Stability of Predictor-Based Dynamic Bandwidth Allocation over EPONs

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***Abstract*— We establish a system model to analyze the stability of the predictor-based dynamic bandwidth allocation (PDBA) scheme over Ethernet passive optical networks (EPONs). We prove that an EPON system with PDBA is stable by proper pole placement as the traffic changes dynamically. Our analysis suggests a straightforward framework for designing the DBA algorithm that enables EPONs with stability.**

***Index Terms*— Ethernet passive optical networks (EPONs), prediction-based dynamic bandwidth allocation (PDBA), stability.**

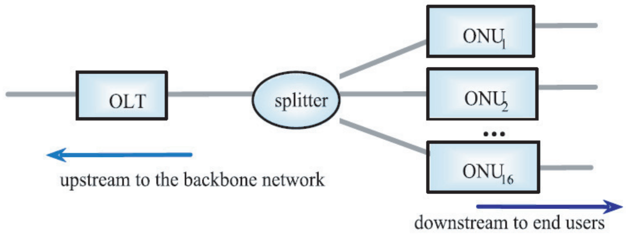

Fig. 1. An Ethernet passive optical network.

I. Introduction

ETHERNET passive optical networks (EPONs) address the first/last mile bottleneck of today's operational network by employing the point-to-multipoint (P2MP) architecture as illustrated in Fig. 1 [1]. In the upstream transmission from multiple optical network units (ONUs) to the associated optical line terminal (OLT), one wavelength is shared among the ONUs, and only one ONU may transmit during a timeslot to avoid data collisions. The upstream channel efficiency is thus determined by the employed bandwidth allocation (BA) scheme. Existing proposals can be categorized into three major types. Static bandwidth allocation (SBA) ignores the bursty nature of network traffic by allocating each ONU a fixed timeslot length. Interleaved Polling with Adaptive Cycle Time (IPACT) [2] and its variations [3]-[5] adopt the REPORT/GATE mechanism for bandwidth negotiation, in which the REPORT message carries the bandwidth request from an ONU, and the GATE message broadcasts the arbitration decision from the OLT. The OLT grants an ONU the number of bytes the ONU requested, not exceeding a maximum timeslot size. In practice, however, each ONU experiences a waiting time, ranging from sending the REPORT message to sending the buffered data, and more data are enqueued during this interval. Predictor-based dynamic bandwidth allocation (PDBA) [6] [7] employs traffic correlation to predict the incoming data of the next time slot.

In [8], we investigated diverse BAs from the viewpoint of EPON system controllability. By "controllable", we mean that, a scheme can meet the dynamic traffic input from multiple ONUs, and steer the bandwidth efficiency over the EPON system from any initial value to the optimum state within a limited time window. We proved that only PDBA is totally controllable, while IPACT and SBA are either partially or totally uncontrollable. In this letter, we further examine the stability characteristics of PDBA, revealing the proper controller that can drive the applied EPON system into the stable state. By "stable", we mean that, when the input traffic load changes dramatically, PDBA is able to provide the upstream bandwidth fair share among the ONUs with optimal bandwidth utilization. Guidelines are also given on designing a stable BA scheme over EPONs. In Section II, we establish a system model to evaluate BAs. We then analyze the stability of PDBA in Section III, and Section IV concludes the letter.

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II. System Model of Bandwidth Allocation

Consider an EPON system with one OLT and y ONUs, the OLT serves each ONU once in a service cycle. Denote $R_i(n)$ as the reported queue length from ONUi $(1 \le i \le y)$ at service cycle $[n-1,\, n]$, $\lambda_i(n)$ as the actually arrived data during the waiting time of ONU at service cycle $[n-1,\, n]$, and $d_i(n)$ as the departed data from ONUi at service cycle n. Then, the queue length at the end of service cycle $(n+1)$ is expressed by

$$R_i(n+1) = R_i(n) + \lambda_i(n) - d_i(n+1). \tag{1}$$

After processing the request, the OLT allocates the timeslot of length $G_i(n)$ to ONUi, and the departed data is expressed by

$$d_i(n) = min\left\{G_i(n), R_i(n-1) + \lambda_i(n-1)\right\}. \tag{2}$$

Denote $G_i^r(n)$ as the bandwidth requirement of ONUi for service cycle n (it may or may not be the same as $R_{i(n)}$, depending on the particular BA scheme), and G_i^{max} as the maximum timeslot length prescribed by the service level agreement (SLA). The granted timeslot is thus represented by

$$G_i(n+1) = min\left\{G_i^r(n+1), G_i^{max}\right\}. \tag{3}$$

When traffic predictor is not employed, $G_i^r(n+1)$ is determined by

$$G_i^r(n+1) = R_i(n). \tag{4a}$$

Otherwise,

$$G_i^r(n+1) = R_i(n) + \hat{\lambda}_i(n), \tag{4b}$$

where $\hat{\lambda}_i(n)$ is the predicted arrival data at ONUi in service cycle $[n-1, n]$. A BA scheme is thus represented by the state space equation as

$$X_i(n+1) = AX_i(n) + BU_i(n), \tag{5}$$

where $X_i(n) = [G_i^r(n)\ R_i(n)]^T$ is the state vector, indicating the bandwidth requirement and the queue length of ONUi, and $U_i(n)$ is the input vector, representing the arrived data during the waiting time and the SLA parameter. Hence, Eq. (5) describes the upstream bandwidth allocation over an EPON system. From Eq. (1) we know that $R_i(n)$, the queue length, is determined by $d_i(n)$. By combining Eqs. (2) and (3), we have

$$d_i(n) = min\left\{G_i^r(n),\ R_i(n-1) + \lambda_i(n-1),\ G_i^{max}\right\} \quad . \tag{6}$$

Depending on the loaded traffic from end users, the EPON system falls into one of the following three scenarios,

1) $d_i(n) = G_i^r(n)$,
2) $d_i(n) = R_i(n-1) + \lambda_i(n-1)$, and
3) $d_i(n) = G_i^{max}$.

Once $d_i(n)$ is settled, the system model in Eq. (5) is determined, exhibiting the relationships among the queue length, bandwidth requirement, and output bandwidth allocation decision. An individual BA scheme designs its own A and B in Eq. (5) to arbitrate the upstream bandwidth in a different way.

The open plant denoted by Eq. (5) usually implies an unbounded output. Since PDBA is fully controllable, there always exists a controller, *i.e.*,

$$U_i(n) = -K_iX_i(n) + F_iR_i(n), \tag{7}$$

to drive the system into the stable state, which is known as *pole placement* [9]. K_i is a constant matrix, F_i is an arbitrary matrix, and $R_i(n)$ is a reference vector. Since F_i and $R_i(n)$ have no impact on the system's stability [9], we will focus on K_i which dominates the system stability. We then replace (7) by

$$U_i(n) = -K_iX_i(n). \tag{8}$$

Substituting Eq. (8) into Eq. (5) yields,

$$X_i(n+1) = (A - BK_i)X_i(n). \tag{9}$$

Therefore, by implementing the controller of Eq. (9), $(A,\ B)$ is changed into $(A - BK_i,\ 0)$. The controllability of a BA scheme is unaltered by state feedback [9]. That is, if $(A,\ B)$ is controllable, so is $(A - BK_i,\ 0)$ for any K_i. In the following sections, we will further investigate the conditions of K_i to ensure the EPON system stability.

III. Stability of Predictor-based DBA (PDBA)

The traffic forecast in PDBA can be generalized as

$$\hat{\lambda}_i(n) = \alpha_i\lambda_i(n-1), \tag{10}$$

where α_i is the estimation index to extract the correlation from the history traffic. α_i indicates the impact of the input history data on the output prediction, and is a positive real number [7][10].

A. Scenario 1: $d_i(n) = G_i^r(n)$

In this scenario, the bandwidth requirement is no larger than the SLA specification, and the granted timeslot is no larger than the arrived data in a service interval. This is the case when the end users are well behaved within the SLA specification. We have $G_i(n+1) = G_i^r(n+1)$, and $d_i(n) = G_i^r(n)$. The state space is thus

$$G_i^r(n+1) = R_i(n) + \hat{\lambda}_i(n) = R_i(n) + \alpha_i\lambda_i(n-1), \tag{11a}$$

$$R_i(n+1) = R_i(n) + \lambda_i(n) - G_i^r(n+1), \tag{11b}$$

Substituting Eq.(11a) into Eq.(11b), we obtain,

$$R_i(n+1) = \lambda_i(n) - \alpha_i\lambda_i(n-1), \tag{11c}$$

Eq.(11a) and Eq.(11c) are discrete linear system represented by

$$\begin{bmatrix} G_i^r(n+1) \\ R_i(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} G_i^r(n) \\ R_i(n) \end{bmatrix} + \begin{bmatrix} 0 & \alpha_i \\ 1 & -\alpha_i \end{bmatrix} \begin{bmatrix} \lambda_i(n) \\ \lambda_i(n-1) \end{bmatrix} \tag{12}$$

Let $A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 & \alpha_i \\ 1 & -\alpha_i \end{bmatrix}$, and $U_i(n) = \begin{bmatrix} \lambda_i(n) & \lambda_i(n-1) \end{bmatrix}^T$; we have

$$X_i(n+1) = A_1X_i(n) + B_1U_i(n), \tag{13}$$

Theorem 3.1 In scenario 1, an EPON system with PDBA is stable when implementing the controller

$$U_i(n) = -K_1X_i(n), \tag{14}$$

where $K_1 = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$ **with**

$$\begin{cases} \alpha_ik_{12}k_{21} - \alpha_ik_{11}k_{22} - \alpha_ik_{22} + k_{11} + k_{12} + 1 > 0 \\ \alpha_ik_{12}k_{21} - \alpha_ik_{11}k_{22} - 2\alpha_ik_{21} + \alpha_ik_{22} + \\ k_{11} - k_{12} + 1 > 0 \\ -1 < \alpha_ik_{12}k_{21} - \alpha_ik_{11}k_{22} + k_{11} - \alpha_ik_{21} < 1 \end{cases} .$$

Proof: When implementing controller $U_i(n)$, the system becomes $X_i(n+1) = (A_1 - B_1K_1)X_i(n)$. This discrete system is stable *iff* eigenvalues of the state matrix $(A_1 - B_1K_1)$ fall inside the unit circle [9]. Let $|ZI - (A_1 - B_1K_1)|$; we have

$$\begin{aligned} D(z) &= det[zI - (A_1 - B_1K_1)] \\ &= z^2 + (\alpha_ik_{21} + k_{12} - \alpha_ik_{22})z + \\ &\quad \alpha_ik_{12}k_{21} - \alpha_ik_{11}k_{22} + k_{11} - \alpha_ik_{21}. \end{aligned} \tag{15}$$

Assume $L = \alpha_ik_{21} + k_{12} - \alpha_ik_{22}$, and $M = \alpha_ik_{12}k_{21} - \alpha_ik_{11}k_{22} + k_{11} - \alpha_ik_{21}$, by applying the Jury's criterion [11], this second order system is stable *iff* the following rules are all fulfilled:

Rule 1: $D(1) = 1 + L + M > 0$, *i.e.*, $M + L + 1 > 0$;

Rule 2: $(-1)^2D(-1) = 1 - L + M > 0$, *i.e.*, $M - L + 1 > 0$;

Rule 3: $|M| < 1$, *i.e.*, $-1 < M < 1$.

From rules 1∼ 3, the necessary and sufficient conditions for Eq.(13) to be stable are

$$\begin{cases} M+L+1>0 \\ M-L+1>0 \\ -1<M<1 \end{cases}, i.e.,$$

$$\begin{cases} \alpha_i k_{12}k_{21} - \alpha_i k_{11}k_{22} - \alpha_i k_{22} + k_{11} + k_{12} + 1 > 0 \\ \alpha_i k_{12}k_{21} - \alpha_i k_{11}k_{22} - 2\alpha_i k_{21} + \alpha_i k_{22} + \\ k_{11} - k_{12} + 1 > 0 \\ -1 < \alpha_i k_{12}k_{21} - \alpha_i k_{11}k_{22} + k_{11} - \alpha_i k_{21} < 1 \end{cases}.$$

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B. Scenario 2: $d_i(n) = R_i(n-1) + \lambda_i(n-1)$

In this scenario, the granted timeslot is larger than the bandwidth requirement, i.e., $G_i(n) > R_i(n-1) + \lambda_i(n-1)$. This "over-grant" is adjusted by reporting the difference between the granted timeslot and bandwidth requirement. To facilitate this mechanism, we use "negative" queue length to measure the "over-grant". Hence we will have

$$R_i(n+1) = R_i(n) + \lambda_i(n) - G_i^r(n+1). \quad (16a)$$

Note that the "negative" REPORT indicates "over-grant" with empty queue and thus the bandwidth requirement only contains the estimated arriving data, i.e.,

$$G_i^r(n+1) = \alpha_i \lambda_i(n-1), \quad (16b)$$

From Eqs.(16a) and (16b), we obtain

$$R_i(n+1) = R_i(n) + \lambda_i(n) - \alpha_i \lambda_i(n-1). \quad (16c)$$

Eqs.(16b) and (16c) give the discrete linear system

$$X_i(n+1) = A_2 X_i(n) + B_2 U_i(n), \quad (17)$$

where $A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 & \alpha_i \\ 1 & -\alpha_i \end{bmatrix}$, and $U_i(n) = \begin{bmatrix} \lambda_i(n) & \lambda_i(n-1) \end{bmatrix}^T$. Similar to the previous scenario, the following theorem is established.

Theorem 3.2 In scenario 2, an EPON system with PDBA is stable when implementing the controller

$$U_i(n) = -K_2 X_i(n), \quad (18)$$

where $K_2 = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$ **with**

$$\begin{cases} \alpha_i p_{12}p_{21} - \alpha_i p_{11}p_{22} - \alpha_i p_{22} + p_{12} > 0 \\ \alpha_i p_{12}p_{21} - \alpha_i p_{11}p_{22} - 2\alpha_i p_{21} + \alpha_i p_{22} - p_{12} + 2 > 0 \\ -1 < \alpha_i p_{12}p_{21} - \alpha_i p_{11}p_{22} - \alpha_i p_{21} < 1 \end{cases}$$

C. Scenario 3: $d_i(n) = G_i^{max}$

In this scenario, the incoming traffic is heavy, and the OLT uses the SLA upper bound G_i^{max} to limit the aggressive bandwidth requirement. The state space turns into

$$G_i^r(n+1) = R_i(n) + \alpha_i \lambda_i(n-1), \quad (19a)$$

$$R_i(n+1) = R_i(n) + \lambda_i(n) - G_i^{max}. \quad (19b)$$

The discrete linear system is described by

$$X_i(n+1) = A_3 X_i(n) + B_3 U_i(n), \quad (20)$$

where $A_3 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, $B_3 = \begin{bmatrix} 0 & \alpha_i \\ 1 & 0 \end{bmatrix}$, and $U_i(n) = \begin{bmatrix} \lambda_i(n) - G_i^{max} & \lambda_i(n-1) \end{bmatrix}^T$.

Tehorem 3.3 In scenario 3, an EPON system with PDBA is stable when implementing the controller

$$U_i(n) = -K_3 X_i(n), \quad (21)$$

where $K_3 = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}$ **with**

$$\begin{cases} \alpha_i q_{12}q_{21} - \alpha_i q_{11}q_{22} + q_{11} + q_{12} > 0 \\ \alpha_i q_{12}q_{21} - 2\alpha_i q_{21} - \alpha_i q_{11}q_{22} + q_{11} - q_{12} + 2 > 0 \\ -1 < \alpha_i q_{12}q_{21} - \alpha_i q_{21} - \alpha_i q_{11}q_{22} + q_{11} < 1 \end{cases}$$

Therefore, the controller can be adapted to scenarios as illustrated by the design guideline shown in Theorems 3.1-3.3. The on-line traffic dynamics imply changes of the queue length and bandwidth requirement of an ONU. The OLT thus works as a central controller to tune ONUs accordingly, ensuring that the upstream bandwidth of an EPON system is fairly shared by multiple ONUs. K_1, K_2, and K_3 describe the controller characteristics in different scenarios, and their relationship to the estimation index has been revealed in Theorems 3.1-3.3.

IV. Conclusion

In this letter, we have analyzed and verified through the state space model that the implementation of PDBA with suitable controllers maintains the EPON system stability. The employed traffic predictor is robust to dynamic traffic load, and the bandwidth utilization can be improved by adaptive control at the OLT side. Our further research will focus on optimal controller design and the proper scheduling scheme among ONUs for PDBA.

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