Nonlinear Predictor-Based Dynamic Resource Allocation Over Point-to-Multipoint (P2MP) Networks: A Control Theoretical Approach

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***Abstract*—Most broadband access networks such as passive optical networks (PONs) adopt the point-to-multipoint (P2MP) topology. One critical issue in such networks is the upstream resource management and allocation mechanism. Nonlinear predictor-based dynamic resource allocation (NLPDRA) schemes for improving the P2MP network upstream transmission efficiency have been investigated in an ad hoc manner. In this paper, we establish a general state space model to analyze the controllability and stability of the NLPDRA schemes from the P2MP network system's point of view and propose controller design guidelines to maintain the system stability under different scenarios. Analytical results show that NLPDRA maintains the P2MP network system controllability even when the loaded network traffic changes drastically. We further prove that a P2MP network system with NLPDRA is stable by proper pole placements as the traffic changes. Finally, we provide guidelines to design a optimal compensator to achieve system accuracy.**

***Index Terms*—Passive optical network (PON); Upstream resource management; State space; Controllability; Stability; Controller design; Nonlinear prediction-based dynamic resource allocation (NLPDRA).**

I. Introduction

In access networks, the point-to-multipoint (P2MP) topology is one of the most commonly used topologies. In general, P2MP comprises a root station (RS) and a number of leaf stations (LSs), in which any media with a RS broadcasting packets through a single trunk (such as frequency, wavelength, or wireless channel) to LSs is referred to as downstream and with LSs unicasting packets through branches and the trunk to the RS is referred to as upstream. In addition, LSs may not communicate with each other in a peer-to-peer manner.

Most wireline broadband access networks such as the time division multiplexing (TDM) passive optical networks (PONs) [1], which include Ethernet passive optical networks (EPONs) [2], gigabit passive optical networks (GPONs) [3], and broadband passive optical networks (BPONs) [4], can be generalized into a P2MP architecture. The P2MP architecture of PONs reduces the dominant deployment and maintenance cost and facilitates the central management by utilizing the root station as the central office. As exemplified by Fig. 1, the P2MP architecture of EPONs, ranging between each optical line terminal (OLT) and its associated optical network units (ONUs), facilitates EPONs with the advantages of minimizing the number of optical transceivers and eliminating the intermediate powering. The OLT herein serves as the root station, and ONUs serve as the leaf stations. In the downstream, packets are broadcast through wavelength λ_1 to each LS (i.e., ONU). While in the upstream, each LS (i.e., ONU) unicasts its packets to the RS (i.e., OLT) through a shared wavelength λ_2.

For P2MP networks, one critical issue is the upstream resource management and allocation mechanism. The upstream resource here could be bandwidth in the scenario of TDM, wavelength in the scenario of wavelength-division multiplexing (WDM), or frequency in the scenario of orthogonal frequency-division multiplexing (OFDM). Under the P2MP architecture, multiple LSs share the upstream trunk, and each LS has no knowledge of the transmission condition of the others. To avoid data collision, a request/grant arbitration mechanism, such as the multipoint control protocol (MPCP) in EPON [5], is usually deployed for the upstream resource sharing. The request/grant mechanism is implemented in continuous service cycles. In each service cycle, LSs need to send requests to the RS for the resource grant before any transmission. Thereafter, the RS determines an appropriate portion of the transmission window in the next service cycle to each LS, by considering the requests as well as the available resources, and sends out grants to LSs. Finally, after receiving the grants, LSs begin to transmit their packets until their granted window size is used up. In this way, a dynamic resource allocation is achieved.

Nonlinear predictor-based dynamic resource allocation (NLPDRA) schemes [6,7] employ traffic correlation to predict the incoming data in the next cycle. A nonlinear index is employed to extract the time-dependent correlation among traffic in consecutive cycles. Simulations show that NLPDRA outperforms traditional bandwidth allocation schemes [7,8].

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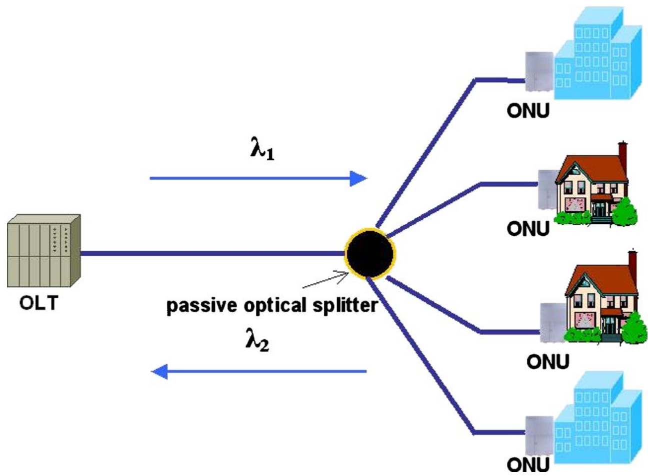

Fig. 1. (Color online) Ethernet passive optical network.

In our earlier works, we analyzed the system-level characteristics of NLPDRA in a TDM-PON system [9,10] at GLOBECOM 2007 and ICC 2008, respectively. In this paper, we extend the investigation of NLPDRA from the vantage point of generic P2MP network system characteristics and expand our discussion to the optimal compensator design by using the proposed state space model. A well-designed resource allocation scheme is expected to maintain the P2MP network performance under dynamic traffic changes and to guarantee the fair share of the available upstream resource among multiple LSs. Towards this end, we start from exploring the NLPDRA controllability and stability. A state space model is introduced as the general representation of the P2MP network system, on which the controllability and stability analysis is conducted to reveal the requirements of maintaining system performance under dynamic traffic input. Following the modeling, guidelines of the controller design, especially the optimal controller design, are further established. These guidelines essentially highlight the framework for designing NLPDRA schemes for P2MP networks that ensure stability and achieve system accuracy. The rest of the paper is organized as follows: Section II describes the system model and associated nonlinear predictor-based dynamic resource allocation schemes; Section III discusses the controllability of NLPDRA; Section IV presents the stability analysis along with the controller design; Section V provides the guidelines to design an optimal compensator to achieve system accuracy; finally, the conclusion is drawn in Section VI.

II. System Model and NLPDRA

In a P2MP network system, the upstream resource allocation is arbitrated by one master, i.e., the RS, over multiple clients, i.e., LSs. Let us assume one RS serves y LSs, and the RS serves each LS once in a service cycle (Fig. 2).

The following notations are adopted for our analysis:

$R_i(n)$ the reported queued length by the piggybacked REPORT message from LS_i $(1 \leqslant i \leqslant y)$ at the beginning of service cycle n;

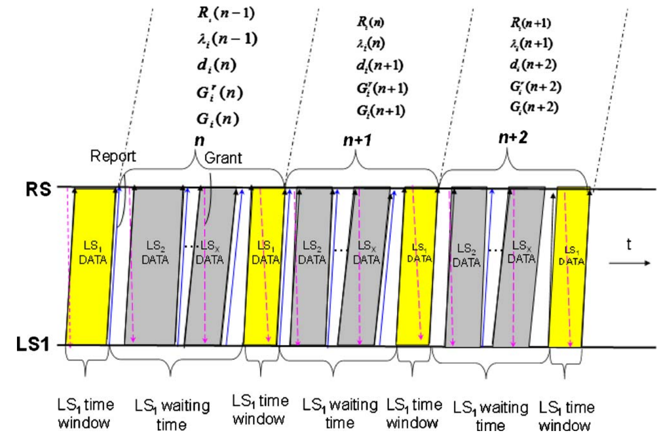

Fig. 2. (Color online) Dynamic resource allocation in P2MP networks.

$\lambda_i(n)$ the actually arrived data of LS_i at the end of service cycle n;

$\hat{\lambda}_i(n)$ the predicted arrival data at LS_i at the beginning of service cycle n;

$d_i(n+1)$ the departed data from LS_i at the end of service cycle n;

$G_i(n+1)$ the allocated timeslot to LS_i at the end of service cycle n;

$G_i^r(n+1)$ the bandwidth requirement of LS_i at the end of service cycle n (it may or may not be the same as $R_i(n)$, depending on the particular bandwidth allocation scheme);

$G_i^{\max}$ the maximum timeslot length prescribed by the service level agreement (SLA).

At the beginning of service cycle $(n+1)$, the queue length of LS_i is the residual of data transmission, and it is described by

$$R_i(n+1) = R_i(n) + \lambda_i(n) - d_i(n+1). \tag{1}$$

A REPORT message is piggybacked at the beginning of timeslot n, indicating the awaiting data, which is the current queue length $R_i(n)$. After processing the request, the RS allocates timeslot $G_i(n)$ to LS_i, and the departed data at the end of service cycle n is

$$d_i(n+1) = \min\{G_i(n+1), R_i(n) + \lambda_i(n)\}. \tag{2}$$

The granted timeslot is thus represented by the smaller value of the bandwidth requirement and the SLA upper bound, i.e.,

$$G_i(n+1) = \min\{G_i^r(n+1), G_i^{\max}\}. \tag{3}$$

When a traffic predictor is employed, $G_i^r(n+1)$ is determined by

$$G_i^r(n+1) = R_i(n) + \hat{\lambda}_i(n), \tag{4}$$

where $\hat{\lambda}_i(n)$ is the predicted arrival data at LS_i in service cycle n.

NLPDRA works as follows: when a queue length report $R_i(n)$ is received, the RS updates the bandwidth requirement from LS_i, i.e., $G_i^r(n)$, according to Eq. (4), and arbitrates the allocated timeslot according to Eq. (3). The traffic forecast by NLPDRA is made according to

$$\hat{\lambda}_i(n+1) = \alpha_i(n)\lambda_i(n), \tag{5}$$

where $\alpha_i(n)$ is the estimation credit, indicating the impact of the input historical data on the output prediction, and $\alpha_i(n)$, for example, is adjusted by the least mean squares (LMS) algorithm [11] as follows:

$$\alpha_i(n+1) = \alpha_i(n) + \tau \times \frac{e_i(n)}{\lambda_i(n)}, \tag{6}$$

where τ is the step size and is defined as a positive real number, $e_i(n)$ is the prediction error, and

$$e_i(n) = \lambda_i(n) - \hat{\lambda}_i(n). \tag{7}$$

Denote $\alpha_i'(n)=\alpha_i(n-1)$ and $\lambda_i'(n)=\lambda_i(n-1)$. From Eqs. (5)–(7), we have

$$\alpha_i(n+1) = \alpha_i(n) + \tau - \tau \times \frac{\alpha_i'(n) \times \lambda_i'(n)}{\lambda_i(n)}. \tag{8a}$$

Obviously, we know from $\alpha_i'(n)=\alpha_i(n-1)$ that

$$\alpha_i'(n+1) = \alpha_i(n). \tag{8b}$$

Equations (1), (4), (8a), and (8b) form the state space of the NLPDRA scheme, which is represented by

$$x_i(n+1) = Ax_i(n) + Bu_i(n), \tag{8}$$

where $x_i(n)=[G_i^r(n)\ R_i(n)\ \alpha_i(n)\ \alpha_i'(n)]^T$ is the P2MP network system state vector, indicating the resource requirement, the queue length of LS_i, and the prediction index. The input vector, $u_i(n)=[\lambda_i(n)\ \lambda_i'(n)]$, represents the arrived data during the waiting time. A and B are the matrices for the state vector and input vector, respectively, that determine the intrinsic characteristics of each scheme at the system level. The state space model represented by Eq. (8) provides a convenient and compact way to model and analyze the upstream resource allocation over P2MP networks. In the next section, the controllability of the P2MP system with NLPDRA will be evaluated based on the model represented by Eq. (8).

III. Controllability of NLPDRA

By "controllability," we mean that the shared upstream bandwidth in a P2MP network system can be arbitrated properly among multiple LSs, even when the loaded traffic changes drastically [12]. We expect that the employed bandwidth allocation scheme is capable of adjusting the bandwidth allocated to each LS in accordance with the traffic dynamics, and the arbitration decision is expected to be fair and efficient.

As formulated in Section II, Eq. (8) describes the upstream bandwidth allocation over a P2MP network system. One element in the state vector, $R_i(n)$, represents the queue length. From Eq. (1), we know that $R_i(n)$ is determined by $d_i(n)$. Combining Eqs. (2) and (3), we have

$$d_i(n) = \min\{G_i^r(n), R_i(n-1) + \lambda_i(n-1), G_i^{\max}\}. \tag{9}$$

Depending on the loaded traffic from end users, the P2MP network system falls in one of the following three scenarios:

1) $d_i(n)=G_i^r(n)$,
2) $d_i(n)=R_i(n-1)+\lambda_i(n-1)$, and
3) $d_i(n)=G_i^{\max}$.

A. Scenario 1: $d_i(n)=G_i^r(n)$

In this scenario, the bandwidth requirement is no larger than the SLA specification, and the granted timeslot is no larger than the total data arrived at LS_i during a service cycle. This is the case when the end users are well behaved under the guidance of the SLA specification. We have $G_i(n+1)=G_i^r(n+1)$ and $d_i(n)=G_i^r(n)$. The state space is thus

$$G_i^r(n+1) = R_i(n) + \hat{\lambda}_i(n) = R_i(n) + \alpha_i'(n)\lambda_i'(n), \tag{10a}$$

$$R_i(n+1) = R_i(n) + \lambda_i(n) - G_i^r(n+1). \tag{10b}$$

Substituting Eq. (10a) into (10b), we obtain

$$R_i(n+1) = \lambda_i(n) - \alpha_i'(n)\lambda_i'(n). \tag{10c}$$

Note that Eqs. (8a), (8b), (10a), and (10c) describe a nonlinear discrete system, and linearization is necessary to analyze the controllability [13,14].

Assume that the equilibrium point is $(G_{i0}^r, R_{i0}, \alpha_{i0}, \alpha_{i0}', \lambda_{i0}, \lambda'_{i0})$, all of which are positive real numbers; linearizing Eqs. (8a), (8b), (10a), and (10c) about the equilibrium point (see Appendix A for details), we obtain the following linearized system:

$$\delta G_i^r(n+1) = \delta R_i(n) + \lambda_{i0}'\delta\alpha_i'(n) + \alpha_{i0}'\delta\lambda_i'(n), \tag{11a}$$

$$\delta R_i(n+1) = \delta\lambda_i(n) - \lambda_{i0}'\delta\alpha_i'(n) - \alpha_{i0}'\delta\lambda_i'(n), \tag{11b}$$

$$\delta\alpha_i(n+1) = \delta\alpha_i(n) - \frac{\tau\lambda_{i0}'}{\lambda_{i0}}\delta\alpha_i'(n) + \frac{\tau\alpha_{i0}'\lambda_{i0}'}{\lambda_{i0}^2}\delta\lambda_i(n) - \frac{\tau\alpha_{i0}'}{\lambda_{i0}}\delta\lambda_i'(n), \tag{11c}$$

$$\delta\alpha_i'(n+1) = \delta\alpha_i(n). \tag{11d}$$

Equations (11a)–(11d) can be further represented by Eq. (11):

$$\delta x_i(n+1) = A_1\delta x_i(n) + B_1\delta u_i(n),$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & \lambda_{i0}' \\ 0 & 0 & 0 & -\lambda_{i0}' \\ 0 & 0 & 1 & -\dfrac{\tau\lambda_{i0}'}{\lambda_{i0}} \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & \alpha_{i0}' \\ 1 & -\alpha_{i0}' \\ \dfrac{\tau\alpha_{i0}'\lambda_{i0}'}{\lambda_{i0}^2} & -\dfrac{\tau\alpha_{i0}'}{\lambda_{i0}} \\ 0 & 0 \end{bmatrix}.$$

Theorem 1: A P2MP network with NLPDRA is controllable when $d_i(n)=G_i^r(n)$.

Proof: A system described by Eq. (11) is controllable *iff* the $n\times nr$ controllability matrix $U=[B\ AB\ \ldots\ A^{n-1}B]$ is full row rank [13,14].

In the above scenario,

$$U=[B_1\quad A_1B_1]=\begin{bmatrix} 0 & \alpha'_{i0} & 1 & -\alpha'_{i0} \\ 1 & -\alpha'_{i0} & 0 & 0 \\ \frac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\frac{\tau\alpha'_{i0}}{\lambda_{i0}} & \frac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\frac{\tau\alpha'_{i0}}{\lambda_{i0}} \\ 0 & 0 & \frac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\frac{\tau\alpha'_{i0}}{\lambda_{i0}} \end{bmatrix}.$$

Notice that U is a square matrix, and it is full rank when $|U|\neq 0$.

Since

$$|U|=\begin{vmatrix} 0 & \alpha'_{i0} & 1 & -\alpha'_{i0} \\ 1 & -\alpha'_{i0} & 0 & 0 \\ \frac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\frac{\tau\alpha'_{i0}}{\lambda_{i0}} & \frac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\frac{\tau\alpha'_{i0}}{\lambda_{i0}} \\ 0 & 0 & \frac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\frac{\tau\alpha'_{i0}}{\lambda_{i0}} \end{vmatrix}=\frac{\tau^2\alpha'^2_{i0}(\lambda_{i0}-\alpha'_{i0}\lambda'_{i0})^2}{\lambda_{i0}^4},$$

and τ, α'_{i0}, λ'_{i0}, and λ_{i0} are defined as positive real numbers, $|U|\neq 0$ iff $\alpha'_{i0}\lambda'_{i0}\neq\lambda_{i0}$ holds. Therefore, when $d_i(n)=G_i^r(n)$, the P2MP network system represented by Eq. (11) is controllable.

B. Scenario 2: $d_i(n)=R_i(n-1)+\lambda_i(n-1)$

In this scenario, the granted timeslot is larger than the bandwidth requirement, i.e., $G_i(n)>R_i(n-1)+\lambda_i(n-1)$. This "overgrant" is adjusted by reporting the difference between the granted timeslot and resource requirement. To facilitate this mechanism, we use "negative" queue length to measure the "overgrant." Hence, we will have

$$R_i(n+1)=R_i(n)+\lambda_i(n)-G_i^r(n+1). \qquad (12a)$$

Note that the "negative" REPORT indicates that the RS overgrants timeslots to LS_i. The prereserved network resource for LS_i is able to deliver all incoming data, and the queue of LS_i is empty after the current service cycle. In this scenario, the resource requirement only contains the estimated arriving data, i.e.,

$$G_i^r(n+1)=\alpha'_i(n)\lambda'_i(n). \qquad (12b)$$

From Eqs. (12a) and (12b), we obtain

$$R_i(n+1)=R_i(n)+\lambda_i(n)-\alpha'_i(n)\lambda'_i(n). \qquad (12c)$$

Following the similar linearization procedure detailed in Appendix B, the state space can be linearized to

$$\delta G_i^r(n+1)=\lambda'_{i0}\delta\alpha'_i(n)+\alpha'_{i0}\delta\lambda'_i(n), \qquad (13a)$$

$$\delta R_i(n+1)=\delta R_i(n)+\delta\lambda_i(n)-\lambda'_{i0}\delta\alpha'_i(n)-\alpha'_{i0}\delta\lambda'_i(n). \qquad (13b)$$

Equations (11c), (11d), (13a), and (13b) are the state space, which can be represented by Eq. (13):

$$\delta x_i(n+1)=A_2\delta x_i(n)+B_2\delta u_i(n),$$

where

$$A_2=\begin{bmatrix} 0 & 0 & 0 & \lambda'_{i0} \\ 0 & 1 & 0 & -\lambda'_{i0} \\ 0 & 0 & 1 & -\frac{\tau\lambda'_{i0}}{\lambda_{i0}} \\ 0 & 0 & 1 & 0 \end{bmatrix},\quad B_2=\begin{bmatrix} 0 & \alpha'_{i0} \\ 1 & -\alpha'_{i0} \\ \frac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\frac{\tau\alpha'_{i0}}{\lambda_{i0}} \\ 0 & 0 \end{bmatrix}.$$

Theorem 2: A P2MP network with NLPDRA is controllable when $d_i(n)=R_i(n-1)+\lambda_i(n-1)$.

Proof: Similarly, we analyze the controllability by evaluating matrix U in this scenario, where

$$U=[B_2\quad A_2B_2]=\begin{bmatrix} 0 & \alpha'_{i0} & 0 & 0 \\ 1 & -\alpha'_{i0} & 1 & -\alpha'_{i0} \\ \frac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\frac{\tau\alpha'_{i0}}{\lambda_{i0}} & \frac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\frac{\tau\alpha'_{i0}}{\lambda_{i0}} \\ 0 & 0 & \frac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\frac{\tau\alpha'_{i0}}{\lambda_{i0}} \end{bmatrix}.$$

Furthermore, we find that

$$|U|=\begin{vmatrix} 0 & \alpha'_{i0} & 0 & 0 \\ 1 & -\alpha'_{i0} & 1 & -\alpha'_{i0} \\ \frac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\frac{\tau\alpha'_{i0}}{\lambda_{i0}} & \frac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\frac{\tau\alpha'_{i0}}{\lambda_{i0}} \\ 0 & 0 & \frac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\frac{\tau\alpha'_{i0}}{\lambda_{i0}} \end{vmatrix}=\frac{\tau^2\alpha'^3_{i0}\lambda'_{i0}(\alpha'_{i0}\lambda'_{i0}-\lambda_{i0})}{\lambda_{i0}^4}.$$

Since τ, α'_{i0}, λ'_{i0}, and λ_{i0} are defined as positive real numbers, $|U|\neq 0$ *iff* $\alpha'_{i0}\lambda'_{i0}\neq\lambda_{i0}$ holds. Therefore, when $d_i(n)=R_i(n-1)+\lambda_i(n-1)$, the P2MP network system represented by Eq. (13) is controllable.

C. Scenario 3: $d_i(n)=G_i^{\max}$

In this scenario, the incoming traffic is heavy, and the RS uses the SLA upper bound $G_i^{\max}$ to limit the aggressive bandwidth requirement. The state space of this scenario turns into

$$G_i^r(n+1)=R_i(n)+\alpha'_i(n)\lambda'_i(n), \qquad (14a)$$

$$R_i(n+1) = R_i(n) - G_i^{\max} + \lambda_i(n). \tag{14b}$$

Following the similar linearization procedure detailed in Appendix C, the above equations can be linearized to

$$\delta G_i^r(n+1) = \delta R_i(n) + \lambda'_{i0}\delta\alpha'_i(n) + \alpha'_{i0}\delta\lambda'_i(n), \tag{15a}$$

$$\delta R_i(n+1) = \delta R_i(n) + \delta\lambda_i(n). \tag{15b}$$

Equations (11c), (11d), (15a), and (15b) are essentially the state space represented by Eq. (15):

$$\delta x_i(n+1) = A_3\delta x_i(n) + B_3\delta u_i(n),$$

where

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & \lambda'_{i0} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\dfrac{\tau\lambda'_{i0}}{\lambda_{i0}} \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 & \alpha'_{i0} \\ 1 & 0 \\ \dfrac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\dfrac{\tau\alpha'_{i0}}{\lambda_{i0}} \\ 0 & 0 \end{bmatrix}.$$

Similar to the previous two scenarios, the following theorem is established.

Theorem 3: A P2MP network system with NLPDRA is controllable when $d_i(n) = G_i^{\max}$.

Proof: Similarly, we check the controllability matrix U, i.e.,

$$U = [B_3 \quad A_3B_3] = \begin{bmatrix} 0 & \alpha'_{i0} & 1 & 0 \\ 1 & 0 & 1 & 0 \\ \dfrac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\dfrac{\tau\alpha'_{i0}}{\lambda_{i0}} & \dfrac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\dfrac{\tau\alpha'_{i0}}{\lambda_{i0}} \\ 0 & 0 & \dfrac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\dfrac{\tau\alpha'_{i0}}{\lambda_{i0}} \end{bmatrix}.$$

Furthermore, the determinant of U is

$$|U| = \begin{vmatrix} 0 & \alpha_{i0} & 1 & 0 \\ 1 & 0 & 1 & 0 \\ \dfrac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\dfrac{\tau\alpha'_{i0}}{\lambda_{i0}} & \dfrac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\dfrac{\tau\alpha'_{i0}}{\lambda_{i0}} \\ 0 & 0 & \dfrac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\dfrac{\tau\alpha'_{i0}}{\lambda_{i0}} \end{vmatrix} = \frac{\tau^2\alpha'^2_{i0}}{\lambda_{i0}^2} + \frac{\tau^2\alpha'^3_{i0}\lambda'_{i0}}{\lambda_{i0}^3}.$$

Since τ, α'_{i0}, λ'_{i0}, and λ_{i0} are defined as positive real numbers, $|U| \neq 0$ always holds. Hence, when $d_i(n) = G_i^{\max}$, the P2MP system represented by Eq. (15) is controllable.

The above three scenarios summarize all of the possible combinations of loaded traffic and granted transmission in a P2MP system. Theorems 1–3 testify that the P2MP network system with NLPDRA is completely controllable.

When the predictor underestimates the traffic, there will be residual data queued up at the LS buffer after one service cycle. This is the so-called "unsatisfied" case, and it falls into Scenario 1. Theorem 1 shows that the P2MP network system with NLPDRA can self-tune to reach the proper state of bandwidth sharing even when prediction inaccuracy occurs.

When the predictor overestimates the traffic, the total data arrived at an LS can be delivered to the RS within the current service cycle, with a small portion of the timeslot being "idle." This falls into Scenario 2. Theorem 2 indicates that the RS with NLPDBS is capable of eliminating the overreserved bandwidth by taking "overgrant" into consideration.

When the users aggressively request the upstream bandwidth, the RS employs the SLA specification to upper bound their transmission, and this falls into Scenario 3. Theorem 3 verifies that the P2MP network system with NLPDRA is capable of limiting the aggressive bandwidth competition among users, and the upstream bandwidth is thus arbitrated fairly.

IV. Stability Analysis and Controller Design

By "stable," we mean that, when the input traffic load changes dramatically, the resource allocation scheme is able to provide the upstream resource fair share among the LSs with optimal bandwidth utilization [15]. For any resource allocation scheme, the stability design is critical because it provides predictability for system behavior and guarantees any generated oscillations to be bounded within a certain range. On the other hand, the instability usually leads to unbounded oscillations that lower the overall network efficiency.

The open plant denoted by Eq. (8) usually implies an unbounded output. For any resource allocation scheme that is controllable, there always exists a controller,

$$u_i(n) = -K_i x_i(n) + F_i r_i(n), \tag{16a}$$

which drives the system into the stable state; this is known as *pole placement* [14]. Here, K_i is a constant matrix, F_i is a predefined matrix, and $r_i(n)$ is a reference vector.

Substituting Eq. (16a) into Eq. (8) yields

$$x_i(n+1) = (A_i - B_iK_i)x_i(n) + B_iF_ir_i(n). \tag{16b}$$

Therefore, by implementing the controller of Eq. (16a), (A_i, B_i) is transformed into $(A_i - B_iK_i, B_iF_i)$. The controllability of a resource allocation scheme is unaltered by state feedback [14]. That is, if (A_i, B_i) is controllable (or uncontrollable), so is $(A_i - B_iK_i, B_iF_i)$ for any K_i and F_i. Since F_i and $r_i(n)$ have no impact on the system's stability [14], we will focus on K_i, which dominates the system stability. Assuming the reference vector $r_i(n) = 0$, we will have

$$u_i(n) = -K_i x_i(n), \tag{17a}$$

$$x_i(n+1) = (A_i - B_iK_i)x_i(n). \tag{17b}$$

Hence, after implementing the controller of Eq. (17a), the system becomes the closed-loop form expressed in Eq. (17b).

Similarly, Eq. (9) gives three different traffic scenarios for the P2MP system, depending on the loaded traffic from end users:

1) $d_i(n) = G_i^r(n)$,
2) $d_i(n) = R_i(n-1) + \lambda_i(n-1)$, and
3) $d_i(n) = G_i^{\max}$.

A. Scenario 1: $d_i(n)=G_i^r(n)$

In the previous section, we developed the linearized state space equation for NLPDRA under Scenario 1 as Eq. (11):

$$\delta x_i(n+1)=A_1\delta x_i(n)+B_1\delta u_i(n),$$

where

$$A_1=\begin{bmatrix}0 & 1 & 0 & \lambda'_{i0}\\ 0 & 0 & 0 & -\lambda'_{i0}\\ 0 & 0 & 1 & -\dfrac{\tau\lambda'_{i0}}{\lambda_{i0}}\\ 0 & 0 & 1 & 0\end{bmatrix},\quad B_1=\begin{bmatrix}0 & \alpha'_{i0}\\ 1 & -\alpha'_{i0}\\ \dfrac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\dfrac{\tau\alpha'_{i0}}{\lambda_{i0}}\\ 0 & 0\end{bmatrix}$$

Theorem 4: In Scenario 1, a P2MP network system with NLPDRA is stable when $0<\tau<\lambda_0/\lambda'_0$.

Proof: The discrete system represented by Eq. (11) is stable *iff* eigenvalues of the state matrix A_1 fall inside the unit circle [14]. Let $|zI-A_1|=0$; we have

$$\det(zI-A_1)=D(z)=z^4-z^3+\frac{\tau\lambda'_0}{\lambda_0}z^2. \tag{18a}$$

According to Jury's criteria [16], a fourth-order system $D(z)=\Sigma_{i=0}^4 a_i z^i$ is stable *iff* the following rules are all fulfilled:

$$\text{Rule 1: } a_0^2-a_4^2-a_0a_3+a_1a_4<0, \tag{18b}$$

$$\text{Rule 2: } a_0^2-a_4^2+a_0a_3-a_1a_4<0, \tag{18c}$$

$$\text{Rule 3: } a_0^3+2a_0a_2a_4+a_1a_3a_4-a_0a_4^2-a_2a_4^2-a_0a_3^2-a_0^2a_4 -a_0^2a_2-a_1^2a_4+a_4^3+a_0a_1a_3>0, \tag{18d}$$

$$\text{Rule 4: } D(1)>0, \tag{18e}$$

$$\text{Rule 5: } D(-1)>0. \tag{18f}$$

Equation (17a) implies $a_0=a_1=0$, $a_2=\tau\lambda'_0/\lambda_0$, $a_3=-1$, and $a_4=1$. By applying Rules 1–5 and considering τ a positive real number, the system is stable when $0<\tau<\lambda_0/\lambda'_0$.

B. Scenario 2: $d_i(n)=R_i(n-1)+\lambda_i(n-1)$

Similarly, we have developed the state space equation for NLPDRA in the previous section under Scenario 2 as Eq. (13):

$$\delta x_i(n+1)=A_2\delta x_i(n)+B_2\delta u_i(n),$$

where

$$A_2=\begin{bmatrix}0 & 0 & 0 & \lambda'_{i0}\\ 0 & 1 & 0 & -\lambda'_{i0}\\ 0 & 0 & 1 & -\dfrac{\tau\lambda'_{i0}}{\lambda_{i0}}\\ 0 & 0 & 1 & 0\end{bmatrix},\quad B_2=\begin{bmatrix}0 & \alpha'_{i0}\\ 1 & -\alpha'_{i0}\\ \dfrac{\tau\alpha'_{i0}\lambda'_{i0}}{\lambda_{i0}^2} & -\dfrac{\tau\alpha'_{i0}}{\lambda_{i0}}\\ 0 & 0\end{bmatrix}.$$

Theorem 5: In scenario 2, a P2MP network system with NLPDRA is stable when implementing the controller

$$u_i(n)=-K_1x_i(n), \tag{19}$$

where

$$K_1=\begin{bmatrix}k_{11} & k_{12} & k_{13} & k_{14}\\ k_{21} & k_{22} & k_{23} & k_{24}\end{bmatrix}.$$

The range of vectors of K_1 is given by

$$\begin{cases}L_0^2-L_0L_3+L_1-1<0\\ L_0^2+L_0L_3-L_1-1<0\\ L_0^3+2L_0L_2+L_1L_3-L_0-L_2-L_0L_3^2-L_0^2-L_0^2L_2-L_1^2+L_0L_1L_3+1>0\\ 1+L_3+L_2+L_1+L_0>0\\ 1-L_3+L_2-L_1+L_0>0\end{cases}, \tag{20}$$

where $L_i|_{i=0-4}$ are given by Eqs. (20c)–(20g).

Proof: Let $|zI-A_2|=0$; we have

$$\det(zI-A_2)=D(z)=z^4-2z^3+\left(1+\frac{\tau\lambda'_0}{\lambda_0}\right)z^2-\frac{\tau\lambda'_0}{\lambda_0}z. \tag{20a}$$

It is easy to check whether the coefficients in Eq. (20a) violate Jury's criteria. However, since NLPDRA is completely controllable [9], there always exists a controller $u_i(n)=-K_1x_i(n)$ that can drive the system into the stable state [14]. After implementing such a controller, the system becomes $x_i(n+1)=(A_2-B_2K_1)x_i(n)$. This discrete system is stable *iff* eigenvalues of the state matrix $(A_2-B_2K_1)$ fall inside the unit circle [16]. Let $|zI-(A_2-B_2K_1)|=0$; by solving this 4×4 matrix, we have

$$\det[(zI-(A_2-B_2K_1)]=D(z)=z^4+L_3z^3+L_2z^2+L_1z+L_0. \tag{20b}$$

The coefficients $L_i|_{i=0-4}$ are given by

$$L_0=\left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}k_{12}-\frac{\tau\alpha_0'}{\lambda_0}k_{22}\right)[(\lambda_0'-\alpha_0'k_{24}+k_{14})\alpha_0'k_{21}+(k_{11}-\alpha_0'k_{21})(\alpha_0'k_{24}-\lambda_0')]-\alpha_0'k_{22}(k_{11}-\alpha_0'k_{21})\left(\frac{\tau\lambda_0'}{\lambda_0}-\frac{\tau\lambda_0'}{\lambda_0}k_{24}+\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}k_{14}\right)$$
$$+\left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}k_{11}-\frac{\tau\alpha_0'}{\lambda_0}k_{21}\right)[\alpha_0'k_{22}(\lambda_0'-\alpha_0'k_{24}+k_{14})-(\alpha_0'k_{22}-1-k_{12})(\alpha_0'k_{24}-\lambda_0')], \tag{20c}$$

$$L_1=\left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}k_{11}-\frac{\tau\alpha_0'}{\lambda_0}k_{21}\right)[\alpha_0'k_{22}(k_{13}-\alpha_0'k_{23})-\alpha_0'k_{23}(\alpha_0'k_{22}-k_{12}-1)]-\left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}k_{12}-\frac{\tau\alpha_0'}{\lambda_0}k_{22}\right)\times[\alpha_0'k_{21}(k_{13}-\alpha_0'k_{23})+\alpha_0'k_{23}$$
$$\times(k_{11}-\alpha_0'k_{21})]+\left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}k_{13}-\frac{\tau\alpha_0'}{\lambda_0}k_{23}-1\right)[\alpha_0'k_{21}(\alpha_0'k_{22}-k_{12}-1)-\alpha_0'k_{22}(k_{11}-\alpha_0'k_{21})]+\left(\frac{\tau\lambda_0'}{\lambda_0}-\frac{\tau\alpha_0'}{\lambda_0}k_{22}+\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}k_{14}\right)$$
$$\times(2\alpha_0'k_{22}-k_{12})-(\lambda_0'-\alpha_0'k_{24}+k_{14})\times\left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}k_{12}-\frac{\tau\alpha_0'}{\lambda_0}k_{22}\right)-\left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}k_{11}-\frac{\tau\alpha_0'}{\lambda_0}k_{21}\right)(\alpha_0'k_{24}-\lambda_0'), \tag{20d}$$

$$L_2=\alpha_0'k_{21}(\alpha_0'k_{22}-k_{12}-1)-\alpha_0'k_{22}(k_{11}-\alpha_0'k_{21})+\left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}k_{13}-\frac{\tau\alpha_0'}{\lambda_0}k_{23}-1\right)\times(\alpha_0'k_{21}+\alpha_0'k_{22}-k_{12}-1)$$
$$-\alpha_0'k_{23}\left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}k_{11}-\frac{\tau\alpha_0'}{\lambda_0}k_{21}\right)-(k_{13}-\alpha_0'k_{23})\left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}k_{12}-\frac{\tau\alpha_0'}{\lambda_0}k_{22}\right)+\frac{\tau\lambda_0'}{\lambda_0}-\frac{\tau\alpha_0'}{\lambda_0}k_{24}+\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}k_{14}, \tag{20e}$$

$$L_3=\alpha_0'k_{21}+\alpha_0'k_{22}-k_{12}+\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}k_{13}-\frac{\tau\alpha_0'}{\lambda_0}k_{23}-2, \tag{20f}$$

$$L_4=1. \tag{20g}$$

By applying Jury's criteria, this fourth-order system is stable *iff* Rules 1–5 [i.e., Eqs. (18b)–(18f)] are all fulfilled as in Eq. (20):

$$\begin{cases} L_0^2-L_0L_3+L_1-1<0 \\ L_0^2+L_0L_3-L_1-1<0 \\ L_0^3+2L_0L_2+L_1L_3-L_0-L_2-L_0L_3^2-L_0^2-L_0^2L_2-L_1^2+L_0L_1L_3+1>0 \\ 1+L_3+L_2+L_1+L_0>0 \\ 1-L_3+L_2-L_1+L_0>0 \end{cases}.$$

C. Scenario 3: $d_i(n)=G_i^{max}$

Similarly, we have developed the state space equation for NLPDRA in the previous section under scenario 3 as Eq. (15):

$$\delta x_i(n+1)=A_3\delta x_i(n)+B_3\delta u_i(n),$$

where

$$A_3=\begin{bmatrix}0&1&0&\lambda_{i0}'\\0&1&0&0\\0&0&1&-\dfrac{\tau\lambda_{i0}'}{\lambda_{i0}}\\0&0&1&0\end{bmatrix},\quad B_3=\begin{bmatrix}0&\alpha_{i0}'\\1&0\\\dfrac{\tau\alpha_{i0}'\lambda_{i0}'}{\lambda_{i0}^2}&-\dfrac{\tau\alpha_{i0}'}{\lambda_{i0}}\\0&0\end{bmatrix}.$$

Theorem 6: In Scenario 3, a P2MP network system with NLPDRA is stable when implementing the controller

$$u_i(n)=-K_2x_i(n), \tag{21}$$

where

$$K_2=\begin{bmatrix}p_{11}&p_{12}&p_{13}&p_{14}\\p_{21}&p_{22}&p_{23}&p_{24}\end{bmatrix}.$$

The range of vectors of K_2 is given by

$$\begin{cases} M_0^2 - M_0M_3 + M_1 - 1 < 0 \\ M_0^2 + M_0M_3 - M_1 - 1 < 0 \\ M_0^3 + 2M_0M_2 + M_1M_3 - M_0 - M_2 - M_0M_3^2 - M_0^2 - M_0^2M_2 - M_1^2 + M_0M_1M_3 + 1 > 0 \\ 1 + M_3 + M_2 + M_1 + M_0 > 0 \\ 1 - M_3 + M_2 - M_1 + M_0 > 0 \end{cases}, \tag{22}$$

where $M|_{i=0-4}$ are given by

$$M_0 = \left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}p_{11} - \frac{\tau\alpha_0'}{\lambda_0}p_{21}\right)[p_{14}(\alpha_0'p_{22} - 1) - (\alpha_0'p_{24} - \lambda_0') \times (p_{12} - 1)] + \left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}p_{12} - \frac{\tau\alpha_0'}{\lambda_0}p_{22}\right)[\alpha_0'p_{14}p_{21} + p_{11} \times (\alpha_0'p_{24} - \lambda_0')] + \left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}p_{14} - \frac{\tau\alpha_0'}{\lambda_0}p_{24} + \frac{\tau\lambda_0'}{\lambda_0}\right) \times [\alpha_0'p_{21}(p_{12} - 1) - p_{11}(\alpha_0'p_{22} - 1)], \tag{22a}$$

$$M_1 = p_{13}(\alpha_0'p_{22} - 1)(\alpha_0'p_{24} - \lambda_0') - \left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}p_{11} - \frac{\tau\alpha_0'}{\lambda_0}p_{21}\right) \times [\alpha_0'p_{23}(p_{12} - 1) + \alpha_0'p_{24} - \lambda_0'] + \left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}p_{12} - \frac{\tau\alpha_0'}{\lambda_0}p_{22}\right) \times (\alpha_0'p_{11}p_{23} - \alpha_0'p_{13}p_{21} - p_{14}), \tag{22b}$$

$$M_2 = \left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}p_{13} - \frac{\tau\alpha_0'}{\lambda_0}p_{23} - 1\right)(\alpha_0'p_{21} + p_{12} - 1) - p_{11}(\alpha_0'p_{22} - 1) - \alpha_0'p_{23}\left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}p_{11} - \frac{\tau\alpha_0'}{\lambda_0}p_{21}\right) - p_{13}\left(\frac{\tau\alpha_0'}{\lambda_0^2}p_{12} - \frac{\tau\alpha_0'}{\lambda_0}p_{22}\right) + \left(\frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}p_{14} - \frac{\tau\alpha_0'}{\lambda_0}p_{24} + \frac{\tau\lambda_0'}{\lambda_0}\right), \tag{22c}$$

$$M_3 = \alpha_0'p_{21} + p_{12} + \frac{\tau\alpha_0'\lambda_0'}{\lambda_0^2}p_{13} - \frac{\tau\alpha_0'}{\lambda_0}p_{23} - 2, \tag{22d}$$

$$M_4 = 1. \tag{22e}$$

The above three scenarios summarize all of the possible combinations of loaded traffic and granted transmission in a P2MP network system.

When the predictor underestimates the traffic, there will be residual data queued up at the LS buffer after one service cycle. This is the so-called "unsatisfied" case and it falls into Scenario 1. Theorem 4 shows that the P2MP network system with NLPDRA can self-tune to reach the stable state of bandwidth sharing even with prediction inaccuracy.

When the predictor overestimates the traffic, the total data arrived at an LS can be delivered to the RS within the current service cycle, with a small portion of the timeslot being "idle." This falls into Scenario 2. Theorem 5 indicates that, by implementing the suitable controller Eq. (19), the RS works as a central controller to tune LSs accordingly, ensuring that the upstream bandwidth of a P2MP network system is fairly shared by multiple LSs.

When the users aggressively request the upstream bandwidth, the RS employs the SLA specification to upper bound their transmission, and this falls into Scenario 3. Theorem 6 shows that the P2MP network system with NLPDRA is capable of guaranteeing the system's stability by implementing the controller Eq. (21). In the last two scenarios, K_1 and K_2 essentially describe the controller characteristics in different scenarios, and their relationship to the estimation index has been revealed in Theorems 5 and 6.

V. Optimal Compensator Design for P2MP Networks

In the previous sections, we discussed the controllability and stability of NLPDRA of a P2MP system. The proposed state space model provides a compact way to reveal the system-level characteristics of P2MP networks. In this section, we will further discuss the optimal compensator design based on the proposed model.

The design objective for resource allocation in a P2MP system is to achieve the system accuracy. A control system is said to be "accurate" if the measured output converges or becomes sufficiently close to the reference input [17]. In a P2MP system, the reference input r can be chosen from various SLA parameters or other predefined parameters. The measured output $Y_i(n)$ is thus required to converge to r in order to ensure that the control objectives are met. In this paper, we choose the desired queue length Q_i^d of LS_i as the reference input, i.e., $r = Q_i^d$. The desired queue length Q_i^d is defined as the efficient queue length to achieve high network resource utilization. Theoretically, each LS needs to maintain a desired queue length Q_i^d to avoid overflow or emptiness [18,19]. If the queue length is too large, data loss and retransmission are inevitable because of the limited available buffers; on the other hand, if the queue length becomes empty, it indicates that the allocated resource for this LS is always more than it actually needs. The network resource is thus wasted with low utilization. Both of the extremes should be avoided by maintaining a desired queue length Q_i^d.

The objective of achieving system accuracy essentially requires that the output $Y_i(n)$, which is the measurement of the report queue length, converges to the system input r, the desired queue length. To reach this objective, a compensator F_i is implemented right after the reference and is added up to the feedback from the state variable to form the controller $U_i(n)$, which is illustrated in Fig. 3. The key issue here is to design a compensator F_i in such a way that it can offset the control error, i.e., $e(n)=0$.

In general, it is difficult to meet optimal queue requirements because of the complexity of mapping these requirements into the corresponding scheduling algorithms and resource management schemes. However, the proposed state space model gives a simple and straightforward framework to achieve this objective by using the state space feedback control techniques.

Consider the measured system output $Y_i(n)=CX_i(n)$, and define matrix $C=[0\ 1]$. The system output is essentially the measurement to the report queue length $Q_i(n)$. The state space system is then described by

$$X_i(n+1)=AX_i(n)+BU_i(n),$$

$$Y_i(n)=CX_i(n). \tag{23}$$

Our target is to design a controller,

$$U_i(n)=-K_iX_i(n)+F_ir, \tag{24}$$

to achieve the design objectives of system accuracy. Figure 3 illustrates our approach by using the controller of Eq. (24) to achieve the prescribed objectives. The reference input r is the desired queue length, and thus $e(n)=Y_i(n)-r$ is the control error. The matrix F_i is a compensator to offset the control error, so that the system output can eventually converge to the input reference [i.e., $e(n)=0$]. Our focus is now on the design of a suitable compensator F_i.

For a particular P2MP system i, the compensator F_i is determined by the state matrix A_i, the input matrix B_i, the output matrix C_i, and the controller gain K_i. In Appendix D, we prove that the compensator F_i that drives the control error $e(n)=Y_i(n)-r$ to zero is given by

$$F_i=[K_i \quad 1]\begin{bmatrix} A_i-I & B_i \\ C_i & 0 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{25}$$

where

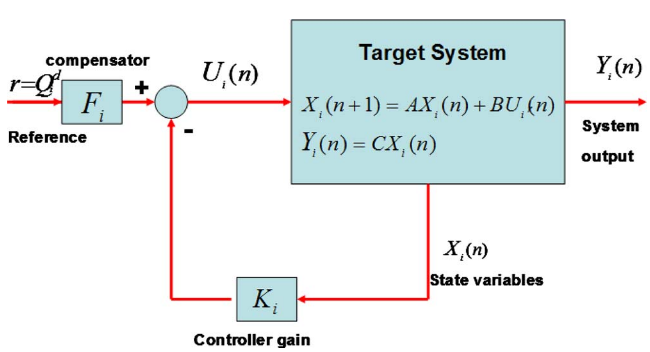

Fig. 3. (Color online) Controller design to meet transient performance objectives.

$$\begin{bmatrix} A_i-I & B_i \\ C_i & 0 \end{bmatrix}$$

is a nonsingular matrix. Please refer to Appendix D for detailed derivation of compensator F_i.

Consequently, by implementing the compensator of Eq. (25), the controller of Eq. (24) is able to force the system output $Y_i(n)$ to track the reference input r, implying that the queue length can be eventually driven into the desired queue length Q_i^d.

VI. Conclusions

In this paper, we have analyzed and verified through the state space model that the implementation of NLPDRA in a P2MP system such as a passive optical network maintains the system controllability. The traffic predictor is robust to dynamic traffic load. A P2MP network system with NLPDRA is able to reach the optimum state of upstream bandwidth allocation, no matter how dynamic the input traffic is. Furthermore, we have also analyzed and verified through the state space model that the implementation of NLPDRA with suitable controllers maintains the P2MP network system stability. The employed traffic predictor is robust to dynamic traffic load, and the resource utilization can be improved by adaptive control at the root station side (OLT for a PON network). Finally, we expanded our discussion of the proposed state space model to the optimal compensator design and provide the guidelines for designing the optimal compensator to reach the system accuracy.

Appendix A: Linearization of the System in Scenario 1

We first define the right-hand sides of Eqs. (10a), (10c), (8a), and (8b) as

$$f_1(R_i,\alpha_i',\lambda_i')=R_i+\alpha_i'\lambda_i', \tag{A.1a}$$

$$f_2(\lambda_i,\alpha_i',\lambda_i')=\lambda_i-\alpha_i'\lambda_i', \tag{A.1b}$$

$$f_3(\alpha_i,\alpha_i',\lambda_i,\lambda_i')=\alpha_i+\tau-\tau\times\frac{\alpha_i'\times\lambda_i'}{\lambda_i}, \tag{A.1c}$$

$$f_4(\alpha_i)=\alpha_i. \tag{A.1d}$$

Taking partial derivatives at the equilibrium point $(G_{i0}^r,R_{i0},\alpha_{i0},\alpha_{i0}',\lambda_{i0},\lambda_{i0}')$ yields

$$\frac{\partial f_1}{\partial R_i}=1,\quad \frac{\partial f_1}{\partial \alpha_i'}=\lambda_{i0}',\quad \frac{\partial f_1}{\partial \lambda_i'}=\alpha_{i0}', \tag{A.1e}$$

$$\frac{\partial f_2}{\partial \lambda_i}=1,\quad \frac{\partial f_2}{\partial \alpha_i'}=-\lambda_{i0}',\quad \frac{\partial f_2}{\partial \lambda_i'}=-\alpha_{i0}', \tag{A.1f}$$

$$\frac{\partial f_3}{\partial \alpha_i}=1,\quad \frac{\partial f_3}{\partial \alpha_i'}=\frac{\tau\lambda_{i0}'}{\lambda_{i0}},\quad \frac{\partial f_3}{\partial \lambda_i}=-\frac{\tau\alpha_{i0}'\lambda_{i0}'}{\lambda_{i0}^2},$$

$$\frac{\partial f_3}{\partial \lambda_i'} = \frac{\tau\alpha_{i0}'}{\lambda_{i0}}, \tag{A.1g}$$

$$\frac{\partial f_4}{\partial \alpha_i} = 1. \tag{A.1h}$$

Therefore, the system denoted by Eqs. (10a), (10c), (8a), and (8b) can be linearized at the equilibrium point $(G_{i0}^r, R_{i0}, \alpha_{i0}, \alpha_{i0}', \lambda_{i0}, \lambda_{i0}')$ as Eqs. (11a)–(11d).

Appendix B. Linearization of the System in Scenario 2

In this scenario, the linearization of Eqs. (8a) and (8b) is unchanged because it follows the same LMS algorithm to update the estimate index $\alpha_i(n+1)$. We then focus on the linearization of loaded traffic and queue length in Scenario 2. We define the right-hand sides of Eqs. (13a) and (13b) by

$$f_5(\alpha_i, \lambda_i) = \alpha_i \lambda_i, \tag{B.1a}$$

$$f_6(G_i^r, \alpha_i', \lambda_i, \lambda_i') = -G_i^r + \lambda_i + (\alpha_i' - 1)\lambda_i'. \tag{B.1b}$$

Taking partial derivatives at the equilibrium point $(G_{i0}^r, R_{i0}, \alpha_{i0}, \alpha_{i0}', \lambda_{i0}, \lambda_{i0}')$ yields

$$\frac{\partial f_5}{\partial \alpha_i} = \lambda_{i0}, \quad \frac{\partial f_5}{\partial \lambda_i} = \alpha_{i0}, \tag{B.1c}$$

$$\frac{\partial f_6}{\partial G_i^r} = -1, \quad \frac{\partial f_6}{\partial \alpha_i'} = \lambda_{i0}', \quad \frac{\partial f_6}{\partial \lambda_i} = 1,$$

$$\frac{\partial f_6}{\partial \lambda_i'} = (\alpha_{i0}' - 1). \tag{B.1d}$$

Therefore, the system denoted by Eqs. (8a), (8b), (13a), and (13b) can be linearized at the equilibrium point $(G_{i0}^r, R_{i0}, \alpha_{i0}, \alpha_{i0}', \lambda_{i0}, \lambda_{i0}')$ as Eqs. (11c), (11d), (14a), and (14b).

Appendix C. Linearization of the System in Scenario 3

Similarly, we define the right-hand sides of Eqs. (15a) and (15b) by

$$f_7(R_i, \alpha_i, \lambda_i) = R_i + \alpha_i \lambda_i, \tag{C.1a}$$

$$f_8(R_i, \lambda_i) = R_i - G_i^{\max} + \lambda_i. \tag{C.1b}$$

Taking partial derivatives at the equilibrium point $(G_{i0}^r, R_{i0}, \alpha_{i0}, \alpha_{i0}', \lambda_{i0}, \lambda_{i0}')$ yields

$$\frac{\partial f_7}{\partial R_i} = 1, \quad \frac{\partial f_7}{\partial \alpha_i} = \lambda_{i0}, \quad \frac{\partial f_7}{\partial \lambda_i} = \alpha_{i0}, \tag{C.1c}$$

$$\frac{\partial f_8}{\partial R_i} = 1, \quad \frac{\partial f_8}{\partial \lambda_i} = 1. \tag{C.1d}$$

The linearization of Eqs. (8a) and (8b) follows the same process as in Scenario 1, resulting in Eqs. (11c) and (11d). The system denoted by Eqs. (8a), (8b), (15a), and (15b) can be linearized at the equilibrium point $(G_{i0}^r, R_{i0}, \alpha_{i0}, \alpha_{i0}', \lambda_{i0}, \lambda_{i0}')$ as Eqs. (11c), (11d), (16a), and (16b).

Appendix D. Derivation of the Compensator F_i

According to feedback control theory [17], when the system output converges to the input reference (i.e., $e(n) = Y_i(n) - r = 0$), the state variable $X_i(n)$ reaches its steady state X_i^{ss}. Assume the associated steady state input is U_i^{ss}, the controller represented by Eq. (24) can thus be rewritten as

$$U_i(n) = -K_i(X_i(n) - X_i^{ss}) + U_i^{ss}. \tag{D.1a}$$

It is obvious from Eq. (D.1a) that the system input $U_i(n)$ reaches its steady state U_i^{ss} when the state variables $X_i(n)$ reach X_i^{ss}. Equation (D.1a) can be further rewritten as $U_i(n) = -K_i X_i(n) + K_i X_i^{ss} + U_i^{ss}$, i.e.,

$$U_i(n) = -K_i X_i(n) + [K_i \quad 1]\begin{bmatrix} X_i^{ss} \\ U_i^{ss} \end{bmatrix}. \tag{D.1b}$$

When the system reaches the steady state, the following equations hold:

$$X_i^{ss} = AX_i^{ss} + BU_i^{ss},$$

$$Y_i^{ss} = CX_i^{ss},$$

$$Y_i^{ss} = r. \tag{D.2a}$$

Equation (D.2a) further yields

$$\begin{matrix} (A-I)X_i^{ss} + BU_i^{ss} = 0 \\ CX_i^{ss} = r \end{matrix}, \quad \text{i.e.,} \quad \begin{bmatrix} A-I & B \\ C & 0 \end{bmatrix}\begin{bmatrix} X_i^{ss} \\ U_i^{ss} \end{bmatrix} = \begin{bmatrix} 0 \\ r \end{bmatrix}. \tag{D.2b}$$

From Eq. (D.2b), we get

$$\begin{bmatrix} X_i^{ss} \\ U_i^{ss} \end{bmatrix} = \begin{bmatrix} A-I & B \\ C & 0 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ r \end{bmatrix}, \tag{D.2c}$$

provided that $\begin{bmatrix} A-1 & B \\ C & 0 \end{bmatrix}$ is a nonsingular matrix.

From Eqs. (D.1b) and (D.2c), we have the controller in the following format:

$$U_i(n) = -K_i X_i(n) + [K_i \quad 1]\begin{bmatrix} A-I & B \\ C & 0 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} r. \tag{D.3}$$

By comparing Eqs. (D.1b) and (D.3), the compensator F_i to offset the control error is

$$F_i = [K_i \quad 1]\begin{bmatrix} A-I & B \\ C & 0 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{D.4}$$

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