A Subspace Method for Blind Channel Identification in OFDM systems

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Abstract—**It has been shown that cyclostationarity in the received signal allows the receiver blindly identify the channel impulse response using only second-order statistics. In orthogonal frequency-division multiplexing (OFDM) systems, cyclostationarity is embedded at the transmitter due to cyclic prefix. In this paper, a subspace approach based on second-order statistics is proposed for blind channel identification in OFDM systems. We derive a sufficient condition that guarantees all the channels to be identifiable no matter what their zero locations are. Computer simulations demonstrate the superior performance of the proposed algorithm over methods reported earlier in the literature.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) has been proposed for different high-bit-rate data transmission systems including digital radio/TV broadcasting systems [1] [8], High-bit-rate Digital Subscriber Loop (HDSL) and Asymmetric Digital Subscriber Loop (ADSL) [2]. In OFDM, the entire channel spectrum is divided into many narrow band subchannels. Data are transmitted parallelly in subchannels. If the number of subchannels is large enough, the symbol duration is much larger than the length of the channel impulse response. Therefore, the intersymbol interference (ISI) can be greatly reduced. In a practical OFDM system, a cyclic prefix is inserted before each transmitted data block. If the length of the cyclic prefix is longer than the length of the channel impulse response, ISI is completely eliminated. However, the length of the channel impulse response is typically not under the control of the designer. It may be longer than the length of the cyclic prefix. To alleviate this problem, a shortened impulse response filter (SIRF) [6] is placed in the receiver to shorten the impulse response of the effective channel. Channel estimation is required for obtaining the optimal SIRF [6].

Traditionally, channel estimation is achieved by sending training sequences through the channel. However, when the channel is varying, even slowly, the training sequence needs to be sent periodically in order to update the channel estimates. Hence, the transmission efficiency is reduced. The increasing demand for high-bit-rate digital mobile communications makes *blind* channel identification and equalization very attractive, since they do not require the transmission of a training sequence. Early methods for blind channel equalization exploit higher-order statistics(HOS) of the outputs that are sampled at symbol rate . These methods require many observations, and have relative slow speed of convergence. More recently, Tong, Xu and Kailath [9] [10] showed that the second-order statistics of the channel output contains sufficient information to estimate most communication channels if it is fractionally sampled. When the sampling rate is higher than the baud rate, the resulting output sequence is wide sense *cyclostationary*. The output second-order statistics contains the phase information of the channel due to the cyclostationarity. Most nonminmum-phase channels can be identified from the second-order statistics of the cyclostationary output sequence. Since this breakthrough by Tong, Xu and Kailath [9], many elegant solutions such as subspace method in [7] and deterministic approach in [13] have been proposed for blind channel identification.

In [4], Giannakis uses a precoder to induce cyclostationarity at the transmitter that guarantees blind identifiability of channels with minimal degradation of information rate. The OFDM with cyclic prefix is a special case of such a precoding scheme [3]. Heath and Giannakis [5] proposed a subspace method using cyclic correlation of the channel output to blindly estimate the channel in OFDM systems. But the estimated channel error in that study is large.

In this paper, a subspace approach based on second-order statistics is proposed to blindly identify the channel in OFDM systems. The channel identifiability is due to the cyclostationarity inherent in the OFDM systems with cyclic prefix. We derive a sufficient condition that guarantees all the channels to be identifiable no matter what their zero locations are. The difference between the proposed algorithm in this paper and the method in [4] is as follows. The approach in [4] uses the cyclic correlation that is defined as the Fourier series expansion of the time-varying correlation of the received data samples. In this paper, we use the time-invariant autocorrlation of the vector that consists of N blocks of the received data. Computer simulations show that the estimated channel error is much smaller than the one reported in [5].

The paper is organized as follows. Section 2 is an overview of OFDM systems. The problem of blind channel identification and the proposed scheme using subspace

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method is presented in section 3. In section 4, we present performance simulations for the proposed algorithms.

II. OFDM Systems

In OFDM systems, the serial data are converted into M parallel streams. Each parallel data stream modulates a different carrier. The frequency separation between the adjacent carriers is $1/T$, where T is the symbol duration for the parallel data that is M times of the symbol duration for the serial data. Let us consider an OFDM signal in the interval $[nT, (n+1)T)$ as

$$s(t) = \sum_{m=0}^{M-1} a_m(n) e^{j\omega_m t}, \tag{1}$$

where $a_m(n)$ are symbols from a constellation such as 16-QAM, ω_m is the frequency of mth carrier that is $m\frac{2\pi}{T}$. The M samples that are sampled at $t = nT + i\frac{1}{T}$, $i = 0, 1, \ldots, M-1$, are given as

$$s(nM+i) = \sum_{m=0}^{M-1} a_m(n) e^{j\frac{2\pi}{M}mi}. \tag{2}$$

It is seen from (2) that the M samples are exactly inverse discrete Fourier transform (IDFT) of a block for M input symbols.

Theoretically speaking, when the number of carriers is large enough, symbol duration T is much larger than the duration of FIR channel; ISI is negligible. However, for the high-bit-rate communications, it is impractical to choose very large M to make ISI negligible. Therefore, a cyclic prefix of length P is added into each block of IDFT output at the transmitter. The length of the prefix is chosen to be longer than the length of the channel impulse response in order to avoid inter-block interference (IBI). That results with total cancellation of ISI and interchannel interference (ICI). The transmitted sequence is expressed as

$$\begin{aligned} s(n(M+P)+i) &= \sum_{m=0}^{M-1} a_m(n) e^{j\frac{2\pi}{M}(i-P)} \\ & \quad i = 0, 1, \ldots, M+P-1, \end{aligned} \tag{3}$$

where $s(n(M+P)+i), i = 0, \ldots, P-1$, is the cyclic prefix.

The received signal $r(n)$ is distorted by the frequency-selective channel and degraded by additive white Gaussian noise (AWGN). It is assumed that the length L of the channel impulse response is known. Assuming that blocks are synchronized and carrier frequency offset is corrected [12], the receiver removes the first P symbols corresponding to the cyclic prefix and performs an M-point DFT on the remaining samples of received signal to obtain $y_i(n), i = 0, \ldots, M-1$. If the cyclic prefix duration is equal or more than the channel duration, i.e. $P \geq L$, it is shown that[14]

$$y_i(n) = a_i(n) H(\frac{2\pi}{M} i) + v_i(n), \tag{4}$$

where $H(\cdot)$ is the frequency response of the channel. It is evident from (4) that the ISI is completely cancelled and the effect of the channel at the receiver is merely a complex gain and AWGN.

In the next section, we develop an algorithm for blind channel estimation that does not require the channel duration to be shorter than that of the cyclic prefix. When the channel duration exceeds the cyclic prefix duration, impulse response shortening [6] can be applied to cancel ISI.

III. Blind Channel Identification

We begin with a data model for a baseband discrete-time OFDM system. Let us denote vector $\mathbf{a}(n) = [a_0(n), a_1(n), \ldots, a_{M-1}(n)]^T$ as the nth block of data and vector $\mathbf{s}(n) = [s_{K-1}(n), s_{K-2}(n), \ldots, s_0(n)]^T$ as a sequence of the nth block of the IDFT output and embedded cyclic prefix, where $K = M + P$ and P is the length of cyclic prefix. Let us define $W \triangleq e^{j\frac{2\pi}{M}}$, and the matrix $[\mathbf{W}]_{ij} \triangleq W^{(M-1-i)j}, i = 0, 1, \ldots, K-1, j = 0, 1, \ldots, M-1$, then the nth transmitted data sequence in (3) can be written as

$$\mathbf{s}(n) = \mathbf{W}\mathbf{a}(n). \tag{5}$$

For a later use, we partition matrix $\mathbf{W}$ as $\mathbf{W} = [\mathbf{W}_1^T \quad \mathbf{W}_2^T]^T$, where size $M \times M$ matrix $\mathbf{W}_1$ can be obtained by flipping the IDFT matrix up down and size $P \times M$ matrix $\mathbf{W}_2$ consists of first P rows of matrix $\mathbf{W}_1$.

Now, consider N blocks of data, $\mathbf{a} = [\mathbf{a}(n)^T, \mathbf{a}(n-1)^T, \ldots, \mathbf{a}(n-N+1)^T]^T$ and $\mathbf{s} = [\mathbf{s}(n)^T, \mathbf{s}(n-1)^T, \ldots, \mathbf{s}(n-N+1)^T]^T$. Let $\tilde{\mathbf{W}} = \mathbf{I_N} \otimes \mathbf{W}$, where $\mathbf{I}_N$ is an $N \times N$ identity matrix and $\otimes$ is the Kronecker product. From (5), we have $\mathbf{s} = \tilde{\mathbf{W}}\mathbf{a}$. Therefore, the received signal is expressed as

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{b} = \mathbf{H}\tilde{\mathbf{W}}\mathbf{a} + \mathbf{b}. \tag{6}$$

where $\mathbf{r}$ is an $(NK-L) \times 1$ vector constructed as follows. Let the nth block of received signal be denoted as $\mathbf{r}(n) = [r_{K-1}(n), r_{K-2}(n), \ldots, r_0(n)]^T$, then $\mathbf{r} = [\mathbf{r}(n)^T, \mathbf{r}(n-1)^T, \ldots, \mathbf{r}(n-N+2)^T, \mathbf{r}(n-N+1)(1:K-L)^T]^T$, where $\mathbf{r}(n-N+1)(1:K-L)$ is a Matlab notation standing for the first $K-L$ elements of $\mathbf{r}(n-N+1)$. $\mathbf{b}$ is a noise vector that is assumed to be zero mean white Gaussian noise with variance matrix $\sigma^2 \mathbf{I}_{NK-L}$ and be mutually independent with the input symbol sequence. $\mathbf{H}$ is an $(NK-L) \times NK$ matrix defined as

$$\mathbf{H} = \begin{bmatrix} h_0 & \ldots & h_L & 0 & \ldots & \ldots & 0 \\ 0 & h_0 & \ldots & h_L & 0 & \ldots & 0 \\ \vdots & & & & & & \vdots \\ 0 & \ldots & \ldots & 0 & h_0 & \ldots & h_L \end{bmatrix}. \tag{7}$$

We define $\mathbf{A} \triangleq \mathbf{H}\tilde{\mathbf{W}}$. Thus, (7) becomes

$$\mathbf{r} = \mathbf{A}\mathbf{a} + \mathbf{b}. \tag{8}$$

Since the matrix $\mathbf{A}$ should be full column rank for the channel to be identified, we give a sufficient condition for full rank requirement as follows.

Theorem 1: If $L \leq PN$, then the matrix $\mathbf{A} = \mathbf{H}\tilde{\mathbf{W}}$ is full column rank.

Proof: $\mathbf{H}$ is a $(KN - L) \times KN$ matrix and $\tilde{\mathbf{W}}$ is of size $KN \times MN$. Hence, $\mathbf{A}$ is of size $(KN - L) \times MN$. If $L \leq PN$, then $\mathbf{A}$ has at least the same number of rows as columns, which is necessary for the matrix $\mathbf{A}$ to be full column rank. We define a unitary $MN \times MN$ matrix $\mathbf{W_3} = \mathbf{I_N} \bigotimes \mathbf{W_1}$, where $\mathbf{W_1}$ is a unitary matrix defined earlier. Then, matrix $\mathbf{AW_3}(= \mathbf{H}\tilde{\mathbf{W}}\mathbf{W_3})$ has the same rank as $\mathbf{A}$. $\tilde{\mathbf{W}}\mathbf{W_3} = \tilde{\mathbf{J}}$ where $\tilde{\mathbf{J}} = \mathbf{I_N} \bigotimes \mathbf{J}$ and $\mathbf{J} = [\mathbf{I_M} \;\; \mathbf{E}^T]^T$, $\mathbf{E} = [\mathbf{I_P} \;\; \mathbf{0_{P\times M-P}}]$. The columns of $\tilde{\mathbf{J}}$ are either $\mathbf{e}_i$ or $\mathbf{e}_i + \mathbf{e}_{i+M}$ where $\mathbf{e}_i$ is the unit vector with a 1 in the ith position and 0's elsewhere. Obviously, The columns of $\tilde{\mathbf{J}}$ are not in the null space of $\mathbf{H}$ as long as $h_0 \neq 0$. Thus, we have $dim(null(A)) = dim(null(\mathbf{H}\tilde{\mathbf{J}}) = 0$. Then, $rank(A) = MN - dim(null(A)) = MN$, i.e. matrix $\mathbf{A}$ is full column rank. □

Remark 1: It is the inserted prefix that makes matrix $\mathbf{A}$ a "tall" matrix and be possible to be full column rank. The full column rank condition can always be satisfied as long as N is chosen to be large enough. Unlike in the multichannel case of[10] in which the transfer function for each subchannel can not have common zeros, any FIR channel here can make $\mathbf{H}$ full column rank no matter what its zero locations are. The condition here and the algorithm developed below is also applied to the case with repeated coding in [11].

As in the methods presented in [7][9] for the multichannel cases, the identification here is based on the $(NK - L) \times (NK - L)$ autocorrelation matrix $\mathbf{R_r}$ of the measurement, where $\mathbf{R_r} = E\{\mathbf{rr}^H\}$. Using (8), $\mathbf{R_r}$ is expressed as

$$\mathbf{R_r} = \mathbf{AR_aA}^H + \sigma^2\mathbf{I}. \tag{9}$$

If matrix $\mathbf{A}$ is full column rank and the autocorrelation of input $\mathbf{R_a}$ is also full rank, then range($\mathbf{A}$)= range($\mathbf{AR_aA}^H$). Let us define the noise subspace for $\mathbf{R_r}$ to be the subspace generated by $PN - L$ eigenvectors corresponding to the smallest eigenvalue, and let $\mathbf{G} = [\mathbf{G}_1, \ldots, \mathbf{G}_{PN-L}]$ be the matrix containing those eigenvectors. Then, $\mathbf{G}$ spans the null space of $\mathbf{AR_aA}^H$ and is orthogonal to its range space

$$\mathbf{G}_i^H\mathbf{A} = 0 \qquad i = 1, \ldots, PN - L, \tag{10}$$

Under the appropriate conditions detailed in the theorem below, the noise subspace determines the channel coefficients $\mathbf{h}$ up to a multiplicative constant.

Theorem 2: Assume that we have $L < \frac{PN}{2}$ for even N and $L < \frac{(N-1)P}{2} - 1$ for odd N. Let $\mathbf{h}'$ be a vector that has the same dimension as $\mathbf{h}$. Let $\mathbf{H}'$ be a nonzero matrix constructed from $\mathbf{h}'$ in the same way as matrix $\mathbf{H}$ constructed from $\mathbf{h}$. Define the matrix $\mathbf{A}' \triangleq \mathbf{H}'\tilde{\mathbf{W}}$. The range of matrix $\mathbf{A}'$ is included in the range of $\mathbf{A}$ iff the corresponding vectors $\mathbf{h}'$ and $\mathbf{h}$ are proportional.

Proof: We define the matrices $\mathbf{B} \triangleq \mathbf{AW_3}$ and $\mathbf{B}' \triangleq \mathbf{A}'\mathbf{W_3}$, where the matrix $\mathbf{W_3}$ is defined in the same way as in the proof of theorem 1. Matrix $\mathbf{B}$ has the same range as matrix $\mathbf{A}$ because $\mathbf{W_3}$ is unitary. Partition matrix $\mathbf{B}$ as

$$\mathbf{B} = \begin{bmatrix} \mathbf{B_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{B_2} & \mathbf{h} & \mathbf{B_3} \\ \mathbf{B_4} & \mathbf{0} & \mathbf{B_5} \end{bmatrix}. \tag{11}$$

The middle column is the $(\lfloor\frac{MN}{2}\rfloor + 1)th$ column, where $\lfloor x \rfloor$ stands for the largest integer less than x. For an even N, $\mathbf{B_1}$ is a $(\frac{KN}{2} - L) \times \frac{MN}{2}$ matrix and $\mathbf{B_5}$ is a $(\frac{KN}{2} - L - 1) \times (\frac{MN}{2} - 1)$ matrix. For an odd N, $\mathbf{B_1}$ is a $\lfloor\frac{KN-P}{2} - L\rfloor \times \lfloor\frac{MN}{2}\rfloor$ matrix and $\mathbf{B_5}$ is a $\lceil\frac{KN+P}{2} - L - 1\rceil \times (\lceil\frac{MN}{2}\rceil - 1)$ matrix, where $\lceil x \rceil$ stands for the smallest integer greater than x. If we take the same column from matrix $\mathbf{B}'$ and since it is in the range of matrix $\mathbf{B}$, we obtain

$$\begin{bmatrix} \mathbf{0} \\ \mathbf{h}' \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{B_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{B_2} & \mathbf{h} & \mathbf{B_3} \\ \mathbf{B_4} & \mathbf{0} & \mathbf{B_5} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha \\ \alpha_2 \end{bmatrix}. \tag{12}$$

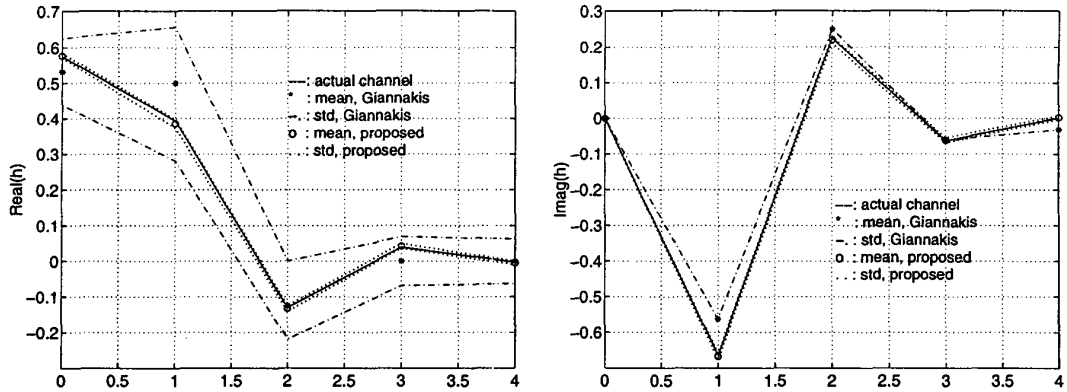
where α_1 and α_2 are vectors and α is a scalar. Using the same procedure in the proof of Theorem 1, we can show that matrices $\mathbf{B_1}$ and $\mathbf{B_5}$ are full column rank as long as the number of rows is equal to or greater than the number of columns. The conditions for both $\mathbf{B_1}$ and $\mathbf{B_5}$ to be full column rank are $L < \frac{PN}{2}$ for even N and $L < \frac{(N-1)P}{2} - 1$ for odd N. Then, both α_1 and α_2 are equal to $\mathbf{0}$ vectors. Therefore, $\mathbf{h}'$ is proportional to $\mathbf{h}$, i.e. $\mathbf{h}' = \alpha\mathbf{h}$. □

Remark 2: The condition given here for the channel to be identifiable is sufficient but not necessary. We can always choose an appropriate N to satisfy the condition. Since in practice only the estimate of $\hat{\mathbf{G}}_k$ is available, we choose to solve (10) in the least square sense. Let us define a matrix $\mathcal{G}_k$ as

$$\mathcal{G}_k = \begin{bmatrix} \mathbf{G}_{k,0} & \ldots & \mathbf{G}_{k,J} & 0 & \ldots & \ldots & 0 \\ 0 & \mathbf{G}_{k,0} & \ldots & \mathbf{G}_{k,J} & 0 & \ldots & 0 \\ \vdots & & & & & & \vdots \\ 0 & \ldots & \ldots & 0 & \mathbf{G}_{k,0} & \ldots & \mathbf{G}_{k,J} \end{bmatrix}, \tag{13}$$

where $J = KN - L - 1$. Then, $\mathbf{G}_k^T\mathbf{H} = \mathbf{h}^T\mathcal{G}_k$. This leads to the following minimization problem

$$\begin{aligned} \hat{\mathbf{h}} &= arg\ min \sum_{k=1}^{PN-L} \hat{\mathbf{G}}_\mathbf{k}^\mathbf{H}\mathbf{H}\tilde{\mathbf{W}}\tilde{\mathbf{W}}^\mathbf{H}\mathbf{H}^\mathbf{H}\hat{\mathbf{G}}_\mathbf{k} \\ &= arg\ min \sum_{k=1}^{PN-L} \mathbf{h}^\mathbf{H}\hat{\mathcal{G}}_\mathbf{k}\tilde{\mathbf{W}}\tilde{\mathbf{W}}^\mathbf{H}\hat{\mathcal{G}}_\mathbf{k}^\mathbf{H}\mathbf{h} \\ &= arg\ min\ \mathbf{h}^\mathbf{H}\Psi\mathbf{h}, \end{aligned} \tag{14}$$



(a) Real part of the channel

(b) Imaginary part of the channel

Fig. 1. Average channel estimates ± standard deviation for M=15, P=4, SNR=20dB

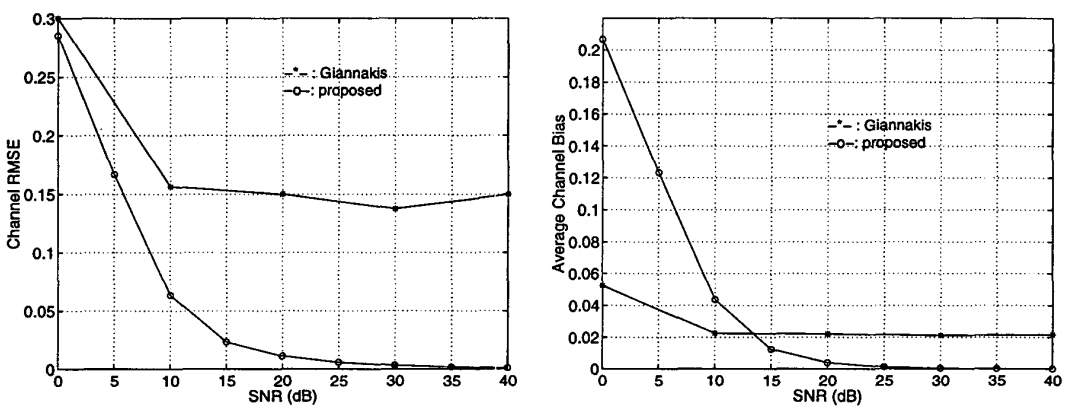


Fig. 2. Channel error versus SNR for M=15, P=4, 120M data

where $\Psi \triangleq \sum_{k=1}^{PN-L} \hat{\mathcal{G}}_{\mathbf{k}} \bar{\mathbf{W}} \bar{\mathbf{W}}^{\mathbf{H}} \hat{\mathcal{G}}_{\mathbf{k}}^{\mathbf{H}}$. Minimization is subject to the constraint $\| \mathbf{h} \| = 1$. Therefore, $\hat{\mathbf{h}}$ is given by the eigenvector corresponding to the smallest eigenvalue of Ψ.

IV. Performance Simulations

In this section, we use simulations to examine the performance of the proposed algorithm. To measure the performance, we define the root-mean-square error (RMSE) as $\frac{1}{\|\mathbf{h}\|}\sqrt{\frac{1}{D(L+1)}\sum_{i=1}^{D} \| \hat{\mathbf{h}}_i - \mathbf{h} \|^2}$ and the channel average bias as $\frac{1}{D(L+1)}\sum_{l=0}^{L} \mid \sum_{i=1}^{D} \hat{h}_i(l) - h(l) \mid$, both averaged over D Monte Carlos to evaluate the channel error. We use the same multipath channel as in [5]. The channel impulse response is $h(t) = e^{-j2\pi(0.15)} r_c(t - T/2, \beta) + 0.8 e^{-j2\pi(0.6)} r_c(t - 1.2T, \beta)$, where $r_c(t)$ is the raised cosine function with roll-off factor $\beta = 0.35$. $h(n)$ is obtained by sampling $h(t)$ at $t = 0, T, \ldots, 4T$. We used $M = 16$ and $16 - QAM$ constellation which are the same as the ones in [5]. The data plotted are averaged over 400 independent runs, i.e. D=400. For comparison, the results in [5], which are denoted as Giannakis, are also displayed. In Fig. 4, with $P = 4$, we show the average channel estimate for $SNR = 20dB$ and 120 blocks of data. It is seen that the estimated channel error of the proposed algorithm is much smaller than that in [5]. The channel error versus SNR is displayed in Fig. 5. Fig. 6 demonstrates that the estimator is consistent. In Fig. 7, for $SNR = 20dB$ and 120 blocks of data, we consider the channel error depending on the size of cyclic prefix when M is fixed. We can observe that the channel is identifiable even when $P \leq L$. It is seen that the performance of the proposed algorithm is better than the one in [5].

V. Conclusions

In this paper, a subspace method is proposed to blindly identify the channel in OFDM systems. The identifiability of a channel is due to the cyclic prefix used in OFDM

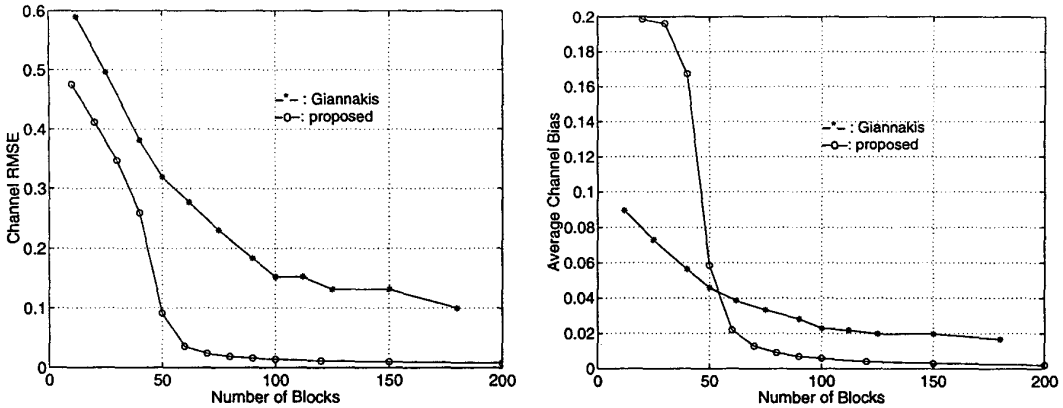


Fig. 3. Channel error versus number of blocks of data for M=15, P=4, SNR=20dB

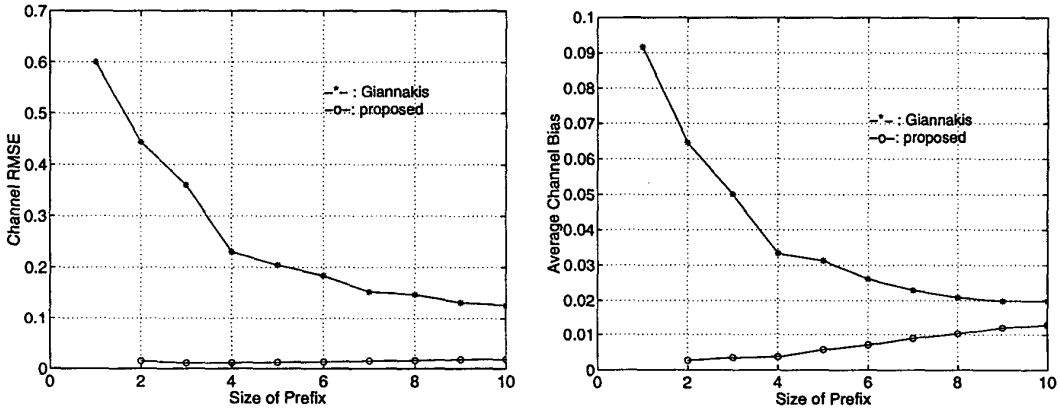


Fig. 4. Channel error versus size of cyclic prefix for M=15, P=4, SNR=20dB, 120 M data

systems. A sufficient condition for the channel to be identifiable is given. This condition is easily satisfied in practical systems. Performance simulations demonstrates the potential of the proposed algorithm.

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