**Proceedings of the ASME 2010 Dynamic Systems and Control Conference**
**DSCC2010**
**September 12-15, 2010, Cambridge, Massachusetts, USA**

**DSCC2010-4006**

# CONTROL OF RECURRENT NEURAL NETWORKS USING DIFFERENTIAL MINIMAX GAME: THE STOCHASTIC CASE

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## ABSTRACT

*As a continuation of our study, this paper extends our research results of optimality-oriented stabilization from deterministic recurrent neural networks to stochastic recurrent neural networks, and presents a new approach to achieve optimally stochastic input-to-state stabilization in probability for stochastic recurrent neural networks driven by noise of unknown covariance. This approach is developed by using stochastic differential minimax game, Hamilton-Jacobi-Isaacs (HJI) equation, inverse optimality, and Lyapunov technique. A numerical example is given to demonstrate the effectiveness of the proposed approach.*

## INTRODUCTION

The past two decades have witnessed enormous advances in engineering and in computer science to build artificial computational systems [1], among which recurrent neural networks have been applied to many scientific and engineering fields, such as system identification and control, pattern recognition, image processing, and modeling biological sensor-motor systems. Therefore, theoretical studies on both stability and controllability of recurrent neural networks have been heavily investigated in the last few years [2] - [10]. However, these studies primarily focused on deterministic recurrent neural networks. In the mathematical models of these aforementioned networks, they do not consider the noise process that is fraught with signal transmission particularly in biological systems.

On the other hand, Werbos [1] pointed out that in order to develop mathematical neural network specifications which have dual uses as models of intelligence in the brain, and as highly effective artificial intelligent systems when implemented in computers and chips, we must consider the stochastic environment. Unfortunately, with regard to the analysis of stochastic recurrent neural networks, there has been little work in the literature until the very recent years [11]. Hence, it is important to analytically explore the characteristics of stabilization and controllability for recurrent neural networks under the influence of stochastic perturbation.

As a continuation of our study in [12], we present in this paper a theoretical analysis for stochastic recurrent neural networks to achieve stochastic input-to-state stabilization in probability under an optimal control strategy, and to attenuate incremental covariance of stochastic perturbation to a predefined level within stability margins. By applying the theory of differential minimax game to the stochastic networks, the approach is developed by considering the vector of external inputs as a player and the vector of stochastic disturbance as the opposing player. Therefore, a minimax equilibrium can be achieved by properly controlling stochastic recurrent neural networks. It should be pointed out that this paper develops a stochastic counterpart of the disturbance attenuation results of those in [12].

## PROBLEM FORMULATION

Based on the standard formulation of stochastic recurrent neural networks [13], we consider the following stochastic recurrent neural network, which is derived from the model of deterministic recurrent neural networks defined in [12] plus an additive white noise. Mathematically, it can be described by the following Ito-type compact form

$$dx = (-Ax + W_1 S(x) + W_2 u)dt + d\Psi \quad (1)$$

 

where $x \in R^n$ is the state of stochastic recurrent neural network, $u \in R^m$ is the input, usually $m \neq n$, $A = diag(-\lambda, \cdots, -\lambda) = -\lambda I \in R^{n \times n}$ and $\lambda > 0$, $S(x) = [s(x_1), \cdots, s(x_n)]^T \in R^n$ is a vector function and its component $s(x_i)$ is a sigmoidal function defined below, $W_1 \in R^{n \times n}$, $W_2 \in R^{n \times m}$ are weight matrices describing the connections of hidden and output layers, and $\Psi$ is an $n$-dimensional independent Wiener process with incremental covariance $\sum(t)\sum(t)^T dt$, i.e., $E\{d\Psi d\Psi^T\} = \sum(t)\sum(t)^T dt$ where $\sum(t)$ is an unknown bounded function taking values in the set of nonnegative definite matrices.

We shall first introduce the following two definitions.

*Definition 1*: Let us define a nonnegative bounded function as follows:

$$\Delta(t) = \sum(t)\sum(t)^T \in R^{n \times n}$$

The function $\Delta(t)$ will be used as a player to oppose the control signal in order to solve a stochastic differential game problem addressed in this paper.

*Definition 2*: The function of $s(x_i)$ possesses the following properties:

(1) $s(x_i)$ is bounded on $R$;

(2) $s(x_i)$ is piecewise analytic and strictly increasing on $R$, *i.e.*, $0 < \dfrac{ds(x_i)}{dx_i} < M_i$ and $M_i < \infty$ for all $x_i \in R$;

(3) $s(x_i) = 0$ when $x_i = 0$.

*Remark*: Based on *Definition 2*, it is important to point out that Model (1) is significantly different from most models reported in the literature. The activation function $s(x_i)$ in this paper represents a class of general nonlinear function that does not have to be the widely used sigmoid function $s(x_i) = a/(1 + e^{-bx_i}) + c$. Therefore, Model (1) encompasses a much larger class of systems.

## DESIGN OF OPTIMALLY STOCHASTIC INPUT-TO-STATE STABILIZATION

We first rewrite the system of the stochastic recurrent neural network (1) as

$$dx = (-Ax + W_1 S(x))dt + d\Psi + W_2 u dt \tag{2}$$

Now let us consider a candidate stochastic Lyapunov function $E$, which is the same as the one given in [12]

$$E = \frac{1}{2} x^T x = \frac{1}{2} \|x\|^2 \tag{3}$$

From [14], the infinitesimal generator of the stochastic differential equation (2) is given as

$$LE = L_f E + \frac{1}{2} T_r \{\sum(t)^T g_1^T \frac{\partial^2 E}{\partial x^2} g_1 \sum(t)\} + L_{g2} Eu \tag{4}$$

where $L_f E = \dfrac{\partial E}{\partial x}(-Ax + W_1 S(x))$, $L_{g2} Eu = \dfrac{\partial E}{\partial x} W_2 u$,

$g_1 = I$, $\dfrac{\partial E}{\partial x} = x^T$, and $\dfrac{\partial^2 E}{\partial x^2} = I$.

Thus

$$LE = -\lambda x^T x + x^T W_1 S(x) + x^T W_2 u + \frac{1}{2} T_r \left\{\sum(t)^T \sum(t)\right\} \tag{5}$$

For the second term $x^T W_1 S(x)$, we can similarly apply the equation (7) in [12] here, and thus resulting in

$$x^T W_1 S(x) \leq x^T \left( \frac{1 + nM^2 \|W_1\|^2}{2} \right) x \tag{6}$$

Substituting (6) into (5), we have

$$LE \leq -\lambda x^T x + x^T \left( \frac{1 + nM^2 \|W_1\|^2}{2} \right) x + x^T W_2 u + \frac{1}{2} T_r \{\sum(t)^T \sum(t)\} \tag{7}$$

We shall next discuss how to find an optimality-oriented control for the stochastic recurrent neural network (2) to achieve stochastic input-to-state stabilization in probability. From the concept of differential minimax game [15], [14], the following general stochastic nonlinear system affined in the noise $\Psi$ and control $u$ is well known

$$dx = f(x)dt + g_1(x)d\Psi + g_2(x)udt \tag{8}$$

If we pursue a differential game problem which uses $\Delta(t)$ defined in *Definition 2* as a player to oppose the control, and suppose that there exists a positive optimal value function $V(t)$, which satisfies the following HJI equation

$$L_f V + \frac{1}{4}\gamma^{-2} \left\| g_1^T(x) \frac{\partial^2 V}{\partial x^2} g_1(x) \right\|^2 - \frac{1}{4} L_{g2} V r^{-1}(x) L_{g2}^T V + q(x) = 0 \tag{9}$$

Then the following control

$$u^{*}(x) = -\frac{1}{2} r^{-1}(x) L g_2^T V \tag{10}$$

is an optimal stabilizing control which minimizes the cost functional

$$J(u,\Delta) = \lim_{t\to\infty} E\left[ V(x(t)) + \int_0^t (q(x) + u^T r(x) u - \gamma^2 \|\Delta\|^2) d\tau \right] \tag{11}$$

where $\gamma > 0$ is a design parameter, both $q(x) \geq 0$ and $r(x) > 0$ for all $x$, and the worst case $\Delta^{*}(t)$ is

$$\Delta^{*}(t) = \frac{1}{2\gamma^2} \left( g_1^T(x) \frac{\partial^2 V}{\partial x^2} g_1(x) \right) \tag{12}$$

We now take the infinitesimal generator of the stochastic differential equation (8) with the optimal value function $V(x)$:

$$LV = L_f V + \frac{1}{2} T_r \left\{ \Sigma(t)^T g_1^T \frac{\partial^2 V}{\partial x^2} g_1 \Sigma(t) \right\} + L_{g_2} V u \tag{13}$$

For the model of the stochastic recurrent neural network, if we consider the Lyapunov function $E$ as the optimal value function $V(x)$, that is the solution $V$ is given by (3), we then have the following equations

$$L_f V = -\lambda x^T x + x^T W_1 S(x) \tag{14}$$

$$\frac{1}{2} T_r \left\{ \Sigma(t)^T g_1^T \frac{\partial^2 V}{\partial x^2} g_1 \Sigma(t) \right\} = \frac{1}{2} T_r \left\{ \Sigma(t)^T \Sigma(t) \right\} \tag{15}$$

$$L g_2 V = x^T W_2 \tag{16}$$

Then the substitution of the equation (3) into the HJI equation (10) yields the next relation

$$-\lambda x^T x + x^T W_1 S(x) + \frac{1}{4}\gamma^{-2} n - \frac{1}{4} x^T W_2 r^{-1}(x) W_2^T x + q(x) = 0 \tag{17}$$

Based on the above, the inequality in (7) can be written as

$$\begin{aligned} LV &\leq -\lambda x^T x + x^T \left( \frac{1 + nM^2 \|W_1\|^2}{2} \right) x + x^T W_2 u + \frac{1}{2} T_r \left\{ \Sigma(t)^T \Sigma(t) \right\} \\ &= -\lambda x^T x + x^T \left( \frac{3 + nM^2 \|W_1\|^2}{2} \right) x + x^T W_2 u - x^T x + \frac{1}{2} T_r \{\Delta(t)\} \end{aligned} \tag{18}$$

From (18), we can set up

$$\mathrm{K}_1(x) = x^T \left( \frac{3 + nM^2 \|W_1\|^2}{2} \right) x \tag{19}$$

$$\mathrm{K}_2^T(x) = x^T W_2 \tag{20}$$

Similar to [12], Let us define the following scalar function

$$\phi(x) = \frac{\mathrm{K}_1(x) + \sqrt{[\mathrm{K}_1(x)]^2 + [\mathrm{K}_2^T(x)\mathrm{K}_2(x)]^2}}{\mathrm{K}_2^T(x)\mathrm{K}_2(x)} \tag{21}$$

Then a control signal can be determined as

$$u = -\phi(x)\mathrm{K}_2(x)$$

with the assumption of $\mathrm{K}_2^T(x)\mathrm{K}_2(x) \neq 0$.
Using the equation (20), the control signal is equivalent to

$$\begin{aligned} u &= -\phi(x) W_2^T x \\ &= -\phi(x) L_{g_2}^T V \end{aligned} \tag{22}$$

By comparing (22) with (10) and using (16), (17) and (20), we obtain

$$r(x) = \frac{1}{2} \phi^{-1}(x) \tag{23}$$

and

$$q(x) = \frac{1}{4} \mathrm{K}_2^T(x) r^{-1}(x) \mathrm{K}_2(x) + \lambda x^T x - x^T W_1 S(x) - \frac{1}{4}\gamma^{-2} n \tag{24}$$

Now assuming that

$$\|x\| \geq \max\left\{ \frac{\sqrt{n}}{2\gamma}, \frac{\gamma}{\sqrt{\lambda}} \|\Delta\| \right\}, \tag{25}$$

we can obtain the following theorem.

*Theorem*: Given the system (1), there exist a positive-definite function $q(x)$ (24) and a strictly positive function $r(x)$ (23) in which $x \neq 0$, such that the feedback control law

$$u = u^* = -\frac{1}{2} r^{-1}(x) W_2^T x \tag{26}$$

achieves both stochastic input-to-state stabilization and inverse optimality with respect to a meaningful cost functional

$$J(u,\Delta) = \lim_{t\to\infty} E\left[ V(x(t)) + \int_0^t (q(x) + u^T r(x) u - \gamma^2 \|\Delta(\tau)\|^2) d\tau \right] \tag{27}$$

for the worst case $\Delta(t)$

$$\Delta(t) = \Delta^*(t) = \frac{1}{2\gamma^2} I \tag{28}$$

*Proof:*

Step 1: Considering a positive-definite stochastic Lyapunov function $V$ that is the same as (3), the infinitesimal generator of the stochastic differential equation (2) is given by

$$LV = -\lambda x^T x + x^T W_1 S(x) + x^T W_2 u + \frac{1}{2} T_r\{\Sigma(t)^T \Sigma(t)\} \tag{29}$$

Substituting (6) into (29), we have

$$\begin{aligned} LV &\le -\lambda x^T x + x^T \left( \frac{1 + nM^2 \|W_1\|^2}{2} \right) x + x^T W_2 u + \frac{1}{2} T_r\left\{\Sigma(t)^T \Sigma(t)\right\} \\ &= -\lambda x^T x + x^T \left( \frac{3 + nM^2 \|W_1\|^2}{2} \right) x + x^T W_2 u - x^T x + \frac{1}{2} T_r\{\Delta(t)\} \end{aligned} \tag{30}$$

The substitution of the control law (26) into $LV$ (30) yields

$$\begin{aligned} LV \le{}& -\lambda x^T x + x^T \left( \frac{3 + nM^2 \|W_1\|^2}{2} \right) x - \mathrm{K}_1(x) \\ & - \sqrt{[\mathrm{K}_1(x)]^2 + [\mathrm{K}_2^T(x)\mathrm{K}_2(x)]^2} - x^T x + \frac{1}{2} T_r\{\Delta(t)\} \end{aligned}$$

By the definition of (19), we obtain

$$LV \le -\lambda x^T x - \sqrt{[\mathrm{K}_1(x)]^2 + [\mathrm{K}_2^T(x)\mathrm{K}_2(x)]^2} - x^T x + \frac{1}{2} T_r\{\Delta(t)\}$$

With the assumption of (25), we know that

$$x^T x = \|x\|^2 \ge \frac{n}{4\gamma^2}$$

Thus

$$\begin{aligned} LV &\le -\lambda x^T x - \sqrt{[\mathrm{K}_1(x)]^2 + [\mathrm{K}_2^T(x)\mathrm{K}_2(x)]^2} - \frac{n}{4\gamma^2} + \frac{1}{2} T_r\{\Delta(t)\} \\ &\le -\lambda x^T x - \sqrt{[\mathrm{K}_1(x)]^2 + [\mathrm{K}_2^T(x)\mathrm{K}_2(x)]^2} \\ &\quad - T_r\left\{ \left( \frac{1}{2\gamma} I - \gamma\Delta(t) \right)^T \left( \frac{1}{2\gamma} I - \gamma\Delta(t) \right) \right\} + \gamma^2 \|\Delta(t)\|^2 \\ &\le -\lambda x^T x + \gamma^2 \|\Delta(t)\|^2 \end{aligned} \tag{31}$$

Therefore $LV \le 0$ whenever $\|x\| \ge \frac{\gamma}{\sqrt{\lambda}} \|\Delta(t)\|$

By the definition of stochastic input-to-state stability [16], we know that the system described by (1) achieves stochastic input-to-state stabilization with the control law (26).

Step 2: Let us consider $q(x)$ and $r(x)$.

By (24), we have

$$q(x) = \frac{1}{4} \mathrm{K}_2^T(x) r^{-1}(x) \mathrm{K}_2(x) + \lambda x^T x - x^T W_1 S(x) - \frac{1}{4} \gamma^{-2} n$$

From (6), we have

$$-x^T W_1 S(x) \ge -x^T \left( \frac{1 + nM^2 \|W_1\|^2}{2} \right) x \tag{32}$$

Using the inequality (32) and relations (23), (21) and (25), the expression $q(x)$ given above can be written as

$$\begin{aligned} q(x) &\ge \frac{1}{2} \left( \mathrm{K}_1(x) + \sqrt{[\mathrm{K}_1(x)]^2 + [\mathrm{K}_2^T(x)\mathrm{K}_2(x)]^2} \right) + \lambda x^T x \\ &\quad - x^T \left( \frac{3 + nM^2 \|W_1\|^2}{2} \right) x + x^T x - \frac{1}{4} \gamma^{-2} n \\ &\ge \lambda x^T x + \frac{1}{2} \left( \sqrt{[\mathrm{K}_1(x)]^2 + [\mathrm{K}_2^T(x)\mathrm{K}_2(x)]^2} - \mathrm{K}_1(x) \right) \\ &\ge 0 \end{aligned}$$

This means that $q(x)$ is positive definite, for all $x \neq 0$, and is radially unbounded.

Also, from (23), it can be seen that $r(x) > 0$ when $K_2^T(x)K_2(x) \neq 0$. That is

$$r(x) = \frac{1}{2}\phi^{-1}(x) = \frac{1}{2}\left(\frac{K_2^T(x)K_2(x)}{K_1(x) + \sqrt{[K_1(x)]^2 + [K_2^T(x)K_2(x)]^2}}\right) > 0$$

By using $q(x)$ and $r(x)$ in (24) and (23), $LV$ can be written into the following form

$$LV = -q(x) - u^T r(x)u + \gamma^2\|\Delta(t)\|^2 - T_r\left\{\left(\frac{1}{2\gamma}I - \gamma\Delta(t)\right)^T\left(\frac{1}{2\gamma}I - \gamma\Delta(t)\right)\right\}$$
$$+ (u - u^*)^T r(x)(u - u^*) \tag{33}$$

According to Dynkin's formula [14], we have

$$J(u, \Delta) = \lim_{t\to\infty} E\left[V(x(t)) + \int_0^t (q(x) + u^T r(x)u - \gamma^2\|\Delta(\tau)\|^2)d\tau\right]$$
$$= \lim_{t\to\infty} E\left[V(x(0)) + \int_0^t (LV + q(x) + u^T r(x)u - \gamma^2\|\Delta(\tau)\|^2)d\tau\right]$$
$$= E[V(x(0))] + \lim_{t\to\infty} E\int_0^t\left[(u-u^*)^T r(x)(u-u^*) - T_r\left\{\left(\frac{1}{2\gamma}I - \gamma\Delta(t)\right)^T\left(\frac{1}{2\gamma}I - \gamma\Delta(t)\right)\right\}\right]dt$$

From the above equation, we know that the optimal control $u = u^*$ is an optimal solution to $J$ (27) for the worst disturbance $\Delta(t) = \Delta(t)^*$ and

$$\min_u \max_\Delta J(u, \Delta) = E[V(x(0))]$$

Therefore, by considering the control $u(t)$ as a player and the noise covariance $\Delta(t)$ as the opposing player, a minimax equilibrium $(u^*, d^*)$ is achieved. This completes the proof.

## NUMERICAL EXAMPLE

In order to effectively describe our results, we present the following second order stochastic recurrent neural network

$$\begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix} = \left(-\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ -7 & 8 \end{bmatrix}\begin{bmatrix} \tanh(x_1) \\ \tanh(x_2) \end{bmatrix}\right)dt + \begin{bmatrix} d\Psi_1 \\ d\Psi_2 \end{bmatrix} + \begin{bmatrix} 4 \\ -4 \end{bmatrix}udt$$

where $x_1(0) = 20$, $x_2(0) = -20$, $\lambda = -2$, $W_1 = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$, $S(x) = \begin{bmatrix} \tanh(x_1) \\ \tanh(x_2) \end{bmatrix}$, $W_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, and $\Psi_1$, $\Psi_2$ are white noises (uniformly random) with the magnitude of $|\Psi_i| = 20$ $(i = 1, 2)$.

Fig. 1 shows the result of time responses of two states ($x_1(t)$ and $x_2(t)$) for this stochastic recurrent neural network without any control inputs. Fig. 2 shows the result of time responses of two states with the implementation of the optimally stochastic input-to-state stabilizing control (26) at $t = 100\,s$. It can be seen that the system achieves the expected performance which conforms to the theoretical analysis in Section III.

## CONCLUSIONS

This paper has presented a new design to achieve optimally stochastic input-to-state stabilization in probability for stochastic recurrent neural networks driven by noise of unknown covariance. The proposed approach is developed by using stochastic differential minimax game, Hamilton-Jacobi-Isaacs (HJI) equation, inverse optimality, and Lyapunov technique. With *Definition 2*, we have extended our previous research [17] to a much larger class of nonlinear stochastic systems. Due to the difficulty to solve the Hamilton-Jacobi-Isaacs equation, for stochastic nonlinear systems, optimal stochastic stabilization seems to be an unachievable goal in feedback design. However, an alternative way has been proposed in this paper to solve the problem and obtain an optimal feedback controller with respect to a meaningful cost functional by using the knowledge of inverse optimality. It is believed that the new design presented in this paper would intensify the applications of stochastic recurrent neural networks.

## REFERENCES


[1] Werbos, P., 2009. "Intelligence in the brain: A theory of how it works and how to build it". *Neural Networks*, 22(3), pp. 200 – 212.

[2] Ensari, T., and Arik, S., 2005. "Global Stability Analysis of Neural Networks with Multiple Time Varying Delays". *IEEE Trans. Automat. Contr.*, 50, pp. 1781-1785.

[3] Chen, B., and Wang, J., 2004. "Global exponential periodicity and global exponential stability of a class of recurrent neural networks". *Physics Letters A*, 329, pp. 36-48.

[4] Marco, M., Forti, M., and Tesi, A., 2004. "On absolute stability of convergence for nonlinear neural network models". W. Kang et al. (Eds.) New Trends in Nonlinear Dynamics and Control and their Applications, Lecture Notes in Control and Information Sciences, Springer, Berlin, 295, pp. 209 – 220.
[5] Sanchez, E., and Perez, J., 2003. "Input-to-State Stabilization of Dynamic Neural Networks". *IEEE Trans. Systems, Man, and Cybernetics – A,* 33, pp. 532-536.
[6] Hu, S., and Wang, J, 2002. "Global Asymptotic Stability and Global Exponential Stability of Continuous-Time Recurrent Neural Networks". *IEEE Trans. Automat. Contr.*, 47, pp. 802-807.
[7] Kulawski, G., and Brdys, M., 2000. "Stable Adaptive Control with Recurrent Networks". *Automatica,* 36, pp. 5-22.
[8] Arik, S., 2000. "Global Asymptotic Stability of a Class of Dynamic Neural Networks". *IEEE Trans. Circuits Syst. I*, 47, pp. 568-571.
[9] Sontag, E., and Qiao, Y., 1999. "Further Results on Controllability of Recurrent Neural Networks". *Systems & Control Letters*, 36, pp. 121-129.
[10] Sontag, E., and Sussmann, H., 1997. "Complete Controllability of Continuous-Time Recurrent Neural Networks". *Systems & Control Letters*, 30, pp. 177-183.
[11] Huang, C., He, Y., Huang, L., and Zhu, W., 2008. "*p*th moment stability analysis of stochastic recurrent neural networks with time-varying delays". *Information Sciences*, 178, pp. 2194-2203.
[12] Liu, Z., and Ansari, N., 2010. "Control of Recurrent Neural Networks Using Differential Minimax Game: the Deterministic Case," Submitted to 2010 ASME Dynamic Systems and Control Conference (DSCC-2010).
[13] Das, S., and Olurotimi, O., 1998. "Noisy Recurrent Neural Networks: The Continuous-Time Case". *IEEE Transactions on Neural Networks*, 9(5), pp. 913-936.
[14] Krstic, M., and Deng, H., 1998. *Stabilization of Nonlinear Uncertain Systems*, Springer-Verlag, New York.
[15] Basar, T., and Bernhard, P. 1995. *$H^{\infty}$-Optimal Control and Related Minimax Design Problems: A Dynamic Game Approach, 2nd Ed.*, Birkhauser, Boston, MA.
[16] Tsinias, J., 1998. "Stochastic input-to-state stability and applications to global feedback stabilization". *International Journal of Control,* 71, pp. 907-930.
[17] Liu, Z., Wang, Q., and Schurz, H., 2009. "Inverse Optimal Noise-to-State Stabilization of Stochastic Recurrent Neural Networks Driven by Noise of Unknown Covariance," *Optimal Control Applications and Methods*, 30(2), pp. 163-178.

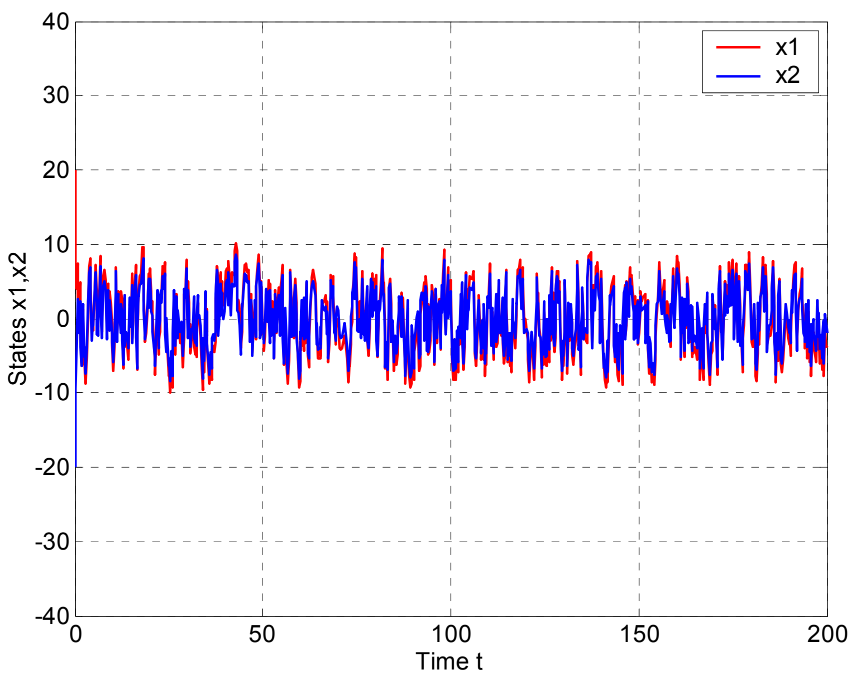


FIGURE 1. SYSTEM RESPONSE ( *u* = 0 )

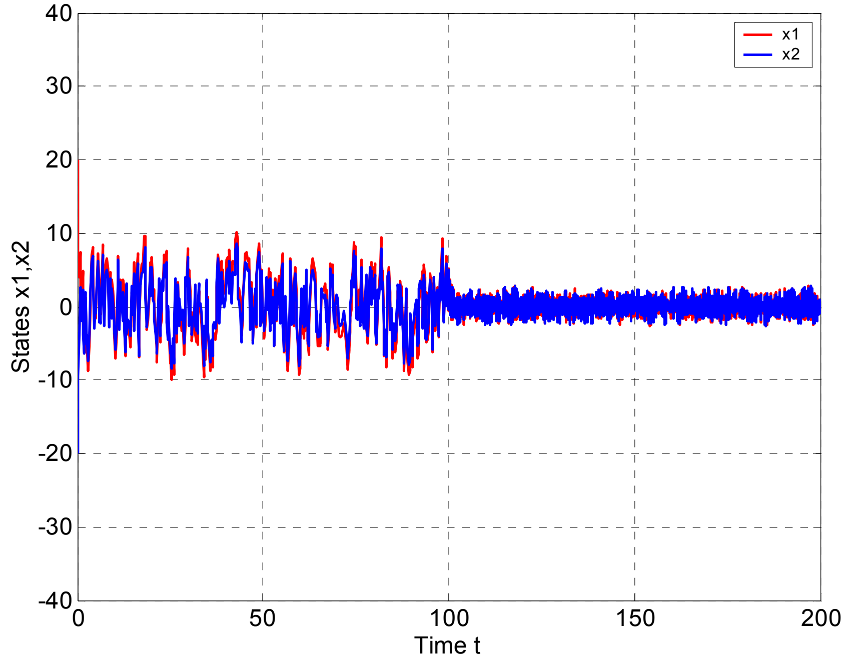


FIGURE 2. SYSTEM RESPONSE *( u = (26) at t = 100 s )*