to be integers and necessarily all distinct, the Moschopoulos representation applies to arbitrary $\{\alpha_n\}_{n=1}^N$ with the possibility of having some of the $\{\alpha_n\}_{n=1}^N$ equal and others distinct. This representation for the PDF of the sum of gamma variates has also the nice feature of being in the from of a *single gamma series*[1], which implies that, after switching the order of summation and integration, all the manipulations that can be performed for the i.i.d. case can also be done for the non-i.i.d case.

B. Sum of Correlated Gamma Variates

We now extend the Moschopoulos result and obtain an exact single gamma-series representation of the sum of arbitrarily correlated gamma variates.

Corollary 1. *Let $\{X_n\}_{n=1}^N$ be a set of N correlated not necessarily identically distributed gamma variates with parameters α and β_n, respectively, (i.e., $X_n \sim \mathcal{G}(\alpha, \beta_n)$) and let ρ_{ij} denote the*

[1] Note that the Sim [28, Appendix] representation of the sum of independent gamma variates possesses this nice feature and could have also been used as a starting point for the unification of the performance analysis of wireless communication systems over Nakagami fading channels.

correlation coefficient between X_i and X_j, i.e.,

$$\rho_{ij} = \rho_{ji} = \frac{\mathrm{Cov}(X_i, X_j)}{\sqrt{\mathrm{Var}(X_i)\ \mathrm{Var}(X_j)}}, \quad 0 \leq \rho_{ij} \leq 1, \qquad i, j = 1, 2, \cdots, N, \tag{4}$$

then the PDF of $Y = \sum_{n=1}^{N} X_n$ can be expressed as

$$p_Y(y) = \prod_{n=1}^{N} \left(\frac{\lambda_1}{\lambda_n}\right)^{\alpha} \sum_{k=0}^{\infty} \frac{\delta_k\ y^{N\alpha+k-1}\ e^{-y/\lambda_1}}{\lambda_1^{N\alpha+k}\ \Gamma(N\alpha+k)}\ U(y), \tag{5}$$

where $\lambda_1 = \min_n\{\lambda_n\}$, $\{\lambda_n\}_{n=1}^{N}$ are the eigenvalues of the matrix $A = D\ C$, where D is the $N \times N$ diagonal matrix with the entries $\{\beta_n\}_{n=1}^{N}$ and C is the $N \times N$ positive definite matrix defined by

$$C = \begin{bmatrix} 1 & \sqrt{\rho_{12}} & \cdots & \sqrt{\rho_{1N}} \\ \sqrt{\rho_{21}} & 1 & \cdots & \sqrt{\rho_{2N}} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \sqrt{\rho_{N1}} & \cdots & \cdots & 1 \end{bmatrix}_{N\times N}, \tag{6}$$

and the coefficients δ_k can be obtained recursively by the formula

$$\begin{cases} \delta_0 & = 1 \\ \delta_{k+1} & = \frac{\alpha}{k+1} \sum_{i=1}^{k+1} \left[\sum_{j=1}^{N} \left(1 - \frac{\lambda_1}{\lambda_j}\right)^i\right] \delta_{k+1-i}, \qquad k = 0, 1, 2, \cdots. \end{cases} \tag{7}$$

Proof: When the gamma variates $\{X_n\}_{n=1}^{N}$ have the same parameter α then based on [31, Eq. (2.1)] (or equivalently [10, Eq. (34)]) the MGF of Y, $\mathcal{M}_Y(s) = E_Y\left[e^{sY}\right]$ can be expressed as

$$\mathcal{M}_Y(s) = |I - sDC|^{-\alpha}, \tag{8}$$

where $|\cdot|$ denotes the determinant operator, I is the $N \times N$ identity matrix, and the matrices D and C are defined in Corollary 1. Note that, as used in [31, Eq. (2.3)], [9, Eq. (15)], and [6, Eq. (33)], (8) can be re-written in terms of the eigenvalues $\{\lambda_n\}_{n=1}^{N}$ of $A = DC$ as

$$\mathcal{M}_Y(s) = \prod_{n=1}^{N} (1 - s\lambda_n)^{-\alpha}, \tag{9}$$

which is in a similar form as the MGF of the sum of independent gamma variates as given in [27, Eq. (2.1)]. Hence the Moschopoulos technique of inverting the MGF of Y (see [27, Eqs. (2.3)-(2.8)]) can be used to invert (9) and to obtain the desired single gamma-series representation (5) for the PDF of the sum of arbitrarily correlated gamma variates. □

III. Applications to the Performance of Diversity Systems

A. System and Channel Models

Consider an MRC or postdetection EGC diversity receiver in which the L diversity paths go through Nakagami fading channels. The instantaneous signal-to-noise (SNR) of the lth path is $\gamma_l \sim \mathcal{G}(m_l, \overline{\gamma}_l/m_l)$, where $\overline{\gamma}_l$ is the average SNR of the lth path and m_l is the Nakagami severity of fading parameter of the lth path. For MRC and postdetection EGC diversity receivers the total SNR at the combiner output $\gamma_t = \sum_{l=1}^{L} \gamma_l$. Hence, to obtain the PDF of γ_t when the diversity paths are uncorrelated, we can use Theorem 1 with the substitutions in (2) and (3) of N by L, y by γ_t, α_l by m_l, and β_1 by $\overline{\gamma}_1/m_1 = \min_l(\beta_l := \overline{\gamma}_l/m_l)$. This result gives an alternative gamma-series representation to the exact integral representation presented in [5, Eq. (29)]. When the diversity paths are correlated so that the correlation coefficient between γ_i and γ_j is $\rho_{ij} = \rho_{ji}$ $(i, j = 1, \cdots L)$, then the PDF of γ_t can be obtained from Corollary 1 by the substitutions in (5) and (7) of y by γ_t, α by m, and β_l by $\overline{\gamma}_l/m$. This result gives an exact gamma-series representation to the approximate gamma solution presented in [12]. Note that in what follows ρ_{ij} denotes the power correlation coefficient between the fading powers in paths i and j and not between the envelope correlation coefficient between the fading envelopes in paths i and j but these two correlation coefficient are related as shown in [1, Eq. (139)].

Example 1 - Constant Correlation: Consider the constant correlation model proposed by Aalo [7, Section II-A] for L identically distributed Nakagami-m channels (i. e., all channels are assumed to have the same average SNR $\overline{\gamma}$ and the same fading parameter m) with constant correlation across all channels. Since this model assumes that the power correlation coefficient ρ is the same between all the channel pairs $(l, l' = 1, 2, \cdots, L)$, i.e.,

$$\rho = \rho_{ll'} = \frac{\mathrm{Cov}(\gamma_l, \gamma_{l'})}{\sqrt{\mathrm{Var}(\gamma_l)\mathrm{Var}(\gamma_{l'})}}, \quad l \neq l', \quad 0 \leq \rho < 1. \tag{10}$$

In the context of antenna diversity the spatial correlation is function of the distance between antennas and this model will then apply to equidistant antennas. This corresponds to size-limited scenarios with diversity reception from an array of 3 antennas placed on an equilateral triangle or from closely placed antennas on other than linear arrays.

Based on the work of Gurland [32], Aalo showed that the PDF of γ_t is given in this case by [7, Eq. (18)][2]

$$p_{\gamma_t}(\gamma_t) = \frac{\left(\frac{m\gamma_t}{\overline{\gamma}}\right)^{Lm-1} \exp\left(-\frac{m\gamma_t}{(1-\sqrt{\rho})\overline{\gamma}}\right) \ {}_1F_1\left(m, Lm; \frac{Lm\sqrt{\rho}\gamma_t}{(1-\sqrt{\rho})(1-\sqrt{\rho}+L\sqrt{\rho})\overline{\gamma}}\right)}{\left(\frac{\overline{\gamma}}{m}\right)\ (1-\sqrt{\rho})^{m(L-1)}\ (1-\sqrt{\rho}+L\sqrt{\rho})^m\ \Gamma(Lm)}; \quad \gamma_t \geq 0, \qquad (11)$$

where ${}_1F_1(\cdot,\cdot;\cdot)$ is the confluent hypergeometric function [33, Chapter 13, p. 503].

For constant correlation, it can be shown that the eigenvalues of the matrix A defined in Corollary 1 are given by [34, Eq. (2.8.3), p. 29]

$$\begin{cases} \lambda_1 & = \cdots = \lambda_{L-1} = \frac{\overline{\gamma}}{m}\left(1-\sqrt{\rho}\right) \\ \lambda_L & = \frac{\overline{\gamma}}{m}\left(1+\sqrt{\rho}(L-1)\right). \end{cases} \qquad (12)$$

Substituting these eigenvalues in (5) we obtain the distribution of the output combined SNR with the the substitutions in (5) and (7) of N by L, y by γ_t, α by m, and β_l by $\overline{\gamma}_l/m$. As an example, Fig 1 shows a comparison of the PDF of the output SNR obtained from the exact expression (11), the exact expression (5) with (12), and the gamma approximate expression offered in [12] for a constant correlation model with $L = 5$, $m = 2.5$, $\overline{\gamma} = 1$, and $\rho = 0.64$. On one hand the two exact expressions (Eqs. (11) and (5)) match perfectly, as expected. On the other hand, although the approximate solution matches quite well with the exact solutions in the high SNR region, it tends to deviate in the lower tail of the PDF. Hence the approximate solution has to be used with caution as far as outage probability and average probability of errors calculations are concerned since the lower tail of the PDF is very critical for these calculations.

Example 2 - Circular Correlation: Consider a circular correlation model for which the correlation matrix C defined in (6) is not only symmetric (as it is the case for any correlation matrix) but has also Lth order circular symmetry, i. e.,

$$C = \begin{bmatrix} 1 & \sqrt{\rho_2} & \cdot & \cdot & \cdot & \sqrt{\rho_L} \\ \sqrt{\rho_L} & 1 & \sqrt{\rho_2} & \cdot & \cdot & \sqrt{\rho_{L-1}} \\ \sqrt{\rho_{L-1}} & \sqrt{\rho_L} & 1 & \sqrt{\rho_2} & \cdot & \sqrt{\rho_{L-2}} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \sqrt{\rho_2} & \cdot & \cdot & \cdot & \sqrt{\rho_L} & 1 \end{bmatrix}_{L\times L}, \qquad (13)$$

[2]It should be noted at this point that in [7, Eq. (18)] the symbol ρ is used to denote the correlation coefficient of the underlying Gaussian processes that produce the fading on the channels. This correlation coefficient is equal to the square root of the power correlation coefficient.

which implies that $\rho_2 = \rho_L$, $\rho_3 = \rho_{L-1}$, $\cdots$. This model may apply to antennas lying on a circle or 4 antennas placed on a square. For L identically distributed Nakagami-m channels, it can be shown that the eigenvalues of the matrix A defined in Corollary 1 are given by [34, Eq. (2.8.4), p. 29]

$$\lambda_l = \frac{\overline{\gamma}}{m} \sum_{k=1}^{L} \sqrt{\rho_l} \;\exp\left(\frac{2\pi j}{L}(l-1)(k-1)\right), \tag{14}$$

where $j^2 = -1$ and $\rho_1 = 1$. Substituting these eigenvalues in (5) we obtain the distribution of the output combined SNR with the substitutions in (5) and (7) of y by γ_t, α by m, and β_l by $\overline{\gamma}_l/m$. As an example, Fig. 2 shows a good match between the PDF obtained analytically (based on (5)) and the one obtained via Monte-Carlo simulations (based on the procedure devised in [35, Appendix]) for the circular correlation case with $L = 5$, $\overline{\gamma} = 1$, $m = 2$, and power correlation coefficients $\rho_2 = \rho_{12} = \rho_{15} = 0.8$ and $\rho_3 = \rho_{13} = \rho_{14} = 0.6$. Fig. 3 compares the exact analytical result (5) with the approximation proposed in [12] for the circular correlation case with $L = 5$, $\overline{\gamma} = 1$, $m = 2.7$, and power correlation coefficients $\rho_2 = \rho_{12} = \rho_{15} = 0.64$ and $\rho_3 = \rho_{13} = \rho_{14} = 0.36$. Again note the overall good match in particular for high SNR but the relatively important deviation between the exact and the approximate result in the sensitive low SNR region.

Based on Theorem 1 and Corollary 1, the performance of MRC and postdetection EGC can now be obtained in the form of rapidly convergent series. Moschopoulos [27] provides a rigorous proof for the uniform convergence of (2) and a bound on the truncation error. Since uniform convergence is sufficient for interchanging the order of summation and integration [36] all the subsequent manipulation in the series which we obtain below are justified. In addition our numerical experiments show that this series along with these other subsequent series are indeed numerically stable and rapidly converging for various scenarios of practical interest. Furthermore, these numerical experiments confirm Moschopoulos bound on the truncation error and show that the convergence rate depends on the maximum of $\left(1 - \frac{\overline{\gamma}_1 m_l}{\overline{\gamma}_l m_1}\right)$, $l = 1, 2, \cdots, L$ for uncorrelated diversity paths and on the maximum of $\left(1 - \frac{\lambda_1}{\lambda_l}\right)$, $l = 1, 2, \cdots, L$ for correlated diversity paths. Therefore, these infinite series can be used in practice by truncating them to a certain order say K, (i.e., K first terms in the infinite series (2) and (5)) in order to meet a specified accuracy. Although the results that are presented below are given for the case of uncorrelated diversity paths, they also apply equally well to the correlated case with the substitutions of m_l by m and

$\overline{\gamma}_l/m_l$ by λ_l, $l=1,2,\cdots,L$.

B. Outage Probability

The probability that γ_t falls below a predetermined threshold γ_{th} can be shown with the help of [30, Eq. (8.356.3)] to be given by

$$P_{\text{out}}=\prod_{l=1}^{L}\left(\frac{m_l\overline{\gamma}_1}{m_1\overline{\gamma}_l}\right)^{m_l}\sum_{k=0}^{\infty}\delta_k\left[1-\frac{\Gamma\left(\sum_{l=1}^{L}m_l+k,\frac{m_1\gamma_{\text{th}}}{\overline{\gamma}_1}\right)}{\Gamma\left(\sum_{l=1}^{L}m_l+k\right)}\right], \tag{15}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function [30]. For the correlated case, (15) provides an extension to the formulas given in [7, Eq. (45)] valid for the equal correlation model, in [7, Eq. (49)] valid (approximately) for the exponential correlation model, and in [10, Eq. (31)] for arbitrary correlation when m is restricted to integer values. Equation (15) can also serve as an alternative formula to the MGF-based numerical technique presented in [37] (which requires the selection of three additional numerical parameters to control the accuracy). If $\sum_{l=1}^{L}m_l$ is restricted to integer values, then (15) reduces to

$$P_{\text{out}}=\prod_{l=1}^{L}\left(\frac{m_l\overline{\gamma}_1}{m_1\overline{\gamma}_l}\right)^{m_l}\sum_{k=0}^{\infty}\delta_k\left[1-\exp\left(-\frac{m_1\gamma_{\text{th}}}{\overline{\gamma}_1}\right)\sum_{i=0}^{\sum_{l=1}^{L}m_l+k-1}\frac{1}{i!}\left(\frac{m_1\gamma_{\text{th}}}{\overline{\gamma}_1}\right)^i\right]. \tag{16}$$

C. Average Bit Error Rate

We limit ourselves to binary modulation but similar results can also be obtained for M-ary modulations with the help of [38, Appendix 5A]. For MRC with coherent detection, the average BER is obtained with the help of [2, Appendix] as

$$P_b(E)=\frac{1}{2}\left[\prod_{l=1}^{L}\left(\frac{m_l\overline{\gamma}_1}{m_1\overline{\gamma}_l}\right)^{m_l}\right]\sqrt{\frac{g\overline{\gamma}_1}{\pi m_1}}\sum_{m=0}^{\infty}\delta_k\frac{\Gamma\left(\sum_{l=1}^{L}m_l+k+\frac{1}{2}\right)}{\Gamma\left(\sum_{l=1}^{L}m_l+k+1\right)}\frac{{}_2F_1\left[1,\sum_{l=1}^{L}m_l+k+\frac{1}{2},\sum_{l=1}^{L}m_l+k+1;\frac{1}{1+\frac{g\overline{\gamma}_1}{m_1}}\right]}{\left(1+\frac{g\overline{\gamma}_1}{m_1}\right)^{\sum_{l=1}^{L}m_l+k+\frac{1}{2}}}, \tag{17}$$

where g is a modulation dependent parameter (e.g., $g=1$ for binary phase shift keying (BPSK)). For the independent diversity paths case, (17) is equivalent to [5, Eq. (33)] (involves a single infinite-range integral) whereas for the correlated case it is equivalent to [14, Eq. (32)] (involves a single finite-range integral), [10, Eq. (22)] (involves a single infinite-range integral), and [9, Eq. (34)] (involves a single infinite-range integral). If $\sum_{l=1}^{L}m_l$ is restricted to integer values, then

(17) reduces to

$$P_b(E) = \frac{1}{2}\left[\prod_{l=1}^{L}\left(\frac{m_l\overline{\gamma}_1}{m_1\overline{\gamma}_l}\right)^{m_l}\right]\sum_{k=0}^{\infty}\delta_k\left[1-\sqrt{\frac{g\overline{\gamma}_1}{m_1+g\overline{\gamma}_1}}\sum_{i=0}^{\sum_{l=1}^{L}m_l+k-1}\frac{1}{4^i}\binom{2i}{i}\left(\frac{m_1}{m_1+g\overline{\gamma}_1}\right)^i\right], \quad (18)$$

which is equivalent to [9, Eq. (23)] (which is in the form of finite sums with multiple derivatives) for the correlated case.

For postdetection EGC with noncoherent detection, the average BER is given by

$$P_b(E) = \frac{1}{2^{2L-1}}\left[\prod_{l=1}^{L}\left(\frac{m_l\overline{\gamma}_1}{m_1\overline{\gamma}_l}\right)^{m_l}\right]\sum_{k=0}^{\infty}\frac{\delta_k}{\Gamma\left(\sum_{l=1}^{L}m_l+k\right)\left(\frac{g\overline{\gamma}_1}{m_1}\right)^{\sum_{l=1}^{L}m_l+k}}\sum_{l=0}^{L-1}c_l\frac{\Gamma\left(l+\sum_{l=1}^{L}m_l+k\right)}{\left(1+\frac{m_1}{g\overline{\gamma}_1}\right)^{l+\sum_{l=1}^{L}m_l+k}}, \quad (19)$$

where $c_l = \frac{1}{l!}\sum_{j=0}^{L-1-l}\binom{2L-1}{j}$ and g is a modulation dependent parameter (e.g., $g = 1/2$ for noncoherent binary frequency shift keying (BFSK)). Eq. (19) is equivalent to [4, Eq. (33)] (involves a single infinite-range integral) for the independent diversity paths case and to [14, Eq. (39)] (involves a single finite-range integral), [8, Eqs. (25) and (48)] (involves multiple derivatives), and [11, Eq. (18)] (in the form of a finite sum and multiple derivatives) for the arbitrarily correlated case.

D. Shannon Capacity

For fading channels, the Shannon capacity characterizes the long-term achievable rate averaged over the fading distribution and depends on the amount of available channel state information (CSI) at the receiver and transmitter [39, 40, 41]. With MRC and optimal power and rate adaptation (OPRA) that requires both transmitter and receiver CSI, it can be shown that the capacity C_{opra} is given by

$$\frac{C_{\text{opra}}}{W} = \log_2(e)\left[\prod_{l=1}^{L}\left(\frac{m_l\overline{\gamma}_1}{m_1\overline{\gamma}_l}\right)^{m_l}\right]\sum_{k=0}^{\infty}\delta_k\sum_{n=0}^{\sum_{l=1}^{L}m_l+k-1}\frac{\Gamma\left(n,\frac{m_1\gamma_0}{\overline{\gamma}_1}\right)}{n!}, \quad (20)$$

where W is the channel bandwidth and γ_0 is the cut-off SNR below which transmission is suspended [39]. With MRC and optimal rate adaptation (ORA) that requires only receiver CSI, it can be shown that the capacity C_{ora} is given by

$$\frac{C_{\text{ora}}}{W} = \log_2(e)\left[\prod_{l=1}^{L}\left(\frac{m_l}{\overline{\gamma}_l}\right)^{m_l}\right]\exp\left(\frac{m_1}{\overline{\gamma}_1}\right)\sum_{k=0}^{\infty}\delta_k\sum_{n=1}^{\sum_{l=1}^{L}m_l+k}\frac{\Gamma\left(-\sum_{l=1}^{L}m_l-k+n,\frac{m_1}{\overline{\gamma}_1}\right)}{\left(\frac{\overline{\gamma}_1}{m_1}\right)^{k-n}}. \quad (21)$$

With MRC and total channel inversion and fixed rate (TIFR) the zero-outage capacity C_{tifr} can be shown to be given by

$$\frac{C_{\text{tifr}}}{W}=\log_2\left[1+\frac{1}{\left[\prod_{l=1}^{L}\left(\frac{m_l}{\overline{\gamma}_l}\right)^{m_l}\right]\left(\frac{\overline{\gamma}_1}{m_1}\right)^{\sum_{l=1}^{L}m_l-1}\sum_{k=0}^{\infty}\frac{\delta_k}{\sum_{l=1}^{L}m_l+k-1}}\right]. \tag{22}$$

For the independent diversity paths case, (20), (21), and (22) provide extension to the corresponding formulas in [13], which are limited to i.i.d. Nakagami diversity paths and [42] which are restricted to unbalanced (necessarily distinct) independent Rayleigh diversity paths. For the correlated case, (20), (21), and (22) generalize the corresponding formulas in [42], which are limited to identically distributed correlated Rayleigh diversity paths with distinct eigenvalues and [43] which is valid for identically distributed Rayleigh diversity paths with equal correlation.

IV. Application to the Performance of Cellular Mobile Radio Systems

A. System and Channel Model

Consider a cellular mobile radio system in which the desired and the N_I interfering signals may have different fading statistics. The desired signal s_0 is assumed to be independent from the N_I interfering signals $\{s_n\}_{n=1}^{N_I}$ and to be subject to Nakagami fading with average fading power $\overline{s}_0$ and fading parameter m_0 or Rician fading with average fading power $\overline{s}_0$ and Rician factor K_0. The N_I active interferers are assumed to be subject to Nakagami fading with their instantaneous signal power $s_n \sim \mathcal{G}(m_n, \overline{s}_n/m_n)$, $n=1,\cdots,N_I$. We can then apply Theorem 1 to find the PDF of the total interference power $s_I=\sum_{n=1}^{N_I} s_n$ with the substitutions in (2) and (3) of N by L, y by s_I, α_l by m_l, and β_1 by $\overline{s}_1/m_1=\min_l(\beta_l:=\overline{s}_l/m_l)$ for the independent interferers case. This result gives an alternative gamma-series representation to approximate single gamma solution given in [44, Appendix A]. For the correlated interferers case, such that the correlation coefficient between s_i and s_j is $\rho_{ij}=\rho_{ji}$ $(i,j=1,\cdots N_I)$ then, the PDF of s_I can be obtained from Corollary 1 by the substitutions in (5) and (7) of N by L, y by s_I, α by m, and β_l by $\overline{\gamma}_l/m$.

Again based on Theorem 1 and Corollary 1, performance results for cellular mobile radio systems operating over Nakagami fading channels can be obtained in the form of rapidly convergent series as we present next. Although the results presented below are given for the uncorrelated interferers case they also apply equally well to the correlated case with the substitutions of m_l by m and $\overline{s}_l/m_l$ by λ_l, $l=1,2,\cdots,L$.

B. Distribution and Moments of CIR

If the desired user is subject to Nakagami fading, the PDF of the CIR $\lambda = s_0/s_I$ can be found to be given by

$$p_\lambda(\lambda) = \prod_{n=0}^{N_I} \left(\frac{\overline{s}_1\ m_n}{\overline{s}_n\ m_1}\right)^{m_n} \sum_{k=0}^{\infty} \frac{\lambda^{m_0-1}\ \delta_k}{B\Big(m_0, \sum_{n=1}^{N_I} m_n + k\Big)\ \Big(1+\lambda \frac{\overline{s}_1\ m_0}{\overline{s}_0\ m_1}\Big)^{(\sum_{n=0}^{N_I} m_n + k)}}\ U(\lambda), \qquad (23)$$

where $B(\cdot,\cdot)$ is the beta function [30]. In addition, it can be shown with the help of [30, Eq. (3.194.3)] that the moments of the CIR are given by

$$E_\lambda[\lambda^p] = \prod_{n=1}^{N_I} \left(\frac{\overline{s}_1\ m_n}{\overline{s}_n\ m_1}\right)^{m_n} \left(\frac{\overline{s}_0\ m_1}{\overline{s}_1\ m_0}\right)^{p} \sum_{k=0}^{\infty} \delta_k\ \frac{B\Big(p+m_0, \sum_{n=1}^{N_I} m_n + k - p\Big)}{B\Big(m_0, \sum_{n=1}^{N_I} m_n + k\Big)}, \qquad p = 1, 2, \cdots. \qquad (24)$$

In particular, the average CIR is obtained from (24) with $p = 1$ yielding

$$\overline{\lambda} = E_\lambda[\lambda] = \prod_{n=1}^{N_I} \left(\frac{\overline{s}_1\ m_n}{\overline{s}_n\ m_1}\right)^{m_n} \left(\frac{\overline{s}_0\ m_1}{\overline{s}_1\ m_0}\right) \sum_{k=0}^{\infty} \delta_k\ \frac{B\Big(1+m_0, \sum_{n=1}^{N_I} m_n + k - 1\Big)}{B\Big(m_0, \sum_{n=1}^{N_I} m_n + k\Big)}. \qquad (25)$$

C. Outage Probability

When the desired user is subject to Nakagami fading, the probability that the CIR falls below a predetermined protection ratio λ_{th} can be shown with the help of [30, Eq. (3.194.1)] to be given by

$$P_{\text{out}} = \prod_{n=0}^{N_I} \left(\frac{\overline{s}_1\ m_n}{\overline{s}_n\ m_1}\right)^{m_n} \sum_{k=0}^{\infty} \frac{\lambda_{\text{th}}^{m_0}\ \delta_k}{m_0\ B\Big(m_0, \sum_{n=1}^{N_I} m_n + k\Big)}\ {}_2F_1\left[\sum_{n=0}^{N_I} m_n + k, m_0; 1+m_0; -\lambda_{\text{th}} \frac{\overline{s}_1\ m_0}{\overline{s}_0\ m_1}\right], \qquad (26)$$

which is equivalent to [18, Eq. (21)] (which involves a single multiple derivative but is restricted to integer values for the desired user m_0), [19, Eq. (21)] (which involves a single infinite-range integral), [20, Eq. (14)] (which involves multiple series), [21, Eq. (4)] (which involves a single series and the selection of an additional numerical parameter h to control the accuracy), and [24, Eq. (2)] (which involves multiple series). When m_0 and $\sum_{n=1}^{N_I} m_n$ are both restricted to integer values, (26) reduces to

$$
\begin{aligned}
P_{\text{out}} &= \prod_{n=1}^{N_I} \left(\frac{\overline{s}_1\ m_n}{\overline{s}_n\ m_1}\right)^{m_n} \sum_{k=0}^{\infty} \delta_k \left(\sum_{n=0}^{N_I} m_n + k - 1\right)! \\
&\times \sum_{n=0}^{\sum_{n=1}^{N_I} m_n + k - 1} \frac{\left(\lambda_{\text{th}} \frac{\overline{s}_1 m_0}{\overline{s}_0 m_1}\right)^{n+m_0}}{\left(\sum_{i=1}^{N_I} m_i + k - 1 - n\right)!\ (m_0+n)!\ \left(1+\lambda_{\text{th}}\frac{\overline{s}_1 m_0}{\overline{s}_0 m_1}\right)^{\sum_{n=0}^{N_I} m_n + k - 1}}. \qquad (27)
\end{aligned}
$$

When the desired user is subject to Rician fading and $\sum_{n=1}^{N_I} m_n$ is restricted to integer values, this outage probability can be found after tedious manipulations and with the help of [30, Eq. (9.220.2)] to be given by

$$
\begin{aligned}
P_{\text{out}} &= \prod_{n=1}^{N_I} \left(\frac{\overline{s}_1\ m_n}{\overline{s}_n\ m_1}\right)^{m_n} \sum_{k=0}^{\infty} \delta_k \left(1 + \frac{\overline{s}_0 m_1}{\lambda_{\text{th}} \overline{s}_1 (1+K_0)}\right)^{-1} \exp\left(-\frac{K_0}{1+\frac{\lambda_{\text{th}}\overline{s}_1(1+K_0)}{\overline{s}_0 m_1}}\right) \\
&\sum_{i=0}^{\sum_{n=1}^{N_I} m_n + k - 1} \left(1+\frac{\lambda_{\text{th}}\overline{s}_1(1+K_0)}{\overline{s}_0 m_1}\right)^{-i} \sum_{j=0}^{i} \binom{i}{j} \frac{1}{j!} \left(\frac{K_0}{1+\frac{\overline{s}_0 m_1}{\lambda_{\text{th}}\overline{s}_1(1+K_0)}}\right)^{j}, \qquad (28)
\end{aligned}
$$

which is equivalent to [22, Eq. (15)] (which involves multiple derivatives and is in the form of finite sums but requires necessarily distinct $\{m_n \overline{s}_n\}_{n=1}^{N_I}$ and integer values for $\{m_n\}_{n=1}^{N_I}$) and [23, Eq. (16)] (which involves multiple series).

D. Average Bit Error Rate

The average BER for several modulation schemes can be obtained from the MGF of the CIR which can be found with the help of [30, Eq. (3.383.5)] to be given by

$$
\mathcal{M}_\lambda(s) = \prod_{n=1}^{N_I} \left(\frac{\overline{s}_1\ m_n}{\overline{s}_n\ m_1}\right)^{m_n} \sum_{k=0}^{\infty} \delta_k \frac{\Gamma\left(\sum_{n=0}^{N_I} m_n + k\right)}{\Gamma\left(\sum_{n=1}^{N_I} m_n + k\right)} \Psi\left(m_0, 1 - \sum_{n=1}^{N_I} m_n + k; -\frac{\overline{s}_0 m_1}{\overline{s}_1\ m_0} s\right), \qquad (29)
$$

where $\Psi(\cdot,\cdot;\cdot)$ is the degenerate hypergeometric function [30]. Eq. (29) generalizes [25, Eq. (17)] to non-i.i.d. Nakagami interferers.

V. Conclusion

In this paper, we relied on the Moschopoulos representation for the PDF of the sum of independent gamma variates and its extension to the PDF of the sum of correlated gamma variates to provide an *exact* PDF-based approach for the performance analysis of many wireless communication systems over not necessarily independent nor identically distributed Nakagami fading channels. The key feature of this representation is that it is in the from of a single gamma series which implies that all the manipulations that can be performed for the i.i.d. case can now also be done for the non-i.i.d case. The coverage is broad in that various performance measures of MRC and postdetection EGC are treated as well as the performance of cellular mobile radio systems with co-channel interference. Aside from putting under the same umbrella many of the past results obtained via CF or MGF-based approaches, the proposed PDF-based approach also allows the derivation of additional performance measures which are harder to analyze via CF or MGF-based approaches.

Acknowledgment

The authors would like to thank Tyrone Roach of the University of Nevada, Las Vegas and Kai-Chuan Chang and Young-Chai Ko of the University of Minnesota, Minneapolis for their validation of all the results presented in this paper by numerical computations or Monte-Carlo simulations.

Appendix A

A MATHEMATICA Program for the PDF of the Sum of Gamma Variates

The following MATHEMATICA program was developed to find the PDF of the sum of gamma variates as per [27].

```
SumOfGammaPDF[ParametersMatrix-, UpperLimitOfSummation-, y-] := Module[ {N, a,
b, C, gamma, delta, rho} ,
(* ''ParametersMatrix'' contains the parameters of the gamma variates.  For example,
if we have three gamma variates with parameters (a1, b1), (a2, b2), and (a3, b3),
then:  ParametersMatrix = { {a1, b1}, {a2, b2}, , b3} }.  Note that b1 has to be the
smallest among b's, otherwise the program yields an error and further computations
will be aborted.
''UpperLimitOfSummation'' is a finite number that we choose to truncate the infinite
series to meet an acceptable precision.
''y'' is the variable which represents the sum of gamma variates.  *)
Print[MatrixForm[ParametersMatrix]];
N = Dimensions[ParametersMatrix][[1]];
a = Table[0, {N}];
Do[a[[i]] = ParametersMatrix[[i, 1]], {i, 1, N} ];
b = Table[0, {N}];
Do[b[[i]] = ParametersMatrix[[i, 2]], {i, 1, N} ];
If[b[[1]] != Min[b],
Print["Error!  b1 has to be the smallest among b's"]; Abort[]];


C = Product[ (b[[1]]/b[[i]])^a[[i]], {i, 1, N}];

gamma[k-] := Sum[(1/k)a[[i]](1 - (b[[1]]/b[[i]]))^k, {i, 1, N}];

(* This recursion calculates the sequence of delta's *)
delta = Table[0, {k, 1, UpperLimitOfSummation}];
delta[[1]] = gamma[1];
```

```
Do[delta[[k]] = Sum[i gamma[i] delta[[k - i]]/k, {i, 1, k - 1}]
+ gamma[k], {k, 2, UpperLimitOfSummation}];
rho = Sum[a[[i]], {i, 1, n}];

(* Note that the zeroth term in the summation is computed separately *)


N[C(y^(rho - 1) Exp[-y/b[[1]]]/(Gamma[rho] b[[1]]^rho) +
Sum[delta[[k]] y^(rho + k - 1) Exp[-y/b[[1]]]/(Gamma[rho + k]
b[[1]]^(rho +k)), {k, 1, UpperLimitOfSummation} ]) ] ]
```

Similar programs have been used to find the PDF of the sum of correlated gamma variates as well as the various other series for the performance measures (outage probability, average BER, and capacity) presented in this paper.

References

[1] M. Nakagami, "The m-distribution- A general formula of intensity distribution of rapid fading," in *Statistical Methods in Radio Wave Propagation*, pp. 3–36, Pergamon Press, Oxford, U. K., 1960.

[2] T. Eng and L. B. Milstein, "Coherent DS-CDMA performance in Nakagami multipath fading," *IEEE Trans. Commun.*, vol. COM-43, pp. 1134–1143, February/March/April 1995.

[3] G. P. Efthymoglou and V. Aalo, "Performance of RAKE receivers in Nakagami fading channel with arbitary fading parameters," *Electron. Lett.*, vol. 31, pp. 1610–1612, August 1995.

[4] G. P. Efthymoglou, V. A. Aalo, and H. Helmken, "Performance analysis of noncoherent binary DS/CDMA systems in a Nakagami multipath channel with arbitrary parameters," *IEE Proc. Commun.*, vol. 144, pp. 166–172, June 1997. See also Proc. IEEE Global Commun. Conf. (GLOBECOM'96), London, U. K., pp. 1296-1300, November 1996.

[5] G. P. Efthymoglou, V. A. Aalo, and H. Helmken, "Performance analysis of coherent DS-CDMA systems in a Nakagami fading channel with arbitrary parameters," *IEEE Trans. Veh. Technol.*, vol. VT-46, pp. 289–297, May 1997.

[6] A. Annamalai, C. Tellambura, and V. K. Bhargava, "Exact evaluation of maximal-ratio and equal-gain diversity receivers for M-ary QAM on Nakagami fading channels," *IEEE Trans. Commun.*, vol. COM-47, pp. 1335–1344, September 1999.

[7] V. A. Aalo, "Performance of maximal-ratio diversity systems in a correlated Nakagami-fading environment," *IEEE Trans. Commun.*, vol. COM-43, pp. 2360–2369, August 1995.

[8] Q. T. Zhang, "Exact analysis of postdetection combining for DPSK and NFSK systems over arbitrary correlated Nakagami channels," *IEEE Trans. Commun.*, vol. COM-46, pp. 1459–1467, November 1998.

[9] Q. T. Zhang, "Maximal-ratio combining over Nakagami fading channels with an arbitrary branch covariance matrix," *IEEE Trans. Veh. Technol.*, vol. VT-48, pp. 1141–1150, July 1999.

[10] P. Lombardo, G. Fedele, and M. M. Rao, "MRC performance for binary signals in Nakagami fading with general branch correlation," *IEEE Trans. Commun.*, vol. COM-47, pp. 44–52, January 1999.

[11] P. Lombardo and G. Fedele, "Post-detection diversity in Nakagami fading chanels with correlated branches," *IEEE Commun. Lett.*, vol. 3, pp. 132–135, May 1999.

[12] C. Mun, C. -H. Kang, and H. -K. Park, "Approximation of SNR statistics for MRC diversity in arbitrarily correlated Nakagami fading channels," *IEE Electron. Lett.*, vol. 35, pp. 266–267, February 1999.

[13] M. -S. Alouini and A. J. Goldsmith, "Capacity of Nakagami multipath fading channels," in *IEEE Veh. Technol. Conf. (VTC'97), Phoenix, AZ*, pp. 358–362, May 1997.

[14] M. -S. Alouini and M. K. Simon, "Multichannel reception of digital signals over correlated Nakagami fading channels," in *Proc. of the 36th Allerton Conference on Communication, Control, and Computing (Allerton'98), Allerton Park, IL*, pp. 146–155, September 1998.

[15] M. Z. Win and J. H. Winters, "Exact error probability expressions for MRC in correlated Nakagami channels with unequal fading parameters and branch powers," in *IEEE Global Commun. Conf. (GLOBECOM'99)*, pp. 2331–2335, December 1999.

[16] A. A. Abu-Dayya and N. C. Beaulieu, "Outage probabilities of cellular mobile radio systems with multiple Nakagami interferers," *IEEE Trans. Veh. Technol.*, vol. VT-40, pp. 757–768, November 1991.

[17] Y. -D. Yao and A. U. H. Sheikh, "Investigation into cochannel interference in microcellular mobile radio systems," *IEEE Trans. Veh. Technol.*, vol. VT-41, pp. 114–123, May 1992.

[18] Q. T. Zhang, "Outage probability of cellular mobile radio in the presence of multiple Nakagami interferers with arbitrary fading parameters," *IEEE Trans. Veh. Technol.*, vol. VT-44, pp. 661–667, August 1995.

[19] Q. T. Zhang, "Outage probability in cellular mobile radio due to Nakagami signal and interferers with arbitrary parameters," *IEEE Trans. on Veh. Technol.*, vol. 45, pp. 364–372, May 1996.

[20] C. Tellambura and V. K. Bhargava, "Outage probability analysis for cellular mobile radio systems subject to Nakagami fading and shadowing," *Trans. IECE of Japan*, vol. E 78-B, pp. 1416–1423, October 1995.

[21] C. Tellambura, "Cochannel interference computation for arbitrary Nakagami fading," *IEEE Trans. on Veh. Technol.*, vol. 48, pp. 487–489, March 1999.

[22] J. -C. Lin, W. -C. Kao, Y. T. Su, and T. -H. Lee, "Outage and coverage considerations for microcellular mobile radio systems in a shadowed-Rician/shadowed-Nakagami environment," *IEEE Trans. Veh. Technol.*, vol. VT-48, pp. 66–75, January 1999.

[23] G. Karagiannidis, S. Kotsopoulos, and C. Georgopoulos, "Outage probability analysis for a Rician signal in L Nakagami interferers with arbitrary parameters," *J. Commun. Networks*, vol. 1, pp. 26–30, March 1999.

[24] G. Karagiannidis, C. Georgopoulos, and S. Kotsopoulos, "Outage probability analysis for a Nakagami signal in L Nakagami interferers," *European Trans. Telecommun.*, 2001.

[25] V. A. Aalo and J. Zhang, "On the effect of cochannel interference on average error rates in Nakagami-fading channels," *IEEE Commun. Lett.*, vol. 3, pp. 136–138, May 1999.

[26] A. M. Mathai, "Storage capacity of a dam with gamma type inputs," *Ann. Inst. Statist. Math. (Part A)*, vol. 34, no. 3, pp. 591–597, 1982.

[27] P. G. Moschopoulos, "The distribution of the sum of independent gamma random variables," *Ann. Inst.*

Statist. Math. (Part A), vol. 37, pp. 541–544, 1985.

[28] C. H. Sim, "Point processes with correlated gamma interarrival times," *Statistics & Probability Lett.*, vol. 15, pp. 135–141, 1992.

[29] G. L. Stüber, *Principles of Mobile Communications.* Norwell, MA: Kluwer Academic Publishers, 1996.

[30] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products.* San Diego, CA: Academic Press, fifth ed., 1994.

[31] S. Kotz and J. Adams, "Distribution of sum of identically distributed exponentially correlated gamma variables," *Annals of Math. Statist.*, vol. 35, pp. 277–283, June 1964.

[32] J. Gurland, "Distribution of the maximum of the arithmetic mean of correlated random variables," *Annals of Math. Statist.*, vol. 26, pp. 294–300, 1955.

[33] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables.* New York, NY: Dover Publications, ninth ed., 1970.

[34] S. J. Press, *Applied Multivariate Analysis.* Holt, Rinehart and Winston, Inc., 1972.

[35] Y. C. Ko and M. -S. Alouini, "Estimation of nakagami-m fading channel parameters with application to optimized transmit diversity systems,." In Revision for *IEEE J. Select. Areas Commun.*

[36] J. D. DePree and C. W. Swartz, *Introduction to Real Analysis.* New York, NY: John Wiley & Sons,, 1988.

[37] Y. -C. Ko, M. -S. Alouini, and M. K. Simon, "Outage probability of diversity systems over generalized fading channels," *IEEE Trans. Commun.*, vol. COM-48, pp. 1783–1787, November 2000.

[38] M. K. Simon and M. -S. Alouini, "Digital communications over generalized fading channels: A unified approach to performance analysis." New York, NY: John Wiley & Sons, 2000.

[39] A. J. Goldsmith and P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. on Information Theory*, vol. IT-43, pp. 1896–1992, November 1997.

[40] G. Caire and S. Shamai, "On the capacity of some channels with channel state information," *IEEE Trans. on Information Theory*, vol. IT-45, pp. 2007–2019, September 1999.

[41] G. Caire, G. Taricco, and E. Biglieri, "Optimum power control over fading channels," *IEEE Trans. on Information Theory*, vol. IT-45, pp. 1468–1489, July 1999.

[42] J. W. Shao, M. -S. Alouini, and A. J. Goldsmith, "Impact of fading correlation and unequal branch gains on the capacity of diversity systems," in *IEEE Veh. Technol. Conf. (VTC'99), Houston, TX*, pp. 2159–2163, May 1999.

[43] R. K. Mallik and M. Z. Win, "Channel capacity in evenly correlated Rayleigh fading with different adaptive transmission schemes and maximal ratio combining," in *Proc. IEEE Int. Symposium Inform. Theory (ISIT'2000), Sorrento, Italy*, p. 412, June 2000.

[44] F. Babich and G. Lombardi, "Statitical analysis and characterization of the indoor propagation channel," *IEEE Trans. Commun.*, vol. COM-48, pp. 455–463, March 2000.

Mohamed-Slim Alouini (S'94-M'99) was born in Tunis, Tunisia. He received the "Diplôme d'Ingénieur" degree from the Ecole Nationale Supérieure des Télécommunications (TELECOM Paris), Paris, France, and the "Diplôme d'Etudes Approfondies (D.E.A.)" degree in Electronics from the University of Pierre & Marie Curie (Paris VI), Paris, France, both in 1993. He received the M.S.E.E. degree from the Georgia Institute of Technology (Georgia Tech), Atlanta, GA, USA, in 1995, and the Ph.D. degree in electrical engineering from the California Institute of Technology (Caltech), Pasadena, CA, USA, in 1998.

While completing his D.E.A. thesis, he worked with the optical submarine systems research group of the French national center of telecommunications (CNET-Paris B), on the development of future transatlantic optical links. While at Georgia Tech, he conducted research in the area of K_a-band satellite channel characterization and modeling. From June 1998 to August 1998, he was a Post-Doctoral Fellow with the Communications group at Caltech carrying out research on adaptive modulation techniques and on CDMA mobile communications. He joined the department of Electrical and Computer Engineering of the University of Minnesota, Minneapolis, in September 1998, where his current research interests include statistical modeling of multipath fading channels, adaptive modulation techniques, diversity systems, and digital communication over fading channels.

Dr. Alouini has published several papers on the above subjects and he is co-author of the recent Wiley Interscience textbook *Digital Communication over Fading Channels.* He is a recipient of a National Semiconductor Graduate Fellowship Award, the Charles Wilts Prize for outstanding independent research leading to a Ph.D. degree in electrical engineering at Caltech, and co-recipient of the 1999 Prize Paper Award of the IEEE Vehicular Technology Conference (VTC'99-Fall), Amsterdam, The Netherlands, for his work on the performance evaluation of diversity systems. He was awarded a 1999 CAREER Award from the National Science Foundation and a McKnight Land-Grant Professorship by the Board of Regents of the University of Minnesota in 2001. He is an editor for the IEEE Transactions on Communications (Modulation & Diversity Systems) and for the Wiley Journal on Wireless Systems and Mobile Computing.

Ali Abdi (S'98) is a Ph.D. candidate in the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN. His previous research interests have included stochastic processes, wireless communications, pattern recognition, neural networks, and time series analysis. His current work is mainly focused on different aspects of wireless communications, with special emphasis on channel modeling and estimation, antenna arrays, and system performance evaluation.

Mostafa Kaveh (F'88) received his B.S. and PhD degrees from Purdue University in 1969 and 1974, respectively, and his M.S. degree from the University of California at Berkeley in 1970. He has been at the University of Minnesota since 1975, where he is a professor and, since 1990, the Head of the Department of Electrical and Computer Engineering. He was a design engineer at Scala Radio Corp., San Leadndro, CA, 1970, and has consulted for industry, including the MIT Lincoln Laboratory, 3M, and Honeywell.

Dr. Kaveh has been professionally active in the Signal Processing Society of the Institute for Electrical and Electronic Engineers (IEEE), which he has served as the Vice President for Publications, a member of the Board of Governors, and the General Chair of ICASSP93. He is a Fellow of IEEE, was the recipient (with A. Barabell) of a 1986 ASSP Senior (best paper) Award, the 1988 ASSP Meritorious Service Award, an IEEE Third Millennium Medal in 2000, and the 2000 Society Award from the IEEE Signal Processing Society.

FIGURES CAPTIONS

1. Fig. 1: Comparison between exact and approximate PDFs for constant correlation with $L = 5$, $\overline{\gamma} = 1$, $m = 2.5$, and constant power correlation $\rho = 0.64$.
2. Fig. 2: Comparison between PDFs obtained analytically and by Monte-Carlo simulations for circular correlation with $L = 5$, $\overline{\gamma} = 1$, $m = 2$, and power correlation coefficients $\rho_2 = \rho_{12} = \rho_{15} = 0.8$ and $\rho_3 = \rho_{13} = \rho_{14} = 0.6$.
3. Fig. 3: Comparison between exact and approximate PDFs for circular correlation with $L = 5$, $\overline{\gamma} = 1$, $m = 2.7$, and power correlation coefficients $\rho_2 = \rho_{12} = \rho_{15} = 0.64$ and $\rho_3 = \rho_{13} = \rho_{14} = 0.36$.

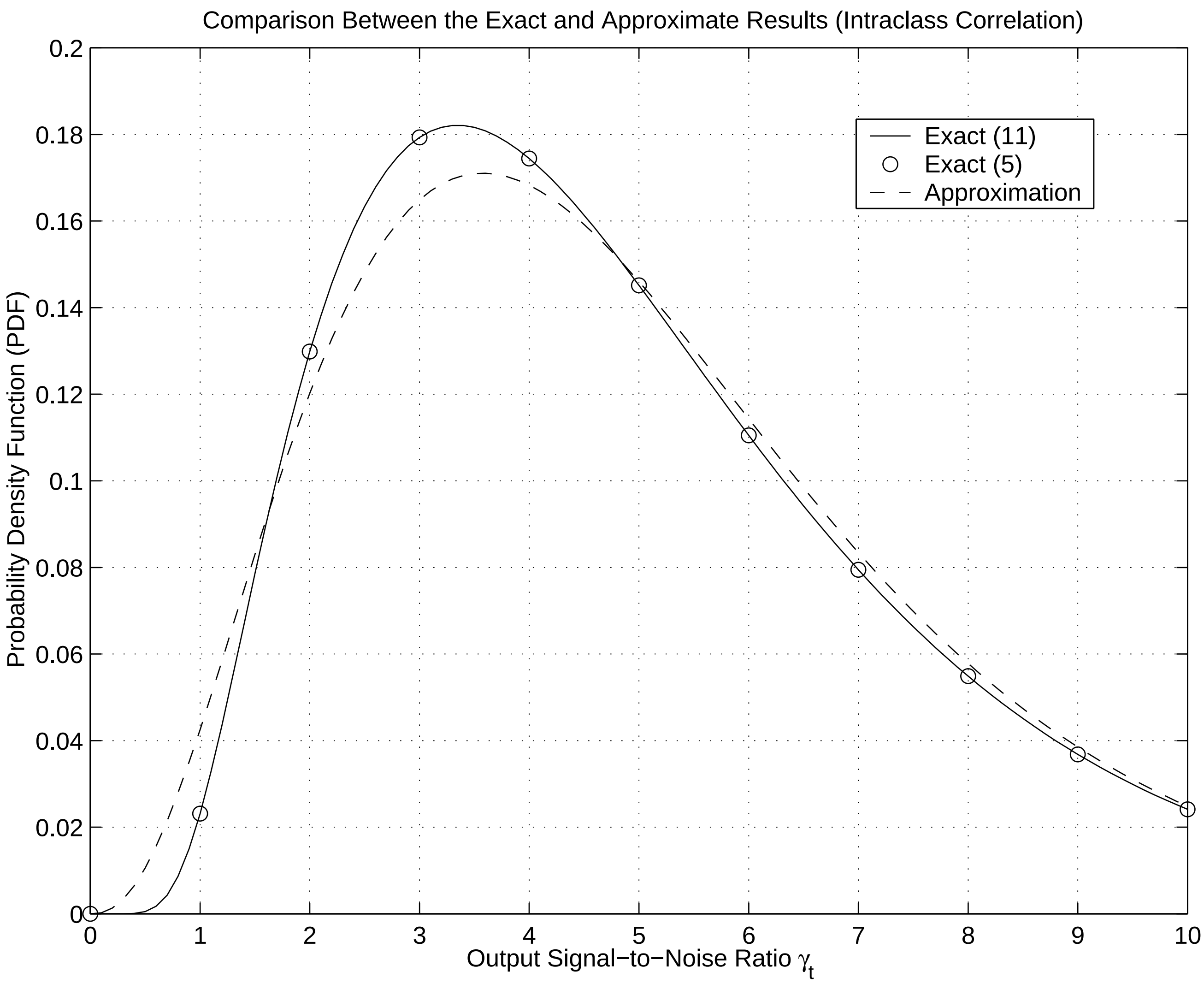
Comparison Between the Exact and Approximate Results (Intraclass Correlation)
Exact (11)
Exact (5)
Approximation
Probability Density Function (PDF)
Output Signal−to−Noise Ratio γ_t
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0 1 2 3 4 5 6 7 8 9 10

Fig. 1.

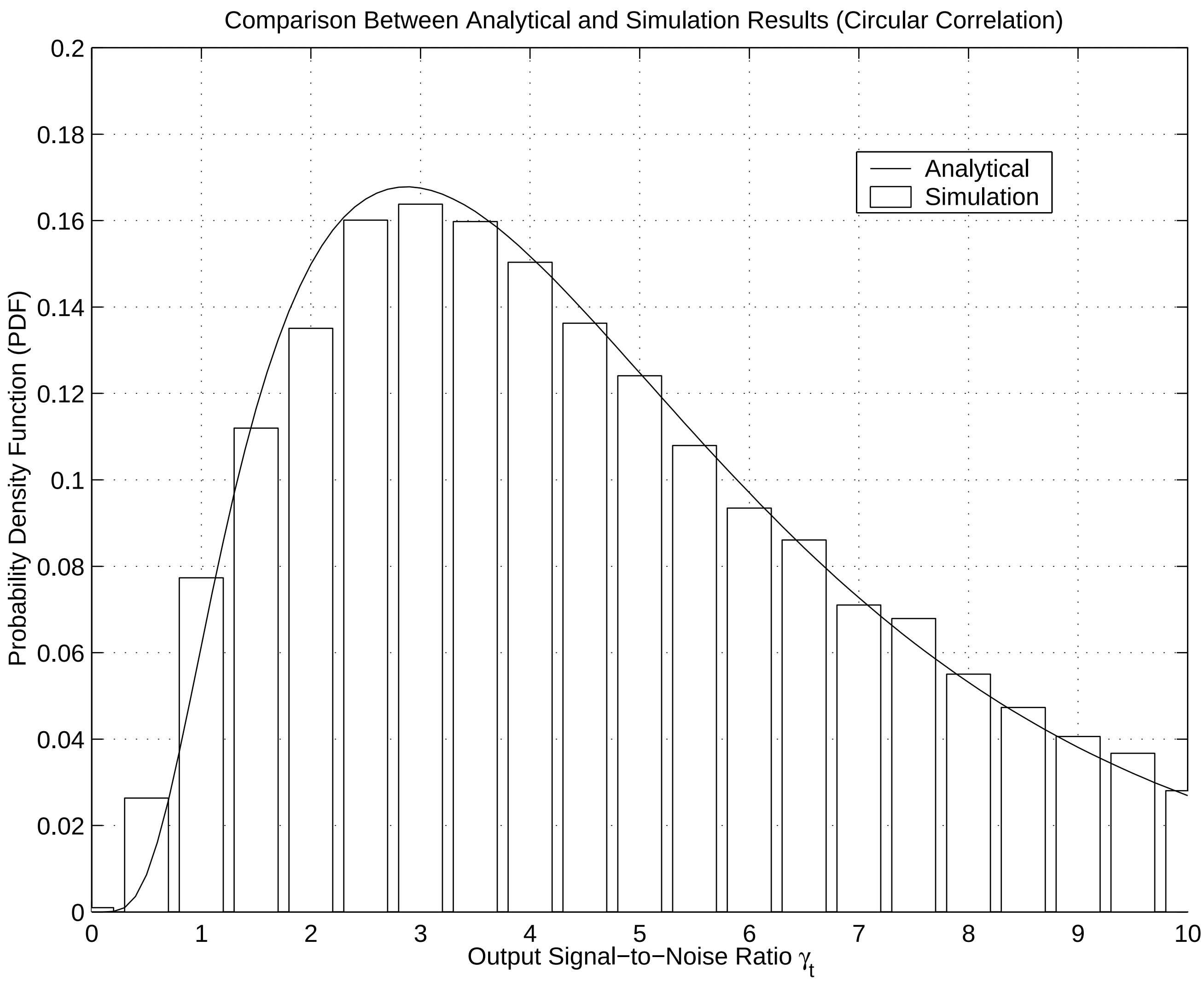
Comparison Between Analytical and Simulation Results (Circular Correlation)
Analytical
Simulation
Probability Density Function (PDF)
Output Signal−to−Noise Ratio γt
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
1
2
3
4
5
6
7
8
9
10

Fig. 2.

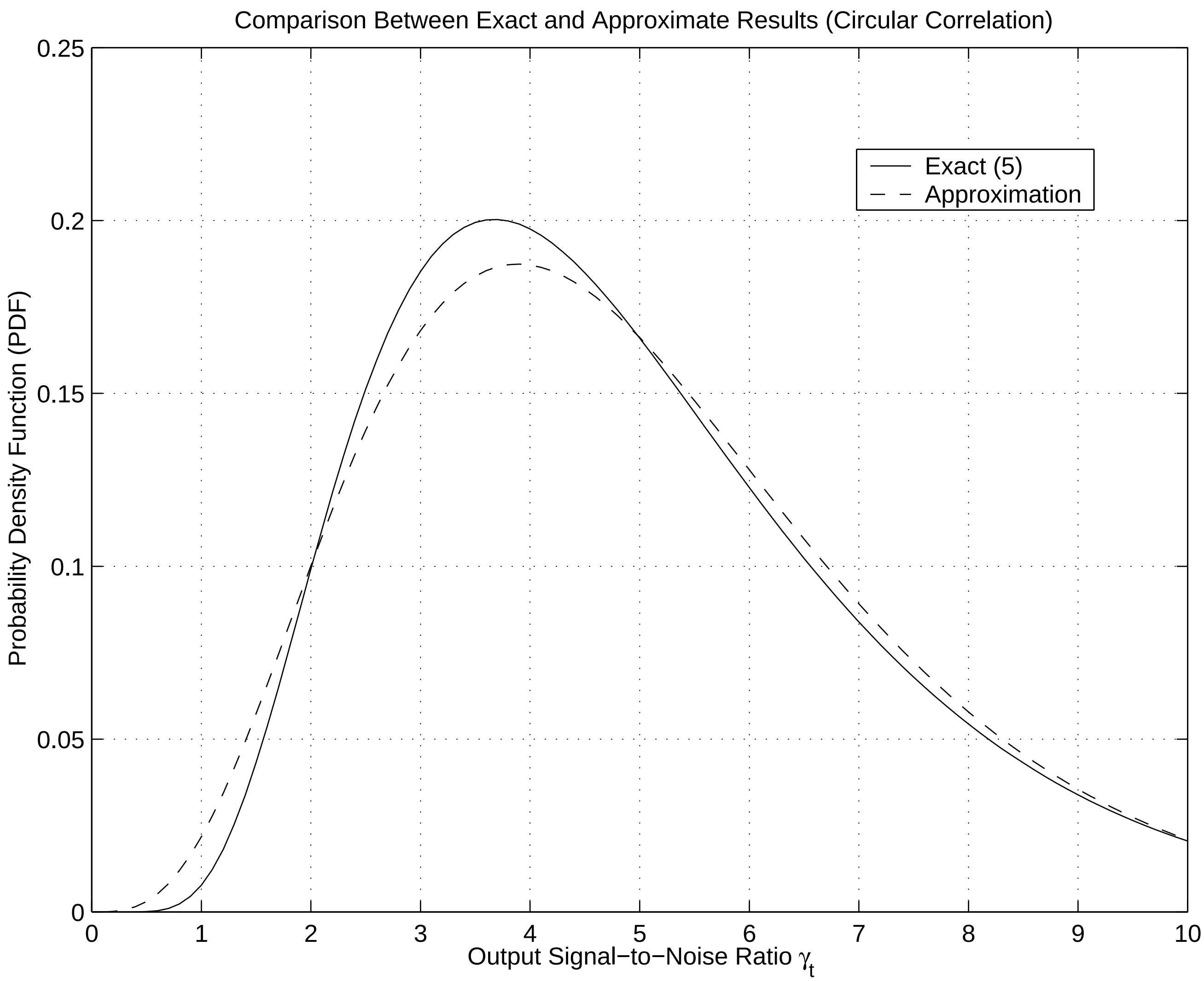
Comparison Between Exact and Approximate Results (Circular Correlation)
Exact (5)
Approximation
Probability Density Function (PDF)
Output Signal−to−Noise Ratio γt
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5 6 7 8 9 10

Fig. 3.