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Adaptive density control in heterogeneous wireless sensor networks with and without power management

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Abstract: The authors study the design of heterogeneous two-tier wireless sensor networks (WSNs), where one tier of nodes is more robust and computationally intensive than the other tier. The authors find the ratios of densities of nodes in each tier to maximise coverage and network lifetime. By employing coverage processes and optimisation theory, the authors show that any topology of WSN derived from random deployments can result in maximum coverage for the given node density and power constraints by satisfying a set of conditions. The authors show that network design in heterogeneous WSNs plays a key role in determining key network performance parameters such as network lifetime. The authors discover a functional relationship between the redundancy, density of nodes in each tier for active coverage and the network lifetime. This relationship is much less pronounced in the absence of heterogeneity. The results of this work can be applied to network design of multi-tier networks and for studying the optimal duty cycles for power saving states for nodes in each tier.

1 Introduction

Network design is an important research area in wireless sensor networks (WSNs). The ability of WSNs to perform large scale, distributed sensing and data processing has led to research in potential applications in monitoring and tracking. The individual nodes that make up the WSNs are equipped with sensors and actuators, non-replenishable battery, transceiver and a microprocessor for data processing functions. Nodes sense and process data gathered from their sensing ranges and transmit these data to a central sink or base station (BS). In spatially large networks, transmitting data from nodes to a distant BS, consumes significant portion of the battery energy. Relaying can be employed to avoid the energy consumption incurred in single-hop networks resulting in considerable latency of data transfer from nodes to the sink. In most environmental monitoring applications, nodes in geographical proximity record similar data, because of high degree of correlation occurring in the sensed phenomenon. One approach that takes advantage of such correlated data is to establish a two-tier network of regular nodes and processing nodes. The regular nodes sense data and transmit it to the nearest processing node. Thus, instead of using single-hop networks or relaying, the processing nodes eliminate/reduce transmissions of redundant data and achieve improvement in network lifetime. Clustering is one such example of a tiered network topology. There are two general ways in which clustering may be facilitated. The first is clustering by selection, which is the most common method cited in current literature on clustering in WSNs. The second way to facilitate clustering is manifested in naturally clustered WSNs, where nodes in the higher tier are not chosen from among the nodes. In a naturally clustered network, the higher tier nodes are a distinct set of nodes scattered over the region with a smaller intensity of distribution than that of regular nodes. Thus we assume a two-tier hierarchy comprising of two distinct sets of nodes: sensor nodes (M-tier nodes) and processing nodes (N-tier nodes). The N-tier nodes are assumed to be robust, less power constrained and larger (akin to localised processing stations) that are capable of intensive processing and computation as compared to those of the M-tier nodes. The M-tier nodes that lie within the communication range of a N-tier node form a cluster. The N-tier nodes are also responsible for communicating to other N-tier nodes and relaying cluster data to central BS. Two-tiered sensor networks are quite common because of the uncontrollable deployment in many situations and thus their characteristics can provide valuable reference for sensor network design. However, our framework can be easily extended to study two-tiered network of homogeneous nodes in the clustering by selection topology, where N-tier nodes and M-tier nodes are homogeneous in sensing range, battery life and computational capacity.

In this paper, we study the problem of designing a two-tier WSN in the context of finding the densities of M-tier and N-tier nodes. Further, given the ratio of M-tier to N-tier nodes, we study the impact of density control to achieve further energy savings. Considerable attention has been given to the issue of density control for power management in dense randomly deployed WSNs [1, 2]. The motivation for this research area arises from the redundancy afforded by dense WSNs, where k-redundancy refers to $k > 1$ sensors sensing any given point (x, y) in the deployment region at all times. Random deployment procedures may result in such topologies with k redundancy; however, power management and reliability constraints require that we do not need all k sensors sensing the point (x, y). This has given rise to the notion of n-coverage, where at least n $(1 < n < k)$ of the k sensors are in the awake mode of operation. This method of scheduling a node to operate in awake/sleep states contributes to power management by reducing the duration of idle mode of transceiver operation in a node. Previous studies [3, 4, 5] have shown that the power consumption in the awake state is at least an order of magnitude greater than that in the sleep state. Within the awake mode of operation, the idle mode of listening for transmissions from the BS or other nodes consumes as much energy as the transmit operation. Clustering of nodes [6, 7] to reduce transmissions of redundant data is one approach for power management in WSNs.

In order to study the impact of network design in tiered networks on energy conservation, we formulate two objectives. The first objective is the minimisation of vacancy, where vacancy in the deployment region is defined as the area which does not lie within the sensing range of any node. The second is improvement of network lifetime. Mathematically, it is equivalent to the following question: What should be the densities of N-tier and M-tier nodes to ensure minimum vacancy and extend network lifetime with given k-redundancy of nodes? Note that, as mentioned earlier in the introduction, k-redundancy refers to the actual redundancy as a result of deployment, while active coverage resulting from n-redundancy $(n < k)$ refers to the actual number of nodes that are in the awake mode sensing a given point.

This paper addresses the problem of density control for active coverage in heterogeneous WSNs. The analysis can also be applied to the case of a single-tier homogeneous WSN, in which case the optimisation constraint of minimising vacancy gives the density of nodes in the network. Assuming that the deployment region is covered with M-tier and N-tier nodes according to Poisson processes with intensity λ_1 and λ_2, respectively, where $\lambda_2 < \lambda_1$, we study density control for the following two cases:

1. All-on network, where all the nodes are continuously on, providing k-active coverage in a network with k-redundancy.

2. Power management, where a node can be in one of two states – 'on' (awake) or 'off' (sleep), where the 'off' state denotes that the node powering down its sensors and actuators, transceivers and computation circuitry. The 'on' state denotes that the state can be transmit, receive or idle state while also performing sensing for the duration of the 'on' state. This power management models more realistic deployment scenarios for WSNs to prolong network lifetime.

We analyse both the 'all-on' case and 'power management' cases for various network configurations resulting from combinations of densities of M-tier and N-tier nodes in the deployment region. Specifically, we analyse the following combinations:

1. Dense networks with high density of M-tier and N-tier nodes.

2. Regular density networks with high density of M-tier nodes but low density of N-tier nodes.

3. Sparse networks with low density of M-tier nodes and low density of N-tier nodes.

4. The fourth case of low densities for M-tier nodes and high density of N-tier nodes is not feasible, and hence will not be investigated.

The 'all-on' and power management cases are analysed with respect to meeting power management and coverage objectives. In the 'all-on' case, the emphasis is on efficient network design and coverage by choosing the optimum ratio of densities for M-tier and N-tier nodes distributions. In the power management case, the emphasis is on minimising vacancy (maximising coverage) by increasing the density of active coverage while satisfying network power constraints to enhance network lifetimes over the 'all-on' case. In this paper, we make the following contributions:

• We provide expressions to optimise coverage in the deployment region. This paper lays the groundwork for analysis of coverage properties and power control in various hierarchical topologies of networks. In particular, the analysis in this paper can be easily extended to a three-tier network comprising of storage nodes, communication backbone and regular nodes. In this case, knowing the sensing and communication abilities of each tier of nodes, the network can be designed such that we have the ratio of nodes in each tier for storage, communication and regular nodes.

- We also analyse the optimisation of active coverage in a k-redundancy WSN with various topologies while ensuring that power constraints of network operation are satisfied. The analysis for adaptive density control can also be modified to obtain the duty cycle for nodes in the active state. This can be easily extended to obtain the optimal duty cycle, for a given density of M-tier and N-tier nodes in the tiered network. There has been similar prior research in [8, 9] to study the probability of nodes in active state. In this paper, we set a given duty cycle for nodes whose densities have been optimised and then study its impact on network lifetime. We validate the proposed model of maximising active coverage for network lifetime activation with the help of numerical simulations.

The rest of the paper is organised as follows: Section 2 presents the coverage model for various densities of nodes in a WSN and develops the analytical model for maximising coverage with and without power management. Section 3 presents the numerical results of the proposed power management model. Section 4 presents related work. Finally, Section 5 concludes the paper and presents future research directions.

2 Coverage model

A process $\mathcal{P}$ is said to be a stationary or homogenous Poisson point process $\mathcal{P}$ with intensity λ [10] if:

1. the number of points ξ_i in any Borel subset $\mathcal{S}$ of $\mathcal{R}$ is Poisson distributed with mean $\lambda\|S\|$, and

2. the numbers of points in any number of disjoint Borel subsets are independent random variables.

A process is called stationary if and only if the function λ is constant almost everywhere. A Boolean model in k-dimension Euclidean space is just the coverage pattern created by a Poisson-distributed sequence of random sets. Specifically, let $P \equiv \{\xi_i, i \geq 1\}$ be a stationary Poisson process of intensity λ in $\mathcal{R}$, the points ξ_i being indexed in any systematic order. Let $S_1, S_2, \ldots,$ be i.i.d. random sets, independent of $\mathcal{P}$. Then

$$C \equiv \{\xi_i + S_i, i \geq 1\} \tag{1}$$

is a Boolean model, where the Poisson process $\mathcal{P}$ is said to drive the Boolean model and the shapes S_i are said to generate the model. The expected vacancy within a region $\mathcal{R}$ denoted by $E(V)$ [10] is

$$E(V) = \|R\| \exp(-\lambda\|S\|) \tag{2}$$

where λ is the intensity of the point process for nodes, $\|R\|$ is the area of the deployment region and $\|S\|$ is the expected area of the node coverage. This vacancy denotes the part of the deployment region that is not covered by any node. In contrast to this moderate distribution of nodes in the deployment region, some WSN applications may call for dense networks with higher concentration of nodes resulting in lesser vacancy in the region. The high intensity of nodes in the deployment region differs from the case of moderate intensity, in that vacant areas of the region are fewer and smaller. The vacancy in a two-dimensional (2-D) deployment region due to high intensity distribution of nodes with circular coverage disks is given by [10]

$$E_{\text{Vd}} = \frac{\sqrt{\pi}\Gamma(3)}{\Gamma(1.5)}\left\{\frac{2\Gamma(1.5)}{\lambda\Gamma(1)}\right\}^2 = \frac{a}{\lambda^2} \tag{3}$$

where a is a constant given by

$$a = \frac{\sqrt{\pi}\Gamma(3)}{\Gamma(1.5)}(2\Gamma(1.5))^2$$

In the other case of sparse networks with low intensity distribution of nodes, where the vacancy in the 2-D deployment region R is almost equal to the area of the region R, the probability that any two coverage disks will not intersect each other is very high. In such a scenario, an approximation to the vacancy in a sparse network is given by [10]

$$E_{\text{V}-\text{sparse}} = \|R\| - E(N)\delta^k E(\|S\|) \tag{4}$$

where N is the number of nodes in the deployment region, $E(\|S\|)$ is the area of the coverage disk of any node S and δ denotes the scale parameter as a function of the intensity λ of distribution of nodes. We will use these results from the theory of coverage processes for varying densities of nodes in a Boolean model for optimising the tradeoff between coverage and network power consumption in the rest of this paper.

2.1 Coverage optimisation in an all-on WSN

In this section, we perform coverage optimisation in WSNs of various topologies to obtain the maximum coverage with given intensities of distribution of M-tier and N-tier nodes in the deployment region. The optimisation for each topology follows the simple procedure below:

1. Obtain the objective function $f(\lambda_1, \lambda_2)$, in each of these cases the objective is to minimise vacancy for the given topology.

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2. Obtain the constraint function $g(\lambda_1, \lambda_2)$. In this section, since we are assuming an 'all-on' network, the constraint is that all nodes are in the 'on' state.

3. Finally, we perform convex optimisation of the vacancy subject to the all-on constraint. In the mathematical analysis some of the objective and constraint functions are non-convex, quadratic and/or conic. We follow the standard procedures outlined in [11, 12] to linearise the optimisation problems. Owing to the space constraint, we omit the conversion procedure and present the final results of the optimisation.

(1) *Dense networks:* Owing to the high density of both M-tier and N-tier nodes, we expect the vacancy in the deployment region to be low (approximately equal to zero). We perform this optimisation subject to the constraint that area no more than that of the sensing region of a node should be vacant

$$E_{\text{V-Cluster}} - E_{\text{V-node}} \le A_{\text{node}} h \tag{5}$$

where h is some constant greater than the number of nodes, $E_{\text{V-Cluster}}$ is the vacancy in the region after deploying the N-tier nodes in the deployment region and $E_{\text{V-Node}}$ is the vacancy in the region after deploying the M-tier nodes, and A_{Node} is the area of the circular coverage disk of a node with radius R_1. The vacancy due to high density λ of nodes in a 2-D deployment region is given by (3) from Section 2. For densities λ_2 for N-tier nodes and λ_1 for M-tier nodes, the objective function of vacancy in the 2-D deployment region becomes

$$a\left\{\frac{1}{n_2\lambda_2^2} - \frac{1}{n_1\lambda_1^2}\right\} < \pi h R_1^2 \tag{6}$$

where n_2 and n_1 are the number of N-tier and M-tier nodes, respectively, in the deployment region. The objective function $f(\lambda_1, \lambda_2)$ is given by

$$f(\lambda_1, \lambda_2) = \frac{1}{n_2\lambda_2^2} - \frac{1}{n_1\lambda_1^2} - bR_1^2 \tag{7}$$

where $b = \pi h/a$ is a constant subject to the constraint that all nodes are 'on'. Applying the Lagrange duality theory for the original problem, we take the constraints into account to formulate the Lagrangian of (5). The Lagrangian optimisation [13] is thus

$$\Delta(\lambda_1, \lambda_2) = \frac{1}{n_2\lambda_2^2} - \frac{1}{n_1\lambda_1^2} - bR_1^2 + \lambda\left(\frac{e^{-\lambda_1}\lambda_1^{n_1}}{n_1!} + \frac{e^{-\lambda_2}\lambda_2^{n_2}}{n_2!}\right) \tag{8}$$

(2) *Regular networks:* We call regular networks as WSNs with high density λ_1 of M-tier nodes and low density of λ_2 of N-tier nodes in the deployment region. In such a network, we approximate the vacancy in the region after deployment of N-tier and M-tier nodes to be approximately equal to zero. To determine the vacancy, we use the equations from vacancy for low density of coverage disks for N-tier nodes and high density of M-tier nodes from Section 2. Thus, the objective function f for vacancy minimisation is $f(\lambda_1, \lambda_2) \simeq 0$.

$$E_{\text{V-Cluster}} - E_{\text{V-node}} \simeq 0 \tag{9}$$

$$f(\lambda_1, \lambda_2) = \|R\| - n_1\lambda_1^2\pi R_1^2 - \frac{a}{\lambda_2^2}$$

s.t.

$$g(\lambda_1, \lambda_2) = \frac{e^{-\lambda_1}\lambda_1^{n_1}}{n_1!} + \frac{e^{-\lambda_2}\lambda_2^{n_2}}{n_2!} \tag{10}$$

Simplifying the constraint function using expressions from inequality theory [14], we obtain

$$\lambda_1 = 1 - \frac{2a}{\|R\|\eta} e^{2/3(n_2-n_1)} + e^{n_2-n_1} \tag{11}$$

for the density of M-tier nodes, and

$$\lambda_2 = \left(\frac{2a}{\|R\| e^{n_2-n_1}\lambda_1^2\pi R_1^2}\right)^{1/3} \tag{12}$$

for the density of N-tier nodes. Thus, the ratio of densities for efficient coverage of the deployment region in WSN applications for regular networks is given by λ_1/λ_2.

(3) *Sparse networks:* Owing to the low density of N-tier and M-tier nodes, we expect the vacancy in the deployment region to be high, but no larger than that of the sensing range of a N-tier node to ensure connectivity. We perform this optimisation subject to the constraint that area no more than that of the sensing region of a N-tier node should be vacant

$$E_{\text{V-Cluster}} - E_{\text{V-node}} \le A_{\text{CH}} c \tag{13}$$

where c is some constant equal to the number of CHs, and A_{CH} is the area of the circular coverage disk of the CH with sensing radius given by R_2.

Using the equations for sparse networks from Section 2, the objective function for minimising vacancy is given by

$$f(\lambda_1, \lambda_2) = \pi(\lambda_1^2 n_1 R_1^2 - \lambda_2^2 n_2 R_2^2) \tag{14}$$

Minimising $f(\lambda_1, \lambda_2)$ subject to $g(\lambda_1, \lambda_2)$ which is the same as those in previous two sections, we obtain

$$\lambda_1 = 1 + \frac{\eta_{\text{node}}}{\eta_{\text{CH}}}(\lambda_2 - 1) \tag{15}$$

where $\eta_{\text{CH}} = \lambda_2^2\pi R_2^2$ and $\eta_{\text{node}} = \lambda_1^2\pi R_1^2$.

This gives us the ratio of densities for the case of all-on WSN for maximising coverage with given topology of sparse nodes.

2.2 Coverage optimisation in a WSN with power management

A key challenge in energy optimisation for densely deployed WSNs is selecting the set of sensors that remain awake for a given cycle. Some of the criteria developed for choosing the set of active nodes are environment probing [15], k-coverage [16] and connectivity-based participation in multi-hop network [17]. In an on-demand network, the BS can query the network on either a random schedule or in response to the changes in the underlying phenomenon monitored by the WSN. For example, a rapidly changing physical parameter calls for higher number of 'awake' nodes that can observe and report the change in phenomenon. In this case, the rate of change of the environmental parameter influences the energy consumption at M-tier nodes, causing a higher number of transmissions from M-tier nodes to N-tier nodes or to the sink through other M-tier nodes that act as relays. While we do not consider the pattern of environment variation that triggers queries from the BS, prior work in [2] develops an energy model which considers reliability of WSN operation and impact of sensing environment variation on network lifetime. However, we use the number of broadcast messages as an indication of network activity, through which we study the latency and network lifetime performance of the WSN with and without power management.

Problem formulation: How do we ensure that the power consumption of the network ψ with n nodes does not exceed a threshold Γ, while still minimising vacancy for different topologies? We assume that a M-tier node j can be in either one of two states: 'on' with a probability p_j or 'off' with a probability $1-p_j$ for an amount of time t. The values of p_j are determined by the application, for example, in a mostly sleeping network, p_j would be close to zero. Recent research in [8, 9] has focused on studying the probability that a node stays in the active state. In this work, however, we obtain the ratio of densities of nodes in M and N tiers for a given probability p_j. We also assume the power consumption for a node j in either state is given by w_j, where $w_{j-\mathrm{off}} \ll w_{j-\mathrm{on}}$, that is, power consumption in 'off' state is much less than that in on state and p_j denotes the probability of node j being in either 'on' state and $1-p_j$ denotes probability of node being in 'off' state.

To proceed with the formulation of the power constraint ψ, we define the power consumption ψ as the sum of the power consumption of every node j in the 'on'/'off' state. The state of every node in the network is represented by X_j, where for all $j = 1, 2 \ldots N$ and the states of any two nodes A and B are mutually independent of each other. We assume this for simplicity of calculation, since in practice the decision to switch a node to the 'on'/'off' state depends on various factors such as the amount of coverage desired for the application, residual battery energy and the reliability constraints. Since the states X_j alternate between one of two states ('on'/'off'), the power constraint ψ can be formulated as a binomial random variable with mean $(\bar{\lambda}) = np$ and variance $\sigma^2 = np(1-p)$. Hence

$$\psi = \sum_j X_j \tag{16}$$

Since the power consumption of the network should satisfy the constraint of being $<\Gamma$, we need to find the probability of $P(\psi < \Gamma)$ which is equivalently given by $1 - P(\psi \geq \Gamma)$.

Since for some t

$$P(\psi \geq \Gamma) = P[\exp(t\psi) \geq \exp(t\Gamma)] \tag{17}$$

$$\leq \frac{E[\exp(t\psi)]}{\exp(t\Gamma)} \tag{18}$$

$$= \frac{\prod_j E[\exp(tw_j\psi_j)]}{\exp(t\Gamma)} \tag{19}$$

where w_j is the power consumption of node j.

But $\forall j \in [n]$, where n is the number of M-tier nodes in the deployment region

$$E[\exp(tw_j\psi_j)] = p_j \exp(tw_j(1-p_j)) + (1-p_j)\exp(-tw_j\psi_j) \tag{20}$$

First we focus on the numerator of equation (19). For a low value of p_j, that is, $p_j \simeq 0$, (19) reduces to

$$\begin{aligned} E[\exp(tw_j\psi_j)] &= p_j \exp(tw_j) + (1-p_j) \\ &= 1 + p_j \exp(tw_j) - p_j \end{aligned} \tag{21}$$

Because $(1+x) \leq \exp(x)$

$$(1 + p_j \exp(tw_j) - p_j) \leq \exp(p_j \exp(tw_j) - p_j) \tag{22}$$

Using the notations for $e^{(.)}$ and exp (.) interchangeably, (22) can be re-written as

$$(1 + p_j \exp(tw_j) - p_j) \leq \exp(p_j(e^{tw_j} - 1)) \tag{23}$$

Hence, the numerator of (19) becomes

$$\prod_j \exp(p_j(e^{tw_j} - 1)) \tag{24}$$

Using the inequality for $r > 1$, $(e^r - 1) > r(e-1)$ in (24),

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the numerator of (19) becomes

$$E[\exp(tw_j\psi_j)] \geq \prod_j p_j tw_j(e-1) \tag{25}$$

$$\geq \prod_j \exp(p_j tw_j(e-1)) \tag{26}$$

$$\leq \exp \sum_j (p_j tw_j(e-1)) \tag{27}$$

Substituting (27) in (17), we obtain

$$P(\psi \geq \Gamma) \leq \exp(t \sum (e-1)p_j w_j) - \exp(t\Gamma) \tag{28}$$

Let $t = \Gamma = 1/(\mathrm{e}-1)/\sum p_j w_j$,

$$P(\psi \geq \Gamma) \leq \left[1 - \left(\frac{1/(e-1)}{\sum_j p_j w_j}\right)^2\right] \tag{29}$$

However, $P(\psi \geq \Gamma)$ can also be written as

$$P(\psi \geq \Gamma) = P\left(\frac{\psi - \overline{\Gamma}}{\sigma^2}\right) \geq \left(\frac{\Gamma - \overline{\Gamma}}{\sigma^2}\right) \tag{30}$$

$$= 1 - \phi\left(\frac{\Gamma - \overline{\Gamma}}{\sigma^2}\right) \tag{31}$$

Let $n_t = \sum_j p_j w_j$. We assume a Poisson distribution of n nodes with density λ_p. Each node stays in the 'on' state with probability p_j and the power consumption in the 'on' state is w_{on}. Therefore neglecting the power consumption of nodes in the 'off' state since it is very small as compared to that in the 'on' state, the operand of the summation in the denominator of the RHS is dominated by the power consumption of nodes in the on state. Denoting w_{on} as the power consumption in the on state, and λ_p as density of nodes in the 'on' state, the RHS can now be re-written as

$$n_t = \frac{\mathrm{e}^{-\lambda_p}\lambda_p^n}{n!} npw_{\mathrm{on}} \tag{32}$$

Therefore

$$P(\psi \geq \Gamma) = \exp\left[1 - \left(\frac{1/(e-1)}{(\mathrm{e}^{-\lambda_p}\lambda_p^n/n!)npw_{\mathrm{on}}}\right)^2\right] \tag{33}$$

$$= \exp\left[1 - \left(\frac{1/(e-1)}{\mathrm{e}^{-\lambda_p}\lambda_p^n pw_{\mathrm{on}}}\right)^2 \frac{n!^2}{n^2}\right] \tag{34}$$

$$= \exp\left[1 - \left(\frac{1/(e-1)}{\mathrm{e}^{-\lambda_p}\lambda_p^n pw_{\mathrm{on}}}\right)^2 (n-1)!^2\right] \tag{35}$$

Let

$$\zeta = \frac{1/(e-1)}{pw_{\mathrm{on}}}(n-1)!$$

Equation (35) becomes

$$P(\psi \geq \Gamma) = \exp[1 - \zeta^2 \mathrm{e}^{2\lambda_p}\lambda_p^{-2n}] \tag{36}$$

Hence from (31) and (36), we obtain

$$1 - \phi\left(\frac{\Gamma - \overline{\Gamma}}{\sigma^2}\right) \leq \exp[1 - \zeta^2 \mathrm{e}^{2\lambda_p}\lambda_p^{-2n}] \tag{37}$$

Rearranging the terms in (37), we obtain

$$\sigma^2 \phi^{-1}(1 - \exp[1 - \zeta^2 \mathrm{e}^{2\lambda_p}\lambda_p^{-2n}]) + \overline{\Gamma} \leq \Gamma \tag{38}$$

Since we are modelling ψ as a binomial random variable with mean $\overline{\Gamma}$ and variance σ^2, (38) becomes

$$np(1-p)\phi^{-1}[1 - \exp(1 - \zeta^2 \mathrm{e}^{2\lambda_p}\lambda_p^{-2n})] + np \leq \Gamma \tag{39}$$

Equation (39) corresponds to the complement of the power constraint, that is $P(\psi \geq \lambda)$. Thus, the power constraint corresponding to the probability of $P(\psi < \lambda)$ which is given by $1 - P(\psi \geq \lambda)$ as follows

$$\Gamma < 1 - np\{(1-p)\phi^{-1}[1 - \exp(1 - \zeta^2 \mathrm{e}^{2\lambda_p}\lambda_p^{-2n})] + 1\} \tag{40}$$

Equation (40) gives us the power constraint for the power management problem for clustered WSNs.

We now summarise the coverage maximisation against power management problem for various densities of N-tier and M-tier nodes. Unlike the case for an all-on network, we cannot provide straightforward closed form equations for the ratios of densities of CHs to that of the nodes in the on state. This is because of the difficulty of obtaining a closed form solution for the problem of minimising vacancy to that of maximising network lifetime. In the next section, we provide numerical results for Monte Carlo simulation of the WSN with the constraints discussed in Section 2 and (40). In each case, we minimise vacancy subject to the power constraint in (40),

- Dense WSNs: From Section 2.1, the vacancy in a dense network of M-tier and N-tier nodes given by (7) is optimised w.r.t. (40), that is

$$f(\lambda_1, \lambda_2) = \frac{1}{n_2\lambda_2^2} - \frac{1}{n_1\lambda_1^2} - K$$

s.t.

$$\Gamma < 1 - np\{(1-p)\phi^{-1}[1 - e\exp(1 - \zeta^2 \mathrm{e}^{2\lambda_p}\lambda_p^{-2n})] + 1\} \tag{41}$$

- Sparse WSNs: From Section 2.1, the vacancy in a sparse network of M-tier and N-tier nodes given by (15) is optimised w.r.t. (40), that is

$$f(\lambda_1, \lambda_2) = \|R\|\{(1 - \lambda_1 \eta_{CH}) - (1 - \lambda_2 \eta_{\text{node}})\}$$

s.t.

$$\Gamma < 1 - np\{(1-p)\phi^{-1}[1 - e\exp(1 - \zeta^2 e^{2\lambda_p} \lambda_p^{-2n})] + 1\} \tag{42}$$

- Regular density WSNs: From Section 2.1, the vacancy in a regular network of M-tier and N-tier nodes given by (2.2) is optimised w.r.t. (40), that is

$$f(\lambda_1, \lambda_2) = 1 - \eta\lambda_1 - \frac{a}{\lambda_2}$$

s.t.

$$\Gamma < 1 - np\{(1-p)\phi^{-1}[1 - \mathrm{e}\exp(1 - \zeta^2 e^{2\lambda_p} \lambda_p^{-2n})] + 1\} \tag{43}$$

- Moderate density WSNs: From Section 2.1, the vacancy in a moderate density network of M-tier and N-tier nodes given by (2) is optimised w.r.t. (40), that is

s.t. $$\|R\|\{\exp(-\lambda_2 R_2^2) - \exp(-\lambda_1 R_1^2)\}$$

$$\Gamma < 1 - np\{(1-p)\phi^{-1}[1 - e\exp(1 - \zeta^2 e^{2\lambda_p} \lambda_p^{-2n})] + 1\} \tag{44}$$

3 Simulation results

3.1 Network performance results after density optimisation in clustered networks without power management

Figs. 1 and 2 show a comparison of network lifetime simulation results for network performance obtained by density optimisation against those in random networks for the case of a mostly 'on' network, where the networks do not perform any power management through energy-saving modes of operation. Here, we use BC to denote the number of broadcast messages. In our model, M-tier nodes relay their data to the nearest N-tier node, which then performs data processing and aggregation, and forwards it to the nearest N-tier node. With clustering, the end users of the data can benefit by reducing the amount of data processing to obtain relevant information at the BS. Fig. 1 shows the results of network lifetime in random and density optimised networks. For all levels of network activity, we see that density optimisation results in higher lifetime than random networks. The improvement in network lifetime is significant for lower network activity (BC = 10). For increased network activity (BC = 30), the improvement in network lifetime is less significant. Within density optimised networks, we see that for low network activity, dense networks have higher network lifetime than sparse networks and networks with high density of M-tier nodes and low density of N-tier nodes. This is because for low network activity, network lifetime is greatly dependent on the M-tier node's radio consumption and microprocessor power consumption is much smaller than the node radio consumption. For higher activity, sparse networks have larger inter-node distances, while networks with high density of M-tier nodes and low density of N-tier nodes have larger cluster sizes. The large inter-node distances in sparse networks prevent communication between nodes, and hence resulting in higher network lifetime. Dense networks have the lowest network lifetime

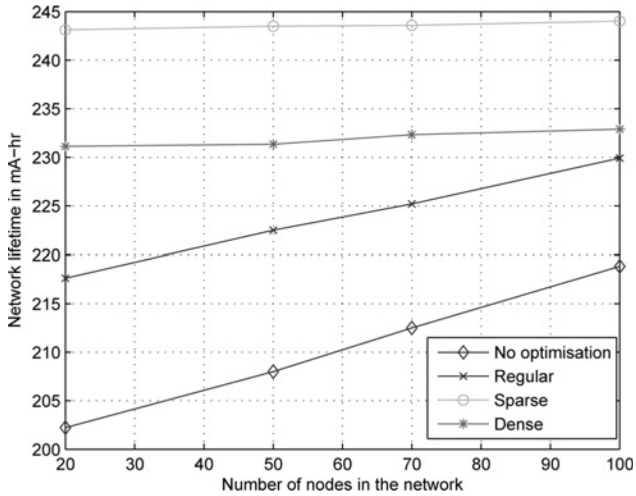

Figure 1 *Network lifetime in a two-tiered WSN without power management, BC = 10*

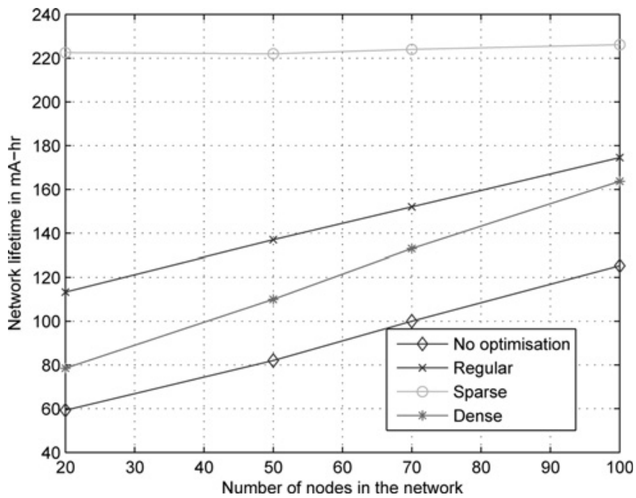

Figure 2 *Network lifetime in a two-tiered WSN without power management, BC = 40*

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for high network activity since the increased number of clusters and higher activity cause faster depletion because of the radio and microprocessor activity. The best case to satisfy coverage and connectivity is a high density of M-tier nodes and low density of N-tier nodes, since it results in a small number of clusters that dense networks and provides the same level of coverage and connectivity.

3.2 Network performance results after density optimisation in clustered networks with power management

In this section, we present results for the case where power management is implemented in the network such that nodes can be in one of two states: 'on' or 'off'. We present simulation results for different probability p that a node is in the 'on' state. The optimisation here is performed for the highest network activity (BC = 40) with respect to minimising the vacancy for each scenario of M-tier and N-tier node densities, and subject to the power constraint imposed by the given value of p.

Figs. 3 and 4 present comparison of network lifetime between random and density optimised networks for different values of p. Similar to WSNs without power management, sparse networks exhibited the highest network lifetime because of the minimum number of node connections as compared to other networks. With the increase in p from 0.2 to 0.7, the network lifetime reduces because of increased activity of nodes in the 'on' state. Dense networks had the lowest network lifetime because of the increased cluster maintenance and intra- and inter-cluster activity. Networks with high density of M-tier nodes and low density of N-tier nodes reported highest network lifetime with power management. A comparison between Figs. 2 and 4 show that power management results in higher network lifetime for WSNs.

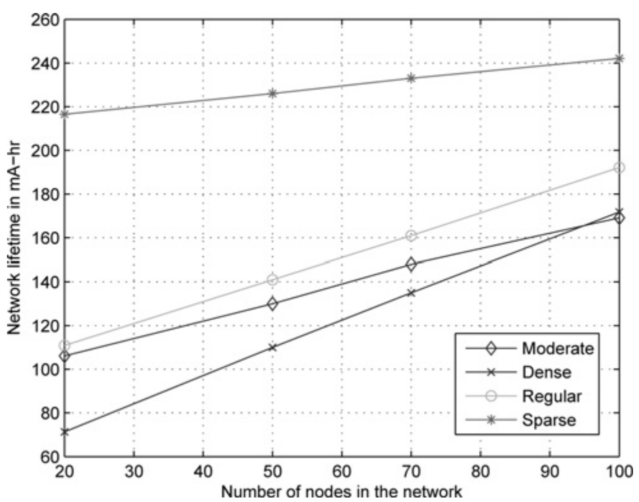

Figure 3 *Network lifetime in a two-tiered WSN with power management for p = 0.2*

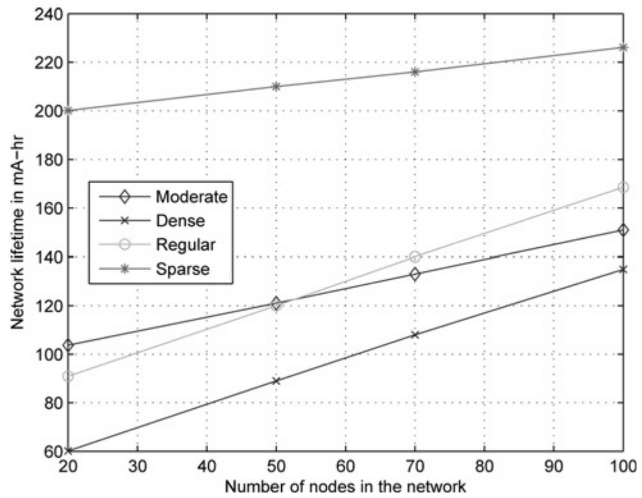

Figure 4 *Network lifetime in a two-tiered WSN with power management for p = 0.9*

4 Related work

The closest related work to ours in the context of network design is [18], where the N-tier nodes are not chosen from among the nodes in a randomised rotation manner [6]. Rather, the N-tier nodes are a separate set of nodes that receive data from regular nodes and perform data aggregation and sensing tasks. The authors presented an energy model for two-tiered clustered WSN, which consists of N-tier nodes randomly and uniformly distributed in the deployment region, and investigated the energy consumption of the network. Another similar work is [19], where the authors studied the impact of cluster density on the capacity of *ad hoc* networks, instead of the widely used assumption of randomly uniformly distributed nodes distributed according to a stationary Poisson point process in the sensing area. They assumed a network model where clustered nodes with density ρ_c, in a 'sea of nodes' of density ρ_s, such that $\rho_s \ll \rho_c$. They showed that the throughput of clustered networks switches at a critical size that is dependent on the sensing area A. Before reaching the critical size, the per-node throughput is almost independent of A, and depends on cluster size and cluster density. They derived bounds on the throughput of clustered networks and helped quantify the concept of 'large' networks, that is, networks whose size exceeds the critical size. Large networks are characterised with increase in capacity as the size decreases further.

Lots of research has been conducted on organising hierarchical sensor networks. For homogeneous networks, many researchers show that hierarchically organising homogeneous sensor networks can improve the energy efficiency [4, 20], scalability [21, 22] and communication capacity. As for heterogeneous networks, Wang *et al.* [23] proposed the sensor networks composed of both static and mobile sensors to achieve balance between sensor cost and coverage. Singh and Prasanna [24] proposed the sensor networks composed of both low power sensor nodes and powerful nodes for energy efficiency.

The approach of tiered architectures offers the convenience and economy of in-node/in-cluster processing of data to reduce transmissions of redundant data, power control, scalability, and improvement in network lifetime. One of the earliest literature on implementing tiered WSN through clustering in WSNs is low energy adaptive clustering hierarchy (LEACH) [6], where cluster formation is designed to achieve prolonged network lifetime by local data processing, rotation of the cluster-head position among nodes and low energy MAC access. In [25], the authors proposed a clustering algorithm, SNOWCLUSTER which creates a three-tiered hierarchy of nodes, clusters and regions. They used a central framework administrator SNOWMAN, proposed in an earlier work to maintaining location information of nodes, monitoring node status and make local decisions and policy allocation for individual nodes. The use of this framework allows nodes to rely on a central framework for policy enaction instead of using its own resources for neighbour discovery and other management tasks.

5 Conclusions

In this paper, we have used the concepts of coverage processes and optimisation theory to explore coverage in various topologies of WSNs generated by combinations of densities of nodes in M and N tiers. In each case, we provide expressions to optimise coverage in the deployment region. We also analyse the optimisation of active coverage in a k-redundancy WSN with various topologies while ensuring that power constraints of network operation are satisfied. While the latter case of power management does not provide closed form solutions to the problem of coverage optimisation against network lifetime extension, we show with the help of numerical simulations that the proposed model increases network lifetime while simultaneously achieving maximum coverage. This paper lays the groundwork for analysis of coverage properties and power control in various topologies of heterogeneous networks and opens research issues for other topologies such as naturally clustered networks. Our future work in this area will be analysing the network lifetime for dense WSNs, where the definition of network lifetime provides a more accurate representation of the residual sensing and communication capacity, as opposed to the conventional definition of network lifetime which uses the time until the first node runs out of battery energy. We also propose to investigate a routing algorithm that utilises the properties of coverage and power control already investigated in this paper to achieve further energy savings and higher reliability of WSN operation.

6 References

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