On Preemptive Multi-wavelength Scheduling in Hybrid WDM/TDM Passive Optical Networks

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***Abstract*—In this paper, we investigate the wavelength scheduling problem in hybrid wavelength division multiplexing/time division multiplexing passive optical networks (WDM/TDM PONs), which can be mapped into multiprocessor scheduling problems with wavelengths and optical network unit (ONU) requests being considered as machines and jobs, respectively. To achieve high bandwidth utilization, guarantee low delay, and ensure short-term fairness, we try to construct a schedule with the minimum latest job completion time. First, we investigate the non-preemptive scheduling problem, which was shown to be NP-hard, and hence requires heuristic algorithms to approximate the optimal solution. The approximation ratio of the best heuristic algorithm is as large as $2-1/m$, where m is the number of wavelengths. Motivated to achieve a smaller latest job completion time, we then investigate the preemptive scheduling problem. Preemption allows jobs to be scheduled more flexibly, and thus may yield a smaller makespan. However, with preemption, jobs may be split into subjobs and scheduled in discontinuous time durations at the expense of more guard time. We show that, with the consideration of guard time, the preemptive scheduling with the objective of minimizing the latest job completion time is NP-hard. To address the problem, we propose an approach by using linear programming with guard time supplement. It is shown that the proposed algorithm can ensure that the latest job completion time is no greater than the optimal value plus $(m-1)g/m$, where g is the guard time between the scheduling of two ONUs. When the network is highly loaded, the approximation ratio is around 1.00061 and 1.002056 for hybrid WDM/TDM Ethernet PON (EPON) and Gigabit-capable PON (GPON), respectively.**

***Index Terms*—Approximation ratio; Delay; Fairness; Hybrid WDM/TDM PON; Wavelength scheduling.**

I. Introduction

By utilizing wavelength division multiplexing (WDM), hybrid WDM/time division multiplexing (TDM) passive optical networks (PONs) [1–3] dramatically increase their bandwidth provisioning, and are potentially able to accommodate bandwidth-demanding applications such as multimedia; by virtue of TDM, hybrid WDM/TDM PONs enable one wavelength to be shared by multiple ONUs, thus facilitating the statistical gain of traffic from multiple optical network units (ONUs) and efficient utilization of wavelength resources. Besides, hybrid WDM/TDM PONs can be deployed by smoothly migrating from the currently deployed TDM PONs [4,5], and thereby bridge the gap between TDM PONs and WDM PONs.

In hybrid WDM/TDM PONs, traffic from multiple ONUs is multiplexed in TDM fashion onto wavelength channels. Owing to the shared nature of the wavelength channel, hybrid WDM/TDM PONs require proper media access control (MAC) protocols to coordinate communications between the optical line terminal (OLT) and ONUs so that the collision of data transmissions from more than one ONU can be avoided. For backward compatibility, the MAC protocol of hybrid WDM/TDM PONs inherits some characteristics from those of Ethernet PON (EPON) and Gigabit-capable PON (GPON), two major flavors of the existing TDM PONs [6,7]. The data transmission processes of these two PONs are similar and can be generalized as follows: ONUs report their queue lengths and send their data packets to the OLT using time slots allocated by OLT; OLT collects queue requests, makes bandwidth allocation decisions, and then notifies ONUs when and on which channel they can transmit packets. We believe that such a request-grant-based transmission mechanism is highly likely to be adopted in hybrid WDM/TDM PONs for consistency [2,8,9]. Following the assumption of the request-grant-based MAC mechanism, OLT assumes most of the intelligence and control of the network, and its functions determine the performance of the network.

One important issue that needs to be addressed in hybrid WDM/TDM PONs is the wavelength scheduling problem, which does not exist in TDM PONs with only one wavelength in each stream, or WDM PONs with dedicated wavelengths for each ONU [10]. Generally, the wavelength assignment problem can be formulated as follows: given ONU requests as well as the supported wavelengths of ONUs, assign wavelengths to ONUs such that the queue requests of ONUs can be satisfied.

In this paper, we map the wavelength assignment problem into a multiprocessor scheduling problem [11], where wavelengths are mapped into machines and ONU requests are mapped into jobs. We try to minimize the latest ONU request (job) scheduling time among all requests for the sake of small delay, fairness, and load balancing. The objective is equivalent to minimizing the makespan in multiprocessor scheduling. Formerly, McGarry *et al.* [12] proposed using the longest processing time (LPT) first rule to minimize the makespan for the case that ONUs can access all the wavelengths. When ONUs can access a limited set of wavelengths, ONUs

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are scheduled with the least flexible job (LFJ) first rule. In terms of multiprocessor scheduling, the scenario that each ONU can access all wavelengths is equivalent to the parallel machine case, while the scenario that ONUs can only access a limited set of wavelengths is equivalent to the scheduling with machine eligibility constraints.

This paper presents the following contributions in addressing the scheduling problem. First, we investigate the scheduling problem without job preemption. The problem was shown to be NP-hard. The smallest approximation ratio, to the best of our knowledge, is $2-1/m$ (m is the number of wavelengths) when ONUs can only access a limited set of wavelengths. The approximation ratio of an algorithm to a minimization problem is the ratio between the result obtained by the algorithm and the optimal solution. Thus, the closer the ratio to 1, the better the algorithm. Motivated to achieve better performances, we investigate the preemptive scheduling problem. Preemption allows jobs to be scheduled more flexibly, and thus may yield a smaller makespan. However, with preemption, jobs may be scheduled in discontinuous time durations, thus requiring more guard time. We show that, with the consideration of guard time, the minimum makespan preemptive scheduling is NP-hard. To address the problem, we propose an approach by using linear programming with guard time supplement. It is shown that the proposed algorithm can ensure that the latest job completion time is no greater than the optimal value plus $(m-1)\mathfrak{g}/m$, where $\mathfrak{g}$ is the guard time between the scheduling of two ONUs. In this paper, we do not consider the impact of laser tuning time on the system performance [13]. When the network is highly loaded, the approximation ratio is around 1.00061 and 1.002056 for hybrid WDM/TDM EPON and GPON, respectively.

The remainder of the paper is organized as follows. Section II discusses two classes of hybrid WDM/TDM PON architectures and their scheduling problems. Section III presents the MAC protocol and time allocation in hybrid WDM/TDM PONs, and formulates the wavelength assignment problems. Section IV discusses the non-preemptive scheduling of the minimum makespan problem. Section V details our proposed preemptive scheduling under various cases. Section VI analyzes and compares the performances of preemptive and non-preemptive scheduling. Section VII presents our concluding remarks.

II. Hybrid WDM/TDM PON Architectures

As its name suggests, the hybrid WDM/TDM PON employs both WDM and TDM technologies, and thus it inherits some advantages of both [14]. WDM PONs provision high bandwidth, but require dedicated wavelengths for each ONU; TDM PONs allow wavelength sharing among ONUs and are more bandwidth efficient, but suffer from low bandwidth provisioning. Owing to the advantages of both high and efficient bandwidth provisioning, hybrid WDM/TDM PONs have received broad research attention recently, and many hybrid WDM/TDM PON architectures have been proposed, but no one is dominant. From the MAC layer's perspective, we can categorize them into two classes: laser-sharing based and wavelength-sharing based.

Laser-sharing-based networks refer to those with ONUs sharing the usage of a set of lasers. One typical example is SUCCESS [15], which equips a central office with tunable lasers and an arrayed waveguide grating (AWG), and ONUs with WDM filters and a burst-mode receiver. The wavelengths from OLT can reach ONUs through different PONs. All these tunable lasers are shared by ONUs, and they communicate with an ONU by tuning to the particular wavelength accommodated by that ONU. In the laser-sharing-based approach, although ONUs have their respective dedicated wavelengths, they cannot transmit data traffic independently, but need to share the usage of lasers in TDM fashion. From the perspective of the MAC layer, an important problem with this network is to dynamically assign these tunable lasers to ONUs to accommodate their respective traffic demands. The architecture discussed in [1] constitutes another example.

Wavelength-sharing-based networks contain those with wavelengths being shared by multiple ONUs. One example is the one proposed in [16], where signals from ONUs are first time division multiplexed onto a wavelength, and then wavelength division multiplexed onto the same fiber. Reference [17] shows another example, in which each ONU supports two wavelengths, among which one wavelength is dedicated for this ONU and the other one is shared by other ONUs. There have been many other architectures proposed for this class, for example, some candidate architectures in next generation access stage 1 [7]. They possess the common characteristic of wavelength sharing. Usually, ONUs are equipped with tunable lasers or a set of fixed-tuned lasers. Each ONU can access some wavelengths depending on the wavelengths supported by its lasers. These networks need to address the issue of dynamically assigning wavelengths to ONUs for their data transmissions [2,6].

Laser-sharing-based networks require dynamic and real-time laser assignment schemes, while wavelength-sharing-based networks need dynamic wavelength assignment algorithms. Both of these two problems can be modeled as multiprocessor scheduling problems by considering lasers and wavelengths as processors in the two respective cases. This paper focuses on addressing the wavelength assignment problem for the latter class of networks. Similar strategies may be applied to solve the laser assignment problem for the former class of networks.

III. MAC Protocol, Time Allocation, and Wavelength Assignment

Before describing the wavelength assignment problem of wavelength-sharing-based hybrid WDM/TDM PONs, we first discuss its MAC layer control protocol, which will affect the formulation of the wavelength assignment problem. Owing to the shared nature of the wavelength channel, MAC is needed for hybrid WDM/TDM PONs to avoid data collision. To be backward compatible, the MAC of hybrid WDM/TDM PONs is likely to inherit from the MAC of TDM PONs.

GPON and EPON constitute two major flavors of currently deployed TDM PONs. As specified in ITU-T G.984.3, GPON supports two MAC mechanisms: status report and non-status report. In status-report MAC, ONUs directly report to OLT about its queue length in the buffer. In non-status-report MAC, OLT infers the queue lengths in ONU buffers based on the historical information of allocated bandwidth and bandwidth utilization. The status-report approach can track the ONU status in a timely fashion, and utilize the resources more efficiently. EPON uses multipoint control protocol (MPCP) specified in IEEE 802.3ah as its MAC. With the MPCP, ONUs piggyback their queue length information onto data packets; OLT collects reports from ONUs, and makes resource allocation decisions, and sends out grants to ONUs. In both PONs, OLT exercises the overall intelligent control, and has to allocate resources including time and wavelengths to ONUs. To be backward compatible, OLT in a hybrid WDM/TDM PON is likely to control the data transmission in the network as well.

In addition to the MAC and time allocation problems in TDM PONs, the central controller OLT needs to address another problem: wavelength assignment, which is unique in hybrid WDM/TDM PONs. Formerly, McGarry *et al.* [6] mapped the wavelength assignment problem into a multiprocessor scheduling problem. They investigated the scheduling with the objective of minimizing the average delay of queues, and employed weighted bipartite matching to address it. McGarry *et al.* [12] also proposed schemes to minimize the minimum latest request scheduling time. More specifically, the LPT first rule was employed when ONUs can access all the wavelengths. When ONUs can access a limited set of wavelengths, they proposed scheduling ONUs with the LFJ first rule. In this paper, we also focus on minimizing the latest ONU request (job) scheduling time. This is equivalent to minimizing the makespan in multiprocessor scheduling.

Minimizing the makespan can achieve high bandwidth utilization, guarantee small delay, and ensure fairness in both GPON and EPON for the following reasons. In GPON, the frame duration is fixed at 125 μs. Scheduling all requests within a frame implies high bandwidth utilization. Additionally, if the incoming traffic cannot be scheduled within a frame, it has to be delayed to the next frame, thus incurring a large delay. Hence, to achieve small delay for queues and ensure fairness among them, it is important to schedule the requests of all queues within a frame, which is equivalent to the decision version of the minimum makespan scheduling problem. In EPON, the cycle duration is not fixed, but is always upper bounded in order to guarantee quality of service (QoS) for applications. When the network is lightly loaded such that the cycle duration is less than the upper bound, minimizing the makespan, i.e., the latest job completion time, can achieve high throughput, small delay, and fairness among ONUs. When the network is highly loaded such that traffic may not be accommodated with the maximum cycle duration, it is important to schedule all the traffic within the cycle in order to avoid a large delay. The problem is equivalent to the decision version of the minimum makespan scheduling problem.

We next address the minimizing makespan scheduling problem in hybrid WDM/TDM PONs. The problem can be generally formulated as follows: *given wavelengths supported by ONUs, the available time of wavelengths, and queue requests from ONUs, schedule ONU requests such that the latest ONU scheduling time is minimized.*

IV. Non-preemptive Multi-wavelength Scheduling

For notational convenience, we denote the case that allows preemption as pr, and $\bar{pr}$ otherwise, the case that ONUs can access all wavelengths as f, and $\bar{f}$ otherwise, and the case that the guard time between scheduling of ONUs is greater than zero as g, and $\bar{g}$ otherwise. In the following, we use $\langle x,y,z\rangle$ to denote each scheduling case, where x can be pr or $\bar{pr}$, y can be f or $\bar{f}$, and z can be g or $\bar{g}$.

In this section, we investigate non-preemptive scheduling, where jobs are scheduled in continuous time durations.

A. ONUs With Full Accessible Wavelength

Following the well-known three-field $\alpha|\beta|\gamma$ classification scheme suggested by Graham *et al.* [18], the scheduling problem under this case is equivalent to the $p\|C_{\max}$ multiprocessor scheduling problem, where p refers to the case with any number of machines.

First, consider the case that the guard time is equal to zero, i.e., Case $\langle\bar{pr},f,\bar{g}\rangle$. Then, the time requirement of ONU i is equal to the time duration for its data transmission, denoted as $\mathbf{r}_i$. Denote $C^*_{\bar{pr},f,\bar{g}}$ and $C^H_{\bar{pr},f,\bar{g}}$ as the minimum makespan and the makespan achieved by heuristic algorithm H, respectively. An algorithm is referred to as a ρ approximation algorithm if its makespan is no greater than ρ times that of the optimal makespan for all instances, i.e., $C^H_{\bar{pr},f,\bar{g}} \le \rho\cdot C^*_{\bar{pr},f,\bar{g}}$. Many heuristic algorithms have been proposed for the $p\|C_{\max}$ problem. For example, list scheduling constructs a list of requests, and schedules these requests in order by using the earliest available wavelength channel; LPT list scheduling modifies list scheduling by ordering requests with the descending order of their sizes first. These two algorithms were shown to have approximation ratios of $2-1/m$ and $4/3-1/(3m)$, respectively. Coffman *et al.* [19] proposed another algorithm referred to as the multifit algorithm. Although the multifit algorithm cannot be guaranteed to obtain a better performance than LPT for all instances, it was shown that multifit has an approximation ratio of 72/61, which is smaller than that of LPT list scheduling. Hence, we suggest using the multifit algorithm in the wavelength scheduling of hybrid WDM/TDM PON.

Considering the guard time between the scheduling of ONUs, i.e., Case $\langle\bar{pr},f,g\rangle$, we change the time requirement of ONU i from $\mathbf{r}_i$ into $\tilde{\mathbf{r}}_i=\mathbf{r}_i+\mathfrak{g}$, where $\mathfrak{g}$ is the guard time between the scheduling of two ONUs, and it is equal to the total time of laser on/off, automatic gain control (AGC), clock and data recovery (CDR), MAC layer overhead, etc. The problem is still equivalent to the $p\|C_{\max}$ problem, which can be addressed by the same algorithm as that in Case $\langle\bar{pr},f,g\rangle$.

B. ONUs With Limited Accessible Wavelength

Similar to the scenario that ONUs have full wavelength access capability, we consider $\mathbf{r}$ as ONU requests in Case $\langle\bar{pr},\bar{f},g\rangle$, and regard $\tilde{\mathbf{r}}=\mathbf{r}+\mathfrak{g}$ as ONU requests in Case $\langle\bar{pr},\bar{f},\bar{g}\rangle$. When ONUs can only access a limited set of wavelengths, the scheduling is equivalent to the $p|M_j|C_{\max}$ multiprocessor scheduling problem, where M_j describes the set of eligible machines for job j. The $p|M_j|C_{\max}$ problem is more general than the $p\|C_{\max}$ problem, and is hence NP-hard.

When the supported wavelengths of ONUs (eligible machines) are nested, the problem is simpler than that with arbitrary wavelength supportability. Formerly, Centeno and Armacost [20] developed a heuristic algorithm for the problem that integrates the least flexible job first rule (LFJ) and the least flexible machine first rule (LFM). Here, the LFJ rule selects the job that can be processed with the smallest number of machine types first, and the LFM rule assigns the job to the most restricted machine. Pinedo [21] stated that the LFJ rule was optimal for $Pm|p_j = 1, M_j|C_{\max}$ when the M_j sets are nested, where $p_j = 1$ denotes that all jobs have unit processing time.

For the general case with arbitrary eligible machine constraints, Centeno and Armacost [22] developed some heuristic algorithms for the $P_m|r_j, M_j|C_{\max}$ problem, where r_j is the release time of job j. They showed that the rule used for job selection affects the performance of heuristic algorithms, and that the LPT rule is superior to the LFJ rule when the machine eligibility sets are not nested. Potts [23] developed a 2-approximation algorithm by using a relaxed linear programming with rounding technique. The rounding process takes 2^m steps. Lenstra *et al.* [24] modified the rounding technique to eliminate the exponential computation. They also showed that no polynomial algorithm can achieve a worst-case ratio less than 3/2 unless P = NP. Shchepin *et al.* [25] further bettered the rounding process and developed a $2-1/m$ approximation algorithm. To the best of our knowledge, this is the best approximation algorithm for this problem.

When $m = 2$, $C^H_{\bar{p}r,\bar{f},\bar{g}}$ is as large as 1.5 times $C^*_{\bar{p}r,\bar{f},\bar{g}}$ in the worst case; $C^H_{\bar{p}r,\bar{f},\bar{g}}$ approaches 2 times $C^*_{\bar{p}r,\bar{f},\bar{g}}$ in the worst case with the increase of the number of wavelengths. Motivated to obtain a smaller makespan, we next investigate the preemption version of the problem.

V. Preemptive Multi-wavelength Scheduling

First, we discuss the feasibility of splitting the granted time duration to an ONU into discontinuous time durations, i.e., the feasibility of job preemption. For our specific multi-wavelength scheduling problem, a job corresponds to the total requests from an ONU, which is the summation of requests of its queues. Hence, a job can be divided into subjobs, each of which corresponds to a queue request of the ONU. In addition, one queue request is constituted by requests of queued packets. In other words, each subjob can be further divided into subjobs, corresponding to requests of packets in the queue. In GPON with the allowance of packet fragmentation, scheduling of a packet can even be divided into scheduling of its partial packets, while in EPON without packet fragmentation, scheduling of a packet cannot be further divided. Therefore, jobs are preemptable in hybrid WDM/TDM GPON and preemptable in a certain degree in hybrid WDM/TDM EPON. This lays the basis for the following preemptive multi-wavelength scheduling problem.

Job preemption may schedule jobs in discontinuous time durations and thus require more guard time between the scheduling of ONUs. In a TDM PON with a single upstream or downstream wavelength, preemption extends the makespan owing to the extra guard time introduced by subjobs. In a hybrid WDM/TDM PON with multiple wavelength channels, preemption allows jobs to be scheduled more flexibly, and thus may yield a smaller makespan. When the guard time $\mathfrak{g}$ is equal to zero, the makespan with preemption is less than that without preemption. When $\mathfrak{g}$ is greater than zero, the extra introduced guard time may be less than the decrease of the makespan caused by the flexibility of preemption. It is also possible that the additional guard time may exceed the decrease of the makespan.

We next investigate preemptive scheduling, analyze its performance, and compare it with non-preemptive scheduling.

A. ONUs With Full Accessible Wavelengths and $\mathfrak{g} = 0$ (Case $\langle pr, f, \bar{g}\rangle$)

Under Case $\langle pr, f, \bar{g}\rangle$, the scheduling is equivalent to the $p|pmtn|C_{\max}$ scheduling problem. It can be solved in polynomial time. Theorem 1 below, which may be readily derived according to Reference [21], gives the minimum makespan $C^*_{pr,f,\bar{g}}$.

Theorem 1. *The scheduling problem under Case $\langle pr, f, \bar{g}\rangle$ is equivalent to the $p|pmtn|C_{\max}$ problem. For a given $\mathbf{r}$, the minimum makespan $C^*_{pr,f,\bar{g}}$ is equal to* $\max[\sum_{i=1}^{N}\mathbf{r}_i/m, \max_i \mathbf{r}_i]$.

If $\sum_{i=1}^{N}\mathbf{r}_i/m > \max_i \mathbf{r}_i$, the minimum makespan $\sum_{i=1}^{N}\mathbf{r}_i/m$ can be achieved by evenly allocating jobs among all machines; if $\max_i \mathbf{r}_i > \sum_{i=1}^{N}\mathbf{r}_i/m$, the minimum makespan $\max_i \mathbf{r}_i$ can be achieved by scheduling jobs on each machine until the total $\max_i \mathbf{r}_i$ time duration on the machine is used up. Algorithm 1 details one means to achieve $C^*_{pr,f,\bar{g}}$. In Algorithm 1, let α_k track the time slots which have been allocated on wavelength k.

Algorithm 1 Achieve $C^*_{pr,f,\bar{g}}$

Initialize $\alpha_k = 0, \forall k$
for $k = 1 : m$ **do**
  Select one unscheduled ONU request and schedule it starting from α_k on wavelength k
  Update α_k as $\alpha_k + \mathbf{r}_i$
  Repeat the process until α_k exceeds $C^*_{pr,f,\bar{g}}$
  Assign the partial request scheduled beyond $C^*_{pr,f,\bar{g}}$ to wavelength $k+1$, starting from α_{k+1}
end for

By Algorithm 1, some jobs may be divided into subjobs and scheduled in discontinuous time durations on different wavelengths. The following properties describe the upper bound of the number of jobs being divided into subjobs, and the makespan of wavelengths.

Property 1. *Each job has at most two subjobs, of which one is scheduled at the end of the time duration on a wavelength, and the other at the beginning of the time duration on another wavelength. The total number of jobs being divided is no greater than* $m-1$.

Proof. It is straightforward to see that subjobs are scheduled either at the beginning or at the end of the time duration on a wavelength. We want to prove that a job can be divided into at most two subjobs. Assume that the remaining time on wavelength w is not enough to schedule job j. Then, the unscheduled part of job j will be scheduled on wavelength $w+1$. Then, job j can occupy the whole time duration of $C^*_{pr,f,\bar{g}}$ on wavelength $w+1$, which is long enough to accommodate any job, since $C^*_{pr,f,\bar{g}} \geq \max_i \mathbf{r}_i$. Hence, a job will be divided into at most two subjobs. Since jobs may be divided into subjobs only at the end of the time duration on wavelength $1, 2, \ldots, m-1$, there are at most $m-1$ jobs being divided into subjobs. □

Property 2. *If one job is scheduled on two wavelengths, these two wavelengths have the same makespan.*

Proof. Assume that job i is divided into two subjobs, in which size $s_{i,w}$ is scheduled on wavelength w, and size $s_{i,w+1}$ is scheduled on wavelength $w+1$. Without loss of generality, assume that the makespan C_{w+1} of wavelength $w+1$ is greater than the makespan C_w of wavelength w. Then, this schedule is not an optimal schedule, since the makespan C_{w+1} of the two wavelengths can be shortened to $(C_{w+1}+C_w)/2$ by changing $s_{i,w}$ into $s_{i,w}+(C_{w+1}-C_w)/2$ and $s_{i,w+1}$ into $s_{i,w+1}-(C_{w+1}-C_w)/2$. □

B. ONUs With Full Accessible Wavelength and $\mathfrak{g} > 0$ (Case $\langle pr, f, g\rangle$)

We first show that, with the consideration of the guard time between the scheduling of ONUs, the minimum makespan scheduling problem is NP-hard even if the jobs are preemptive.

Theorem 2. *The multi-wavelength scheduling problem with the objective of minimizing the makespan under Case $\langle pr, f, g\rangle$ is NP-hard.*

Proof. We prove that the corresponding decision problem is equivalent to the bin-packing problem when there are two wavelengths and $\sum_i \tilde{\mathbf{r}}_i/2 \leq \ell < \sum_i \tilde{\mathbf{r}}_i/2 + \mathfrak{g}/2$, where $\tilde{\mathbf{r}} = \mathbf{r} + \mathfrak{g}$. Assume there is one job being divided into two subjobs. Then, one extra guard time $\mathfrak{g}$ is needed, and hence the makespan is no less than $\sum_i \tilde{\mathbf{r}}_i/2 + \mathfrak{g}/2$. Therefore, no job can be divided to achieve a makespan less than $(\sum_i \tilde{\mathbf{r}}_i + \mathfrak{g})/2$. Since jobs cannot be preempted, the problem is equivalent to the bin-packing problem, which is NP-complete. The original minimization problem is then NP-hard. □

To address the problem, we propose a heuristic algorithm by incorporating Algorithm 1 with guard time supplement. First, with $\tilde{\mathbf{r}}$ being considered as the job request, we use Algorithm 1 to generate an initial schedule, in which jobs may be divided into subjobs. Since each subjob needs guard time with the duration of $\mathfrak{g}$, the allocated time for jobs being divided into subjobs may not be enough to accommodate their data transmission. The second step of our heuristic algorithm is to supplement some more time to these divided subjobs, which will be detailed next.

Without loss of generality, denote the job scheduled at the end of the time duration on wavelength w as job w. Denote $s_{i,w}$ as the time duration job i is scheduled on wavelength w. According to Property 1, only job i $(1 \leq i < m)$ may be divided into subjobs, and all the other jobs are not divided in the initial schedule. Then, only job i $(1 \leq i < m)$ needs be supplemented with some extra time. If job i is divided into subjobs, one part is scheduled on wavelength i and the remaining part is scheduled on wavelength $i+1$, i.e., $0 < s_{i,i}, s_{i,i+1} < \tilde{\mathbf{r}}_j$. Let $x_{i,i}$ and $x_{i,i+1}$ $(1 \leq i < m)$ be the extra time supplemented to $s_{i,i}$ and $s_{i,i+1}$, respectively.

The problem of determining $x_{i,i}$ and $x_{i,i+1}$ for $1 \leq i < m$ with the minimum makespan can be formulated as follows. C denotes the latest job completion time on all wavelengths.

$$\begin{aligned} \text{minimize} \quad & C & (1)\\ \text{subject to} \quad & C_1 + x_{1,1} \leq C, & (2)\\ & C_i + x_{i-1,i} + x_{i,i} \leq C, \quad \forall 1 < i < m, & (3)\\ & C_m + x_{m-1,m} \leq C, & (4)\\ & (s_{i,i} + x_{i,i} - \mathfrak{g})^+ + (s_{i,i+1} + x_{i,i+1} - \mathfrak{g})^+ & \\ & \geq \mathbf{r}_w, \quad \forall 1 \leq i < m. & (5) \end{aligned}$$

Constraints (2)–(4) state that the latest job completion time on each wavelength should be no greater than C. Since only one subjob $x_{1,1}$ is scheduled on wavelength 1, Constraint (2) only concerns the extra time duration allocated to subjob $x_{1,1}$. Similarly, Constraint (4) for wavelength i only concerns the extra time duration allocated to subjob $x_{i-1,i}$. For wavelength i $(1 < i < m-1)$, subjobs of job $i-1$ and i are scheduled, and hence the constraint needs to consider $x_{i-1,i} + x_{i,i}$. Constraint (5) states that the time duration for data transmission is no less than its request.

When the number of wavelength is equal to two, considering different cases of $s_{i,i}+x_{i,i}-\mathfrak{g}$ and $s_{i,i+1}+x_{i,i+1}-\mathfrak{g}$ in Constraint (5), we can solve the problem as follows:

- $s_{1,1} > (\mathfrak{g}+C_1-C_2)/2$ and $s_{1,2} > (\mathfrak{g}+C_2-C_1)/2$: $x_{1,1} = (\mathfrak{g}+C_2-C_1)/2$, $x_{1,2} = (\mathfrak{g}+C_1-C_2)/2$. The time duration for data transmission on the two wavelengths is equal to $(\mathfrak{g}+C_2-C_1)/2 + s_{1,1} - \mathfrak{g}$ and $(\mathfrak{g}+C_1-C_2)/2 + s_{1,2} - \mathfrak{g}$, respectively. The makespan on the two wavelengths is $C = (\mathfrak{g}+C_2+C_1)/2$. The increase of the makespan on the two wavelengths is

  $$\begin{aligned}(\mathfrak{g}+C_2+C_1)/2 - \max\{C_2, C_1\} &= \mathfrak{g}/2 - |C_2 - C_1|/2\\ &\leq \mathfrak{g}/2.\end{aligned}$$

  The equality holds when $C_1 = C_2$.
- $s_{1,1} \leq (\mathfrak{g}+C_1-C_2)/2$: in this case, if the two wavelengths are made to have the same makespan, $x_{1,1} = (\mathfrak{g}+C_2-C_1)/2$, $x_{1,1} + s_{1,1} - \mathfrak{g} \leq 0$, resulting in no time duration being used for data transmission. The optimal solution is to set $x_{1,1} = -s_{1,1}$ and $x_{1,2} = s_{1,1}$. The increase of the latest job completion time on the two wavelengths is

  $$\begin{aligned}&\max\{C_1 - s_{1,1}, C_2 + s_{1,1}\} - \max\{C_2, C_1\}\\ &= \begin{cases} C_2 - C_1 + s_{1,1}, & \text{if } C_1 > C_2\\ s_{1,1}, & \text{otherwise}\end{cases}\\ &\leq \begin{cases} \min\{\mathfrak{g} - s_{1,1}, s_{1,1}\}, & \text{if } C_1 > C_2\\ (\mathfrak{g}+C_1-C_2)/2, & \text{otherwise}\end{cases}\\ &\leq \mathfrak{g}/2.\end{aligned}$$

- $s_{1,2} \le (\mathfrak{g} + C_2 - C_1)/2$: Similar to the situation under the condition that $\mathbf{r}_{1_1} \le (\mathfrak{g} + C_1 - C_2)/2$, $x_{1,1} = s_{1,2}$ and $x_{1,2} = -s_{1,2}$. The increase of the latest job completion time on the two wavelengths is less than $\mathfrak{g}/2$.

Since the increase of the makespan in all the three cases is less than $\mathfrak{g}/2$, we can conclude that $C^H_{pr,f,g} \le C^*_{pr,f,g} + \mathfrak{g}/2$.

When the number of wavelengths is equal to m, the number of variables $x_{i,i}$ and $x_{i,i+1}$ $(1 \le i < m)$ is equal to $2(m-1)$. Considering different cases of Constraint (5), the minimization problem can be transformed into $2^{2(m-1)}$ linear programming (LP) problems. Then, the computation complexity is exponential in m. To reduce the computation, we convert Constraint (5) into the following Constraint (6), and solve the corresponding LP problem instead:

$$x_{i,i} + x_{i,i+1} \ge \mathfrak{g}, \quad \text{if } s_{i,i+1} > 0. \tag{6}$$

Constraint (6) supplements jobs being divided into subjobs with $\mathfrak{g}$ or more time. If Constraint (6) is satisfied, then Constraint (5) can be satisfied with the same variables, since $(s_{i,i} + x_{i,i} - \mathfrak{g})^+ + (s_{i,i+1} + x_{i,i+1} - \mathfrak{g})^+ \ge s_{i,i} + s_{i,i+1} + x_{i,i} + x_{i,i+1} - 2\mathfrak{g} \ge \mathbf{r}_i$.

We can solve this particular linear programming problem described by Eqs. (1)–(4) and (6) by using Algorithm 2 with complexity $O(m)$. We shall illustrate Algorithm 2 by one example; assume there are five wavelengths, and that jobs 1, 2, and 4 are scheduled on machines 1 and 2, machines 2 and 3, and machines 4 and 5, respectively. Then, $\alpha_1 = 3$ and $\alpha_2 = 5$. $x_{1,1} = 2\mathfrak{g}/3$, $x_{1,2} = \mathfrak{g}/3$, $x_{2,2} = \mathfrak{g}/3$, $x_{2,3} = 2\mathfrak{g}/3$, $x_{4,4} = \mathfrak{g}/2$, and $x_{4,5} = \mathfrak{g}/2$.

We next show that Algorithm 2 solves the LP problem described by Eqs. (1)–(4) and (6), and then derive the upper bound of the makespan generated by this heuristic algorithm.

Lemma 1. *Algorithm 2 solves the LP problem described by Eqs. (1)–(4) and (6).*

Proof. Jobs $\alpha_i + 1, \ldots, \alpha_{i+1} - 1$ are divided at the end of the time duration on wavelengths $\alpha_i + 1, \ldots, \alpha_{i+1} - 1$, respectively. Then, wavelengths $\alpha_i + 1, \ldots, \alpha_{i+1}$ have the same makespan in the initial schedule created by Algorithm 1 according to Property 2. After supplementing allocations by Algorithm 2, each of these wavelengths increases its makespan by $\mathfrak{g}(\beta-1)/\beta$. Hence, wavelengths $\alpha_i + 1, \ldots, \alpha_{i+1}$ still have the same makespan after the supplement. On the other hand, Algorithm 2 supplements each job of jobs $\alpha_i + 1, \ldots, \alpha_{i+1} - 1$ with $\mathfrak{g}$ time. Insufficient supplement of jobs $\alpha_i + 1, \ldots, \alpha_{i+1} - 1$ will result if the makespan of wavelengths $\alpha_i + 1, \ldots, \alpha_{i+1}$ is decreased. Therefore, Algorithm 2 solves the LP problem described by Eqs. (1)–(4) and (6). □

Theorem 3. $C^H_{pr,f,g} \le C^*_{pr,f,g} + (m-1)\mathfrak{g}/m$.

Proof. Let μ be the makespan of the initial schedule after using Algorithm 1. Then, $C^H_{pr,f,g} \le \mu + (m-1)\mathfrak{g}/m$, since $(\beta-1)\mathfrak{g}/\beta \le (m-1)\mathfrak{g}/m$. On the other hand, $C^*_{pr,f,g} \ge \mu$. Hence, $C^H_{pr,f,g} \le C^*_{pr,f,g} + (m-1)\mathfrak{g}/m$. □

Algorithm 2 has complexity $O(m)$, since the number of jobs being divided into subjobs is less than m. Our proposed heuristic algorithm is to first use Algorithm 1, and then use Algorithm 2. Algorithm 1 has complexity $O(n)$. Hence, the heuristic algorithm for the scheduling problem under Case $\langle pr, f, g\rangle$ has complexity $O(n+m)$.

Algorithm 2 Supplement guard time under Case $\langle pr, f, g\rangle$

$\alpha_0 = 0$, $\alpha_1, \alpha_2, \ldots$ list all the wavelengths at the end of which no job is divided.
$k = 1$
while α_k exists **do**
  $\beta = \alpha_k - \alpha_{k-1}$
  $x_{\alpha_{k-1}+1,\alpha_{k-1}+1} = (\beta-1)\mathfrak{g}/\beta, x_{\alpha_{k-1}+1,\alpha_{k-1}+2} = \mathfrak{g}/\beta$
  $x_{w,w} = (\beta-2)\mathfrak{g}/\beta, x_{w,w+1} = 2\mathfrak{g}/\beta, \forall \alpha_{k-1} + 1 < w < \alpha_k - 1$
  $x_{\alpha_k-1,\alpha_k-1} = \mathfrak{g}/\beta, x_{\alpha_k-1,\alpha_k} = (\beta-1)\mathfrak{g}/\beta$
  $k = k+1$
end while

C. ONUs With Limited Accessible Wavelengths and $\mathfrak{g} = 0$ (Case $\langle pr, \bar{f}, \bar{g}\rangle$)

Because of the additional eligible machine constraints, the scheduling in Case $\langle pr, \bar{f}, \bar{g}\rangle$ is more complicated as compared to that in Case $\langle pr, f, \bar{g}\rangle$; it can no longer be solved by Algorithm 1. The LP formulation of the problem is similar to that described in [23]. Denote J_w as the set of jobs which can access wavelength w, and M_i as the set of wavelengths to which job i can have access.

$$\text{minimize} \quad C \tag{7}$$

$$\text{subject to} \quad \sum_{i \in J_w} s_{i,w} \le C, \quad \forall w, \tag{8}$$

$$\sum_{m \in M_i} s_{i,w} \ge \mathbf{r}_i, \quad \forall i, \tag{9}$$

$$s_{i,w} \ge 0, \quad \forall i, w. \tag{10}$$

Constraint (8) limits the total time allocated on a wavelength to be less than C. Constraint (9) states that the sum of the time duration allocated to a job should be no less than its request. This LP problem can be solved polynomially.

Potts [23] analyzed this particular LP problem and derived the following properties about the number of non-zero variables and fractional variables.

Property 3. *There are at most $n+m-1$ variables having non-zero values. Among thenjobs from ONUs, there are at least $n-m+1$ jobs without being divided into subjobs in the final schedule, and at most $m-1$ jobs being divided into subjobs. The number of subjobs is at most $2(m-1)$.*

D. ONUs With Limited Accessible Wavelengths and $\mathfrak{g} > 0$ (Case $\langle pr, \bar{f}, g\rangle$)

When $\mathfrak{g} > 0$, the scheduling problem is NP-hard, since it is more general as compared to that under Case $\langle pr, f, g\rangle$, which is NP-hard. The first step of our heuristic algorithm is to update the time requirements for ONUs into $\tilde{\mathbf{r}} = \mathbf{r} + \mathfrak{g}$, and then construct an initial schedule by solving the LP problem described by Eqs. (7)–(10). Since the LP approach may divide some jobs into subjobs, some more time needs to be supplemented to guarantee enough time for the data transmission of jobs being divided into subjobs.

If $0 < s_{i,w} < \tilde{\mathbf{r}}_i$, a subjob of job i is scheduled on wavelength w. We supplement $x_{i,w}$ to this subjob to make sure that job i can get enough time for its data transmission. The problem of supplementing time allocation for jobs with the minimum makespan can be formulated as follows:

$$\text{minimize} \quad C$$
$$\text{subject to} \quad \sum_{\{i|0<s_{i,w}<\tilde{\mathbf{r}}_i\}} x_{i,w} + C_w \le C, \quad \forall 1 \le w \le m, \quad (11)$$
$$\sum_{\{w|0<s_{i,w}<\tilde{\mathbf{r}}_i\}} (s_{i,w} + x_{i,w} - \mathfrak{g})^+ = \mathbf{r}_i, \quad \forall i. \quad (12)$$

Constraint (11) restricts the latest job completion time on any wavelength w to be no greater than C after supplementing. Constraint (12) states that the total time allocated to a job should be enough to complete its data transmission.

To solve this problem, possible cases of $s_{i,w} + x_{i,w} - \mathfrak{g}$ for every subjob in Constraint (12) should be considered. As stated in Property 3, the number of subjobs can be as large as $2(m-1)$. The computation is exponential in m. To relieve the computation burden, we replace Constraint (12) by Constraint (13).

$$\sum_{\{w|0<s_{i,w}<\tilde{\mathbf{r}}_i\}} x_{i,w} \ge (\mathfrak{n}_i - 1)\mathfrak{g}, \quad \forall i, \quad (13)$$

where $\mathfrak{n}_i$ ($\mathfrak{n}_i \ge 1$) is the number of subjobs of job i. The feasible solution satisfying Constraint (13) also satisfies Constraint (12), since, if $\sum_{\{w|0<s_{i,w}<\tilde{\mathbf{r}}_i\}} x_{i,w} \ge (\mathfrak{n}_i - 1)\mathfrak{g}$, then

$$\sum_{\{w|0<s_{i,w}<\tilde{\mathbf{r}}_i\}} (s_{i,w} + x_{i,w} - \mathfrak{g})^+ \ge \sum_{\{w|0<s_{i,w}<\tilde{\mathbf{r}}_i\}} (s_{i,w} + x_{i,w} - \mathfrak{g})$$
$$\ge \sum_{\{w|0<s_{i,w}<\tilde{\mathbf{r}}_i\}} x_{i,w} - \mathfrak{n}_i\mathfrak{g} + \tilde{\mathbf{r}}_i = \mathbf{r}_i.$$

To solve the minimization problem with Constraints (11) and (13), we propose an algorithm with complexity $O(m)$ based on an acyclic preemptive graph proposed by Evgeny *et al.* [25].

Formerly, Evgeny *et al.* [25] constructed a non-preemptive scheduling of the minimal makespan by using linear programming with rounding. To obtain the optimal rounding, they created a preemption graph as follows. In graph G, vertices represent machines and edges represent jobs. An edge connects a pair of vertices in G if the job associated with the edge is shared by the two machines. Since there are at most $m-1$ jobs being divided, the number of edges should be no greater than $m-1$. They showed that graph G is acyclic for the preemptive jobs created by LP. Figure 1(a) illustrates one example of an acyclic preemption graph G with two jobs and four machines. Job 1 is scheduled on machines 1 and 2, and job 2 is scheduled on machines 2, 3, and 4. Using the same preemptive graph, we supplement time allocations to jobs being divided into subjobs.

It is possible that graph G is not connected. Let $G_1, G_2, \ldots$ be connected subgraphs of graph G, $G = \cup_k G_k$, $G_k \cap G_{k'} = \emptyset$. For G_k, we have the following property regarding the makespan of its involved wavelengths.

Lemma 2. *In the initial schedule constructed by LP, all wavelengths corresponding to vertices in G_k have the same makespan.*

Proof. In G_k, the job associated with edge $\{w,v\}$ is shared by machines w and v. Then, machines w and v have the same makespan according to Property 2. All machines have the same makespan, since G_k is connected. □

Based on Lemma 2, we obtain the following property of the makespan of the final schedule.

Lemma 3. *In the final schedule obtained after guard time supplement, the minimum makespan among all wavelengths in graph G_k is no less than $(|V_k|-1)\mathfrak{g}/|V_k| + C^k$, where $|V_k|$ is the number of vertices in G_k and C^k is the makespan of wavelengths in G_k in the initial schedule.*

Proof. Let set Φ contain all the jobs associated with edges in G_k. The number of edges associated with job i ($i \in \Phi$) is equal to $\mathfrak{n}_i - 1$. The total number of edges of G_k is equal to $\sum_{i\in\Phi}(\mathfrak{n}_i - 1)$. Since G_k is connected and acyclic, the total number of edges of G_k is equal to $|V_k|-1$. Hence, $|V_k| - 1 = \sum_{i\in\Phi}(\mathfrak{n}_i - 1)$. Based on Constraint (13), $\sum_{\{w|0<s_{i,w}<\tilde{\mathbf{r}}_i\}} x_{i,w} \ge (\mathfrak{n}_i - 1)\mathfrak{g}$. Then, the total time to be supplemented is no less than $(|V_k|-1)\mathfrak{g}$. When the minimum makespan among all wavelengths in the final schedule is equal to $(|V_k|-1)\mathfrak{g}/|V_k| + C^k$, the total time which can be supplemented is $(|V_k|-1)\mathfrak{g}$. For a smaller makespan, the total time being supplemented must be less than $(|V_k|-1)\mathfrak{g}$. Hence, $(|V_k|-1)\mathfrak{g}/|V_k| + C_k$ is the lower bound of the minimum makespan. □

Lemma 3 gives a lower bound of the makespan of wavelengths in G_k. Algorithm 3 describes one supplement scheme to achieve the lower bound.

Algorithm 3 Supplement guard time under Case $\langle pr, \bar{f}, g\rangle$

Construct a cyclic preemptive graph G
Let $G_1, G_2, \ldots$ be connected subgraphs of graph G
Set $\alpha_i = (\mathfrak{n}_i - 1)\mathfrak{g}$ for job i with subjobs
$k = 1$
while G_k exists **do**
  $\beta_w = (|V_k|-1)\mathfrak{g}/|V_k|, \forall w \in G_k$
  while G_k has vertices **do**
    Select one leaf vertex w; denote the vertex connecting w as v, and the job associated with edge $\{w,v\}$ as i
    $x_{i,w} = \beta_w$, $\alpha_i = \alpha_i - \beta_w$
    if job i is associated with edge $\{w,v\}$ only, **then**
      $x_{i,v} = \alpha_i$, $\beta_v = \beta_v - x_{i,v}$
    end if
    Remove w and $\{w,v\}$ from G_k
  end while
  $k = k+1$
end while

In Algorithm 3, β_w denotes the time available on wavelength w before its makespan reaches $(|V_k|-1)\mathfrak{g}/|V_k| + C_k$, and α_i denotes the time to be supplemented to job i. β_w is initialized as $(|V_k|-1)\mathfrak{g}/|V_k|, \forall w \in G_k$, and α_i is initialized as $(\mathfrak{n}_i - 1)\mathfrak{g}$. The supplement begins from the leaves of the tree. For the leaf node w, denote the vertex connecting w as v, and the job associated with edge $\{w,v\}$ as i. We allocate all the remaining time available β_w on wavelength w to job i, and update the time request of job i to $\alpha_i - \beta_w$. If job i is associated with $\{w,v\}$ only, we schedule the remaining request of job i onto wavelength v, and update the time budget of wavelength v accordingly. Then,

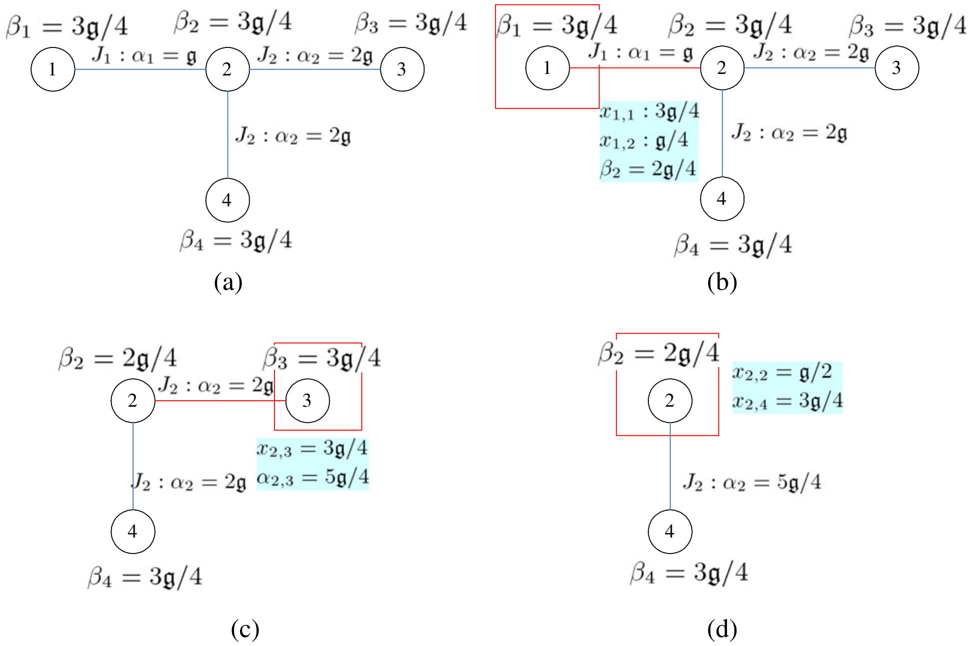

Fig. 1. (Color online) An example of supplementing allocations under Case $\langle pr, \bar{f}, g\rangle$.

we remove vertex w and edge $\{w,v\}$ from G_k. This process is repeated until G_k is empty.

Figures 1(b)–1(d) illustrate one example of supplementing allocations for wavelengths in graph G as shown in Fig. 1(a). Vertex 1 is selected to be processed first. Since job 1 is processed on machines 1 and 2 only, as shown in Fig. 1(b), $x_{1,1} = 3\mathfrak{g}/4$, $x_{1,2} = \mathfrak{g}/4$, and β_2 is updated as $3\mathfrak{g}/4 - \mathfrak{g}/4 = 2\mathfrak{g}/4$. Vertex 3 is the next to be processed. As shown in Fig. 1(c), $x_{2,3} = 3\mathfrak{g}/4$, $\alpha_{2,3}$ is updated as $5\mathfrak{g}/4$. The last step is to process machines 1 and 4, $x_{2,2} = \mathfrak{g}/2$ and $x_{2,4} = 3\mathfrak{g}/4$, as shown in Fig. 1(d).

Theorem 4. $C^H_{pr,f,g} \le C^*_{pr,f,g} + (m-1)/m\mathfrak{g}$.

Proof. Let C_0 be the makespan of the initial schedule after performing LP. Then, since $(|V_k|-1)\mathfrak{g}/|V_k| \le (m-1)\mathfrak{g}/m$, $C^H_{pr,f,g} \le C_0 + \mathfrak{g}(m-1)/m$. On the other hand, $C^*_{pr,f,g} \ge C_0$. Hence, $C^H_{pr,f,g} \le C^*_{pr,f,g} + (m-1)/m\mathfrak{g}$. □

In Algorithm 3, the total number of iterations is equal to the number of vertices in graph G. Hence, Algorithm 3 has complexity $O(m)$.

VI. Comparison Between Preemptive Scheduling and Non-preemptive Scheduling

In this section, we compare the performances of preemptive scheduling with those of non-preemptive scheduling.

When the guard time $\mathfrak{g}$ is equal to zero, both $C^*_{pr,f,\bar{g}}$ and $C^*_{pr,\bar{f},\bar{g}}$ can be obtained in polynomial time, while $C^*_{\bar{p}r,f,\bar{g}}$ and $C^*_{\bar{p}r,\bar{f},\bar{g}}$ need heuristic algorithms to approximate them, and $C^H_{\bar{p}r,f,\bar{g}} \ge C^*_{\bar{p}r,f,\bar{g}}$ and $C^H_{\bar{p}r,\bar{f},\bar{g}} \ge C^*_{\bar{p}r,\bar{f},\bar{g}}$. In addition, owing to the flexibility introduced by preemption, $C^*_{pr,f,\bar{g}} \le C^*_{\bar{p}r,f,\bar{g}}$ and $C^*_{pr,\bar{f},\bar{g}} \le C^*_{\bar{p}r,\bar{f},\bar{g}}$. Therefore, when $\mathfrak{g} = 0$, the preemptive schedule yields a smaller or an equal makespan as compared to the non-preemptive schedule. When $\mathfrak{g} > 0$, $C^*_{pr,f,\bar{g}}$, $C^*_{pr,\bar{f},\bar{g}}$, $C^*_{\bar{p}r,f,\bar{g}}$, and $C^*_{\bar{p}r,\bar{f},\bar{g}}$ cannot be obtained in polynomial time, and heuristic algorithms are needed to approximate these values. As discussed above, $C^H_{pr,f,g} \le C^*_{pr,f,g} + (m-1)\mathfrak{g}/m$, $C^H_{pr,\bar{f},g} \le C^*_{pr,\bar{f},g} + (m-1)\mathfrak{g}/m$, $C^H_{\bar{p}r,f,g} \le 72/61C^*_{\bar{p}r,f,g}$, and $C^H_{\bar{p}r,\bar{f},g} \le (2-1/m)C^*_{pr,\bar{f},g}$. Then,

$$\frac{C^*_{pr,\bar{f},\bar{g}}}{72/61C^*_{\bar{p}r,\bar{f},\bar{g}}} \le \frac{C^H_{pr,\bar{f},\bar{g}}}{C^H_{\bar{p}r,\bar{f},\bar{g}}} \le \frac{C^*_{pr,\bar{f},\bar{g}} + (m-1)\mathfrak{g}/m}{C^*_{\bar{p}r,\bar{f},\bar{g}}}$$

$$\frac{C^*_{pr,f,\bar{g}}}{(2-1/m)C^*_{\bar{p}r,f,\bar{g}}} \le \frac{C^H_{pr,f,\bar{g}}}{C^H_{\bar{p}r,f,\bar{g}}} \le \frac{C^*_{pr,f,\bar{g}} + (m-1)\mathfrak{g}/m}{C^*_{\bar{p}r,f,\bar{g}}}.$$

Owing to the flexibility of preemptive jobs, $C^*_{pr,f,g} \le C^*_{\bar{p}r,f,g}$ and $C^*_{pr,\bar{f},g} \le C^*_{\bar{p}r,\bar{f},g}$. If the guard time $\mathfrak{g}$ is negligible as compared to the makespan, in the worst case, $C^H_{pr,f,g}$ and $C^H_{pr,\bar{f},g}$ approximate $C^H_{\bar{p}r,f,g}$ and $C^H_{\bar{p}r,\bar{f},g}$, respectively. In the best case, $C^H_{pr,f,g}$ is at most $61C^H_{\bar{p}r,f,g}/72$, and $C^H_{pr,\bar{f},g}$ is at most $C^H_{\bar{p}r,\bar{f},g}/(2-1/m)$. Hence, the preemptive scheduling outperforms the non-preemptive scheduling when $\mathfrak{g}$ is negligible as compared to the makespan. Next, we discuss $\mathfrak{g}$ in GPON and EPON.

ITU-T G.984.3 specifies the GPON upstream frame format. The upstream transmission overhead contains the physical layer overhead (PLOu), physical layer operations, administration, and management upstream (PLOAMu), power leveling sequence upstream (PLSu), and dynamic bandwidth report upstream (DBRu). The PLOAMu field has a length of 13 bytes. The DBRu field consists of at most 5 bytes. The PLOu for GPON with 1.244 Gbps upstream data rate is 12 bytes as specified in ITU-T G.984.2. The PLSu field is optional, and is used for power control measurement by the ONU. For each ONU, it is reasonable that the physical layer OAM, power control, and dynamic bandwidth report are carried out at most once in one frame. Then, the extra subjobs introduced by preemption will not need additional guard time for the PLOAMu, PLSu, and DBRu fields. Considering the PLOu field only, the guard time for GPON with 1.244 Gbps data rate is equal to $12 \cdot 8$ bits/1.244 Gbps ≈ 77.17 ns. The guard time $\mathfrak{g}$ between the scheduling of two ONUs is about 0.061% times the GPON frame size, which is 125 µs. Assume that the network is heavily loaded with $C^*_{pr,f,g} = 125$ µs and $C^*_{pr,\bar{f},g} = 125$ µs; then $C^H_{pr,f,g}$ and $C^H_{pr,\bar{f},g}$ have the following property:

$$C^H_{pr,f,g} \le C^*_{pr,f,g} + (m-1)\mathfrak{g}/m < C^*_{pr,f,g} + \mathfrak{g} \approx 1.00061 C^*_{pr,f,g},$$
$$C^H_{pr,\bar{f},g} \le C^*_{pr,\bar{f},g} + (m-1)\mathfrak{g}/m < C^*_{pr,\bar{f},g} + \mathfrak{g} \approx 1.00061 C^*_{pr,\bar{f},g}.$$

Therefore, in hybrid WDM/TDM GPON, preemptive scheduling with our proposed heuristic algorithm has a better performance than non-preemptive scheduling, especially when ONUs can only access a limited set of wavelengths.

For EPON, IEEE 802.3ah specifies a time duration of 400 ns for both AGC and CDR, a time duration of 512 ns for the laser on/off time, and a time duration of 32 ns for the code alignment zone (CAZ) [26]. Therefore, the total guard time is is equal to $400+400+2*512+32 = 2056$ ns. For 10 G EPON, IEEE P802.3av defines an adjustable laser on/off time with the default value of 512 ns, since most of the deployed transceivers nowadays require less than 512 ns on/off time. Hence, the guard time in 10 G EPON is smaller than 2056 ns. Then, assume that the EPON cycle duration is 1 ms, the guard time $\mathfrak{g}$ between the scheduling of two ONUs is about 0.2056% times the cycle duration. Assume that the network is heavily loaded with $C^*_{pr,f,g} = 1$ ms and $C^*_{pr,\bar{f},g} = 1$ ms; then $C^H_{pr,f,g}$ and $C^H_{pr,\bar{f},g}$ have the following properties:

$$C^H_{pr,f,g} \le C^*_{pr,f,g} + (m-1)\mathfrak{g}/m < C^*_{pr,f,g} + \mathfrak{g} \approx 1.002056 C^*_{pr,f,g},$$
$$C^H_{pr,\bar{f},g} \le C^*_{pr,\bar{f},g} + (m-1)\mathfrak{g}/m < C^*_{pr,\bar{f},g} + \mathfrak{g} \approx 1.002056 C^*_{pr,\bar{f},g}.$$

These theoretical analyses demonstrate that, in hybrid WDM/TDM EPON, our proposed preemptive scheduling has better performance than the best non-preemptive scheduling algorithm whose approximation ratio is as large as $2-1/m$.

VII. Conclusion

In this paper, we have discussed the wavelength scheduling problem in hybrid WDM/TDM PONs. We consider two scenarios: the ONUs can access all wavelengths and the ONUs can only access a limited set of wavelengths. When preemption is disallowed, the scheduling problem with the minimum makespan in the two cases can be respectively mapped into $p\|C_{\max}$ and $p|M_j|C_{\max}$ multiprocessor scheduling problems. Both of these two problems are NP-hard, and the best heuristic algorithms have approximation ratios 72/61 and $2-1/m$, respectively, where m is the number of wavelengths. In order to achieve a better performance, especially for the case when ONUs can only access a limited set of wavelengths, we investigate the preemptive version of the scheduling problem. However, we show that, without considering the guard time between the scheduling of ONUs, denoted by $\mathfrak{g}$, the preemptive scheduling is NP-hard under both cases. We have proposed heuristic algorithms and shown that the makespan produced by our heuristic algorithms is no greater than the minimum value plus $(m-1)\mathfrak{g}/m$. When the network is highly loaded, the approximation ratio is around 1.00061 and 1.002056 for hybrid WDM/TDM EPON and GPON, respectively.

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