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Optimal Cooperative Power Allocation for Energy Harvesting Enabled Relay Networks

Xueqing Huang, *Student Member, IEEE,* and Nirwan Ansari, *Fellow, IEEE*

***Abstract*—In this paper, we present a new power allocation scheme for a decode-and-forward (DF) relaying-enhanced cooperative wireless system. While both the source and relay nodes have limited energy storage, the source node can also draw power from the surrounding radio frequency (RF) signals. In particular, we assume a deterministic RF energy harvesting (EH) model under which the signals transmitted by the relay serve as the renewable energy source for the source node. The amount of harvested energy is known for a given transmission power of the forwarding signal and channel condition between the source and relay nodes. To maximize the overall throughput while meeting the constraints imposed by the initially stored energy and the renewable RF energy source, an optimization problem is formulated and solved. Based on different harvesting efficiencies and channel conditions, closed form solutions are derived to obtain the optimal joint source and relay power allocation. It is shown that instead of demanding high on-grid power supply or high green energy availability, the system can achieve compatible or higher throughput by utilizing the harvested energy.**

***Index Terms*—Power allocation, DF-relay, Cooperative communications, RF energy harvesting.**

I. Introduction

Although current wireless networks still primarily rely on the on-grid or un-rechargeable energy sources, continuous advances in green energy technology has motivated the research of the *green powered wireless network* [1], [2]. The concept of energy harvesting (EH) has been proposed to capture and store energy from readily available ambient sources that are free for users, including wind, solar, biomass, hydro, geothermal, tides, and even radio frequency signals [3], [4]. EH is capable of generating electricity or other energy form, which is renewable and more environmentally friendly than that derived from fossil fuels [5].

The generic green energy harvesting model adopts the *harvest-store-use* architecture with a storage component (e.g., rechargeable batteries) to hoard the harvested energy for future use. Except for the storage unit, the energy harvester and the energy usage components can be either 1) separated, which allows *simultaneous* energy harvesting and wireless functionality, such as data transmission or reception, or 2) co-located, which adopts *time switching* scheduling between energy harvesting and consumption processes. Furthermore, the existing literature assumes that the current harvested energy can only flows to latter slots, owing to the *energy half-duplex constraint* [6]. So, before performing the wireless functionality, the available residual energy is observable in both architectures, similar to the traditional on-grid powered wireless networks [7].

It is, however, not trivial to design and optimize the green energy enabled networks owing to the fact that the *energy-arrival rate* of the free energy is determined by the surrounding environment, such as the power generators' geo-locations and weather conditions. Since energy cannot be consumed before it is harvested, the *opportunistic energy harvesting* results in fluctuating power budget, namely, *energy causality constraint* (EC-constraint). The EC-constraint mandates that, at any time, the total consumed energy should be equal to or less than the total harvested energy, which maybe further limited by the finite battery capacity [8], [9].

For the architecture with separated energy harvester and information transmitter, green power management is essential to maximize the system performance without violating the EC-constraint. Ho and Zhang [10] considered the point-to-point wireless system with the energy harvesting transmitter. Optimal energy allocation algorithms are developed to maximize the throughput over a finite time horizon. Similarly, the throughput by a deadline is maximized and the transmission completion time of the communication session is minimized [11], [12]. Moreover, the works in [13] and [14] explored the joint source and relay power allocation over time to maximize the throughput of the three node decode-and-forward (DF) relay system, in which both the source and relay nodes transmit with power drawn from independent energy-harvesting sources.

For the green relay enhanced cooperative wireless network, radio frequency (RF) harvesting is an energy form of particular potential because it enables simultaneous wireless information and power transfer [15]. For

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the relay node (RN) with co-located data and energy reception components, it can either split the received signals between data detector and energy harvester (*power splitting*), or perform the above mentioned two processes sequentially (*time switching*) [16].

Furthermore, since the half duplex relay is required to transmit and receive over orthogonal time slots [17], the source node (SN) can harness energy from the forwarding signals transmitted by the relay. Inherently, the data transmission and energy harvesting will occur alternately. So, the co-located time switching architecture can be adopted by SN.

As compared with RF-EH RN where two individual RF chains are needed [18] - [19], we study the less hardware demanding scenario where RF energy harvesting capability is introduced into SN rather than RN. The advantage of RF-EH SN is studied by addressing the joint energy management policies for the source and relay nodes. The DF-relay node is equipped with limited energy storage, and the source node can harvest energy from the relaying signals. To guarantee a certain level of stability in energy provisioning, a certain amount of energy storage is also available for SN in case the power provided by the RF energy harvester is insufficient for immediate data transmission. By utilizing the power available at SN and RN wisely, the system throughput is maximized for a given amount of time. The derived results can be extended to scenarios where both SN and RN can harvest energy from each other's signals, as shown in Appendix.

II. System Model and Problem Formulation

Considering the half duplex DF-relay enhanced system with RF-EH SN as shown in Fig. 1 (a), SN will transmit data to the relay node in the first time slot (TS), while DF-RN will decode and re-encode the received signal, and then forward it to the destination node (DN) in the subsequent TS. We refer to this complete data flow from SN-RN-DN, i.e., data flow in two consecutive TSs, as one *phase*. As far as the energy flow is concerned, SN can harvest energy from the forwarding signal in the second TS, and the stored energy can facilitate further data transmission in the following phases, as shown in Fig. 1 (b).

To measure the Shannon capacity of the system over N phases, where N can be the delay requirements of data traffic, we assume at the beginning of the data transmission that the amounts of energy already acquired by SN and RN are $P_{1,0}$ and $P_{2,0}$, respectively. The total bandwidth occupied by the system is B. For the sake of convenience, we assume the constant channel power gains across N phases [14], where h_i is the channel gain of the SN-RN link ($i = 1$) and the RN-DN link ($i = 2$). $\gamma_i = |h_i|^2/(N_0 B)$ denotes the corresponding normalized signal-noise-ratio (SNR) associated with the channel between SN and RN ($i = 1$) as well as that associated with the channel between RN and DN ($i = 2$). $N_0 B$ represents the power of additive white Gaussian noise. Without loss of generality, for now, we assume no direct link exists between SN and DN, i.e., the corresponding SNR $\gamma_1' = 0$. The case with the direct link will be discussed in the next section.

The goal is to design the optimal power allocation $P_{i,j},\ i = \{1,2\}, j \in \mathscr{N} = \{1,\cdots,N\}$ such that the overall system throughput across N phases is maximized.

$$
\begin{aligned}
C^* = \max_{P_{i,j}} C = \tfrac{B}{2} \sum_{j=1}^{N} \min_{i=1,2} \{\log(1+P_{i,j}\gamma_i)\} \\
s.t.\quad EC_j^*: \quad & \sum_{k=1}^{j} P_{1,k} \le P_{1,0} + \beta \sum_{k=1}^{j-1} P_{2,k}, j \in \mathscr{N} \\
C_1^*: \quad & \sum_{j=1}^{N} P_{2,j} \le P_{2,0} \\
C_2^*: \quad & P_{i,j} \ge 0, i \in \{1,2\}, j \in \mathscr{N}
\end{aligned} \tag{1}
$$

where in the energy causality constraint EC_j^*, $\beta P_{2,j}$ is the amount of power harvested in phase j and used after the j-th phase. $\beta = \eta |h_1|^2$ with η denoting the energy harvesting efficiency factor [20]. C_1^* represents the budget of the transmission power in RN. C_2^* represents the non-negative power allocation. $B/2$ is attributed to the half-duplex of the relay channel. Note that the terms of power and energy are used interchangeably in this paper, since the power consumption and energy storage are all measured within each TS.

III. Power Allocation Analysis

In the objective function of Eq. (1), the throughput of phase j is determined by $\min\limits_{i=1,2}\{P_{i,j}\gamma_i\}$. Meanwhile, as shown in the energy causality constraint EC_j^*, SN can harvest energy from the signals transmitted by RN, while RN does not have the energy harvesting capability. So, it is reasonable to assume $P_{1,j}\gamma_1 \le P_{2,j}\gamma_2, j \in \mathscr{N}$. We present this result in the following proposition.

Proposition 1: For any optimal solution, we can always find an equivalent power allocation scheme such that for any $j \in \mathscr{N}$, $P_{1,j}\gamma_1 \le P_{2,j}\gamma_2$.

Proof. Suppose there exists an optimal solution with $P_{1,j}\gamma_1 > P_{2,j}\gamma_2$, then we can always reduce $P_{1,j}$ to $P'_{1,j} = P_{2,j}\gamma_2/\gamma_1$. As we can see from the objective function in Eq. (1), $\min\limits_{i=1,2}\{P_{i,j}\gamma_i\} = \min\left\{P'_{1,j}\gamma_1, P_{2,j}\gamma_2\right\}$, i.e., the throughput in phase j is the same as the given optimal solution. Since the power allocation of RN, $P_{2,j}$, is not changed, C_1^* is still satisfied, and $\beta \sum\limits_{k=1}^{j-1} P_{2,k}$, the total energy harvested

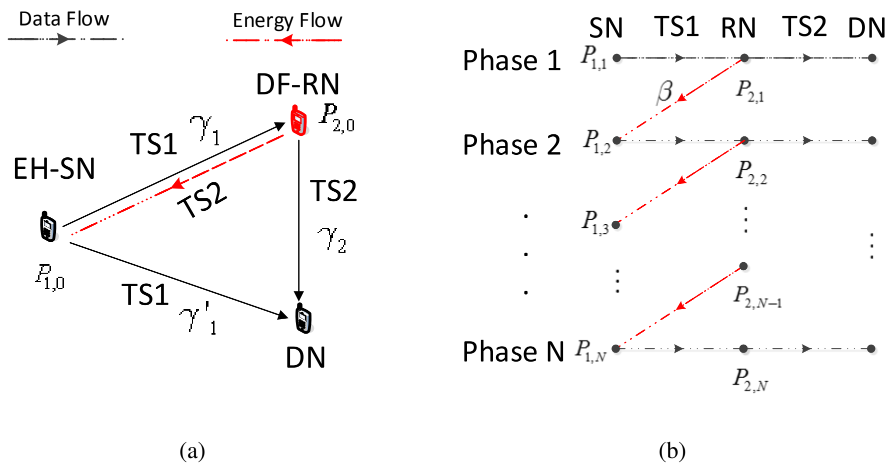


Fig. 1: Energy and data flows in the DF-relay enhanced system with RF-EH SN

by SN up to phase j, remains the same. Furthermore, as $P'_{1,j} < P_{1,j}$, the energy causality in phase j is not violated either. □

Adopting the characteristic of the optimal solution given in *Proposition 1*, at the j-th phase, $j \in \mathcal{N}$, we can divide $P_{2,j}$ into two parts as illustrated below

$$P_{2,j} = p_j + \alpha_j \tag{2}$$

where $p_j = P_{1,j}\gamma_1/\gamma_2$ is for **data forwarding**, and $P_{1,j}$ is solely used for **data transmission**. α_j is the **power supplement** provided by RN to increase the energy storage of SN, and the harvested $\alpha_j\beta$ will be used by SN for future data transmission.

As we can see, based on Eq. (2), the variables in Eq. (1) have become $\{p_j, \alpha_j\}$. To reveal more insights of the optimal solution, we first define the **residual power** of SN and RN at the beginning of the j-th phase as $\overline{P_{1,j}}$ and $\overline{P_{2,j}}$, respectively. Then, the following propositions are presented.

Proposition 2: In the optimal power allocation $\{p_j, \alpha_j\}$, there exists an optimal profile which satisfies the following equality.

$$\begin{cases} \alpha_1 \geq 0 \\ \alpha_j = 0, j \in \{2, \cdots, N\} \end{cases}$$

Proof. Since the residual energy of RN $\overline{P_{2,j}}$ is non-increasing with j, it is feasible to aggregate all the power supplement to the first phase. More specifically, for any feasible solution with $\alpha_j > 0, j \in \{2, \cdots, N\}$, an equivalent solution $\{P^*_{1,j}, p^*_j, \alpha^*_j\}$ can always be found with 1) the same power allocation for data transmission and forwarding, i.e., $P^*_{1,j} = P_{1,j}$, $p^*_j = p_j$, $\forall j \in \{1, \cdots, N\}$; 2) power supplements are aggregated to the first phase, i.e., $\alpha^*_1 = \sum_{j=1}^{N} \alpha_j$, $\alpha^*_j = 0$, $\forall j \in \{2, \cdots, N\}$. □

Proposition 3: To maximize the throughput, the power budget constraint C^*_1 in Eq. (1) is satisfied with equality.

$$\sum_{j=1}^{N} p_j + \alpha_1 = P_{2,0}, N \geq 2 \tag{3}$$

Proof. We will prove this statement by contradiction. Assume for an optimal profile, Eq. (3) is not satisfied with equality. Then, we can find another feasible policy that sends more data than the optimal profile which is a contradiction.

Since in the last phase, at least one of the nodes (SN or RN) will use all of its residual energy, $\alpha_1 + \sum_{j=1}^{N} p_j < P_{2,0}$ implies $p_N < \overline{P_{2,N}}$ and $P_{1,N} = \overline{P_{1,N}}$.

a) If $\overline{P_{1,N}}\gamma_1 \geq \overline{P_{2,N}}\gamma_2$, then obviously $p_N < \overline{P_{2,N}}$ is not the optimal solution.

b) If $\overline{P_{1,N}}\gamma_1 < \overline{P_{2,N}}\gamma_2$, then, there exists a positive α_{N-1} such that

$$\left(\overline{P_{1,N}} + \alpha_{N-1}\beta\right)\gamma_1 = (\overline{P_{2,N}} - \alpha_{N-1})\gamma_2$$

where $\alpha_{N-1} = \frac{\overline{P_{2,N}}\gamma_2 - \overline{P_{1,N}}\gamma_1}{\beta\gamma_1+\gamma_2}$.

This means at phase N-1, in addition to p_{N-1}, RN will provide power supplement α_{N-1} such that $P^*_{1,N} = \overline{P_{1,N}} + \alpha_{N-1}\beta$, $p^*_N = \overline{P_{2,N}} - \alpha_{N-1}$. Since $\min_{i=1,2} \{P^*_{i,N}\gamma_i\} \geq \min_{i=1,2} \{P_{i,N}\gamma_i\}$, the total throughput will increase when $p^*_N = \overline{P^*_{2,N}}$. □

By adopting the previous propositions, the solution to

Eq. (4) must be the solution to Eq. (1).

$$\begin{aligned} C^* = \max_{\{p_j,\alpha\}} & C = \frac{B}{2}\sum_{j=1}^{N}\log(1+p_j\gamma_2) \\ s.t. \quad EC_1: & \ p_1 \le P_{1,0}\gamma \\ EC_j: & \ \sum_{k=1}^{j} p_k \le P_{1,0}\gamma + \beta\gamma(\alpha + \sum_{k=1}^{j-1} p_k) \\ C1: & \ \sum_{j=1}^{N} p_j + \alpha = P_{2,0} \\ C2: & \ \alpha, p_j \ge 0, j \in \mathcal{N} \end{aligned} \tag{4}$$

where α is the aggregated power supplement provided by RN in the first phase. EC_j, $j \in \{2, \cdots, N\}$, is the corresponding EC-constraint, and the SNR ratio $\gamma = \gamma_1/\gamma_2$.

Owing to the linearity of the energy causality constraints [14], the optimization problem given in Eq. (4) is convex, which can be solved by existing approaches given in [21]. However, to reveal some insights on the optimal solution, we exploit the special structure of our energy harvesting system and tailor the method to solve Eq. (4). Note that the following method can be extended to scenarios where both SN and RN can harvest energy from each other, where the specific extension is described in Appendix.

We first consider the scenario where there is only the constant power budget, i.e., constraint C_1. According to the water-filling algorithm [21], the relaxed optimal solution for this scenario is

$$\alpha = 0, p_j = P_{2,0}/N, j \in \{1, \cdots, N\} \tag{5}$$

Remark 1: With the introduction of the energy-causality constraints $EC_j, j \in \{1, \cdots, N\}$ in Eq. (4), the relaxed optimal solution in Eq. (5) is feasible if the following inequalities are satisfied.

$$\begin{cases} P_{1,0} \ge P_{2,0}\frac{1}{N\gamma}, & \beta\gamma \ge 1 \\ P_{1,0} \ge P_{2,0}\frac{N-(N-1)\beta\gamma}{N\gamma}, & \beta\gamma < 1 \end{cases} \tag{6}$$

Proof. To check the feasibility of the relaxed optimal solution for the scenario with all of the EC-constraints, we substitute $\{\alpha, p_j\}$ of Eq. (5) into $EC_j, j \in \{1, \cdots, N\}$, and the following results are obtained.

$$\frac{P_{2,0}}{N}[j-(j-1)\beta\gamma] \le P_{1,0}\gamma, j \in \{1, \cdots, N\} \tag{7}$$

With $\beta\gamma \ge 1$, if Eq. (7) is satisfied for $j = 1$, then the relaxed optimal solution is feasible. With $\beta\gamma < 1$, if Eq. (7) is satisfied for $j = N$, then the relaxed optimal solution is feasible. □

Based on *Remark 1*, we can find the closed form solutions to Eq. (4) with various settings of the system parameters β, γ and the initial power storage $P_{i,0}$, $i = \{1, 2\}$. Note that although normally the overall harvesting efficiency $\beta < 1$, $\beta\gamma \ge 1$ may occur in practice, since γ is the ratio of two normalized SNRs.

IV. Scenario I: $\beta\gamma \ge 1$

With $\beta\gamma \ge 1$, the residual powers of SN and RN in phase j, $j \in \{2, \cdots, N\}$, are

$$\begin{cases} \overline{P_{i,1}} = P_{i,0}, i \in \{1, 2\} \\ \overline{P_{1,j}} = \overline{P_{1,j-1}} + \beta P_{2,j-1} - P_{1,j-1} \\ \overline{P_{2,j}} = \overline{P_{2,j-1}} - P_{2,j-1} \end{cases} \tag{8}$$

where $\beta P_{2,j-1} - P_{1,j-1} \ge (\beta\gamma - 1)P_{1,j-1} > 0$.

Remark 2: Since $\overline{P_{1,j}}$ in Eq. (8) is a non-decreasing function of j, while $\overline{P_{2,j}}$ is a non-increasing function of j, we can always find an optimal solution such that p_j and $P_{1,j}$ are non-decreasing functions of j.

To find the solution in *Remark 2*, we first suppose the residual energy of RN is sufficient to match the residual power of SN, i.e., $\overline{P_{2,j}} \ge \overline{P_{1,j}}\gamma$, then to increase $\overline{P_{1,j+1}}$, SN will prefer to adopt the *greedy power allocation*, i.e., transmits with all of the residual power in phase j. However, when the residual energy of RN is insufficient to match the residual power of SN, then obviously RN could not afford SN's greedy power allocation policy. The following proposition will decide how to allocate $\{p_j, \alpha\}$ based on the relationship between $\overline{P_{1,j}}$ and $\overline{P_{2,j}}$.

Proposition 4: With $\beta\gamma \ge 1$, there exists a certain phase l, $(0 \le l \le N)$, such that the maximum throughput can be guaranteed when SN adopts greedy power allocation before and in phase l, and RN adopts equal power allocation after phase l.

$$\begin{cases} P_{1,j} = \overline{P_{1,j}}, j \in \{1, \cdots, l\} & (a) \\ P_{2,j} = \frac{\overline{P_{2,l+1}}}{N-l}, j \in \{l+1, \cdots, N\} & (b) \end{cases} \tag{9}$$

where $l+1$ is the first phase that the residual energy of RN is insufficient to match the residual power of SN.

Proof. According to *Remark 1*, when $\overline{P_{1,j}}$ and $\overline{P_{2,j}}$ satisfy Eq. (10), the residual power of RN will be insufficient to match the residual power in SN. So, RN will divide $\overline{P_{2,j}}$ equally from phase j to phase N.

$$\overline{P_{1,j}} \ge \begin{cases} (P_{2,0} - \alpha)/(N\gamma), & j = 1 \\ \overline{P_{2,j}}/[(N-j+1)\gamma], & j \ge 1 \end{cases} \tag{10}$$

Suppose $l+1$ is the first phase that does satisfy Eq. (10); so far we have known that Eq. (9b) must be satisfied from phase $l+1$ to phase N. Next, to show that Eq. (9b) is the solution that satisfies *Remark 2*, we will prove $P_{1,l} \le P_{1,j}$, $j \ge l+1$ by contradiction.

Suppose $P_{1,l} > P_{1,l+1} = \overline{P_{2,l+1}}/[(N-l)\gamma]$, then we have

$$p_l \ge \begin{cases} (P_{2,0} - \alpha)/N, & j = 1 \\ \overline{P_{2,l}}/(N-l+1), & j \ge 1 \end{cases} \tag{11}$$

Since $P_{1,l} = p_l/\gamma$, Eq. (11) indicates that phase l also satisfies Eq. (10), which contradicts with the assumption.

Consequently, to prove *Proposition 4*, we only need to prove that Eq. (9a) must be satisfied from phase 1 to phase l, where $P_{1,j}$ is inherently non-decreasing function of j.

Since for any $j \leq l$, we have $\overline{P_{1,j}} < \overline{P_{2,l+1}}/[(N-l)\gamma]$, then according to the water-filling algorithm, any solution with $P_{1,j} < \overline{P_{1,j}}$ will yield less sum throughput than Eq. (9a). □

According to *Proposition 4*, the optimal power allocation is given in Table I. The following proposition facilitates finding the optimal l.

Proposition 5: With $\beta\gamma \geq 1$ and $P_{1,0}$ is between $\left[P_k^{th}, P_{k-1}^{th}\right)$, $k \in \{1, \cdots, N\}$, then, $l \leq k$, where

$$\begin{cases} P_k^{th} = \frac{P_{2,0}}{\gamma[(N-k)(\beta\gamma)^k + \sum_{j=1}^{k}(\beta\gamma)^{j-1}]} \\ P_N^{th} = 0 \end{cases} \tag{13}$$

Proof. As we can see in Table I, with $\alpha = 0$, let P_k^{th} be the threshold of $P_{1,0}$ such that RN's residual power is just enough to match SN's residual power, i.e., Eq. (12) in phase $k+1$ is satisfied with equality.

$$P_k^{th}\gamma(\beta\gamma)^k = \frac{P_{2,0} - \sum_{j=1}^{k} P_{2,j}}{N-k}$$

where $P_{2,j} = P_k^{th}\gamma(\beta\gamma)^{j-1}$, $j = \{1, \cdots, k\}$.

Then, $P_{1,0} \geq P_k^{th}$ or $\alpha \geq 0$ means the insufficiency of RN's residual power will occur no later than phase $k+1$. Since $l+1$ is the first phase that the residual energy of RN is insufficient to match the residual power of SN, $l \leq k$. □

A. $P_{1,0} \geq P_0^{th} = \frac{P_{2,0}}{N\gamma}$

According to *Remark 1*, EC_1 is satisfied and the relaxed solution in Eq. (5) is optimal.

$$\begin{cases} P_{2,j} = p_j = \frac{P_{2,0}}{N}, j \in \{1, \cdots, N\} \\ P_{1,j} = \frac{p_j}{\gamma} = \frac{P_{2,0}}{N\gamma}, j \in \{1, \cdots, N\} \end{cases} \tag{14}$$

B. $P_{k-1}^{th} > P_{1,0} \geq P_k^{th}$, $k \in \{1, \cdots, N\}$

For each $l \in \{1, \cdots, k\}$, as shown in Table I, p_j, $j \in \{2, \cdots, N\}$, in Eq. (4) are all determined by α. The feasible domain of α for each l is given by the threshold α_l^{th}, which is defined as the minimum positive value such that Eq. (12) is satisfied with equality.

$$\begin{cases} \alpha_l^{th} = \left\{\frac{P_{2,0}}{(N-l)(\beta\gamma)^l + \sum_{j=1}^{l}(\beta\gamma)^{j-1}} - P_{1,0}\gamma\right\}^+ \\ \alpha_0^{th} = P_{2,0} \end{cases}$$

where $\{\bullet\}^+ = max\{\bullet, 0\}$.

When $\alpha_{l-1}^{th} > \alpha \geq \alpha_l^{th}$, the insufficiency of RN's power will occur at phase $l+1$. As illustrated in Eq. (15), the simplified form of Eq. (4) allows us to use the Lagrange method to get the optimal solution by setting the first derivative of the objective function in Eq. (15) to zero, as shown in Eq. (16). The corresponding optimal solution is

$$\begin{cases} P_{2,1} = p_1 + \alpha^* = P_{1,0}\gamma + \alpha^* \\ P_{2,j} = (P_{1,0}\gamma + \alpha^*)(\beta\gamma)^j, j \in \{2, \cdots, l\} \\ P_{2,j} = \frac{(P_{2,0} - \sum_{t=1}^{l} P_{2,t})}{N-l}, j \in \{l+1, \cdots, N\} \\ P_{1,1} = P_{1,0}, P_{1,j} = \frac{P_{2,j}}{\gamma}, j \in \{2, \cdots, N\} \end{cases} \tag{17}$$

where α^* is the solution to Eq. (16), if it falls within $(\alpha_{l-1}^{th}, \alpha_l^{th}]$. Otherwise, $\alpha^* = \alpha_l^{th}$.

Note: For the case with $P_0^{th} > P_{1,0} \geq P_1^{th}$, the optimal solution is $\alpha^* = 0$, since $k = l = 1$ and $\alpha_l^{th} = 0$.

V. Scenario II: $\beta\gamma < 1$

Unlike the scenario where SN can rely solely on the harvested energy after the first phase, SN needs to spare part of the initial power storage $P_{1,0}$ for future data transmission with $\beta\gamma < 1$.

From the throughput's perspective, as compared with $P_{i,j} > P_{i,j+1}$, $P_{i,j}^* = P_{i,j+1}^*$ is always a preferable solution (*Remark 1*). However, $P_{i,j}^* = P_{i,j+1}^*$ may not be feasible. The reason is that from the energy's point of view, $P_{i,j}^* \leq P_{i,j+1}^*$ will bring less harvested energy to phase $j+1$ than $P_{i,j} > P_{i,j+1}$.

To untangle the above mentioned relationship between energy and throughput, we have the following remarks.

Remark 3: With $\beta\gamma < 1$, we can always find an optimal solution such that $P_{2,j}$ is a non-increasing function of j.

Proof. For $j \geq 2$, $\overline{P_{1,j}}$ and $\overline{P_{2,j}}$ are both non-increasing functions of j, and so for any feasible solution, we can always find an equivalence with p_j and $P_{1,j}$ being non-increasing functions of j. For $j = 1$, $\overline{P_{1,2}} > \overline{P_{1,1}}$ would only occur when the power supplement α satisfies $(P_{1,1}\gamma + \alpha)\beta > P_{1,1}$.

When $(P_{1,1}\gamma + \alpha)\beta < P_{1,2}$, suppose $P_{2,2} > P_{2,1}$ is in the optimal solution, then increasing $P_{1,1}$ and $P_{2,1}$ would not only be feasible, but always yield greater sum throughput in phase 1 and 2. This contradicts with the assumption.

Consequently, $P_{2,2} > P_{2,1}$ is only possible when $(P_{1,1}\gamma + \alpha)\beta \geq \{P_{1,1}, P_{1,2}\}$.

$$(P_{1,1}\gamma + \alpha)\beta\gamma \geq P_{1,2}\gamma > P_{1,1}\gamma + \alpha$$

Since $\beta\gamma > 1$ contradicts with the assumption, we can always find an optimal solution such that $P_{2,j}$ is a non-increasing function of j. □

Proposition 6: Based on *Remark 3*, since $\overline{P_{1,j}}$ and $\overline{P_{2,j}}$ in Eq. (8) are decreasing functions of j, $j \in \{3, \cdots, N-1\}$, there exists an optimal solution with the following characteristics.

$$P_{2,1} \geq P_{2,j-1} = P_{2,j} \geq P_{2,N}, \forall j \in \{3, \cdots, N-1\}$$

TABLE I: Optimal strategy with $\beta\gamma \geq 1$

SN	DN
$P_{1,1} = P_{1,0}$	$P_{2,1} = P_{1,0}\gamma + \alpha < \frac{P_{2,0}}{N}$
$P_{1,2} = (P_{1,0}\gamma + \alpha)\beta$	$P_{2,2} = (P_{1,0}\gamma + \alpha)\beta\gamma$
$\vdots$	$\vdots$
$P_{1,l} = (P_{1,0}\gamma + \alpha)(\beta\gamma)^{l-2}\beta$	$P_{2,l} = (P_{1,0}\gamma + \alpha)(\beta\gamma)^{l-1} < \frac{P_{2,0} - \sum_{j=1}^{l-1} P_{2,j}}{N-(l-1)}$
$P_{1,l+1} = \frac{P_{2,l+1}}{\gamma}$	$P_{2,l+1} = \dfrac{P_{2,0} - \sum_{j=1}^{l} P_{2,j}}{N-l} \leq (P_{1,0}\gamma + \alpha)(\beta\gamma)^{l} \quad (12)$
$P_{i,j} = P_{i,l+1},\ i = \{1,2\},\ j \in \{l+2, \cdots, N\}$	

$$\max_{\{\alpha\}} \sum_{j=2}^{l} \log[1 + (P_{1,0}\gamma + \alpha)(\beta\gamma)^{j-1}\gamma_2] + (N-l)\log\{1 + \frac{[P_{2,0} - (P_{1,0}\gamma + \alpha)\sum_{j=1}^{l}(\beta\gamma)^{j-1}]\gamma_2}{(N-l)}\} \quad s.t.\ \alpha_{l-1}^{th} > \alpha \geq \alpha_l^{th} \tag{15}$$

$$\begin{cases} -\sum_{j=2}^{l} \frac{(\beta\gamma)^{t-1}(1-\beta\gamma)}{1+(P_{1,0}\gamma+\alpha)(\beta\gamma)^{j-1}\gamma_2} + \frac{(N-l)[1-(\beta\gamma)^{l-1}]}{N-l+[P_{2,0}-(P_{1,0}\gamma+\alpha)\sum_{j=1}^{l}(\beta\gamma)^{j-1}]\gamma_2} = 0, & \beta\gamma > 1 \\ -\frac{l-1}{1+(P_{1,0}\gamma+\alpha)\gamma_2} + \frac{(N-l)l}{N-l+[P_{2,0}-(P_{1,0}\gamma+\alpha)l]\gamma_2} = 0, & \beta\gamma = 1 \end{cases} \tag{16}$$

Remark 4: For the optimal solution in *Proposition 6*, 1) if $P_{2,N-1} > P_{2,N}$, there must be $P_{2,N} = \overline{P_{1,N}}\gamma$; 2) if $p_1 < P_{2,2}$, there must be $p_1 = P_{1,0}\gamma$.

Proof. We only prove the first part here, and the second part can be proved similarly. According to *Proposition 6* and *Remark 1*,

$$P_{2,N-1} \geq P_{2,N}$$

where the inequality is only possible when $\overline{P_{1,N-1}} < \frac{2-\beta\gamma}{2\gamma}(P_{2,N-1} + P_{2,N})$.

Suppose in the optimal solution, both $P_{2,N-1} > P_{2,N}$ and $P_{2,N} < \overline{P_{1,N}}\gamma$ are satisfied. Then, we will have

$$P_{1,N} < \overline{P_{1,N}}$$

However, a new power allocation scheme exists.

$$\begin{cases} P_{1,N-1}^* = P_{1,N-1} - \frac{\overline{P_{1,N}} - P_{1,N}}{\beta\gamma} \\ P_{1,N}^* = P_{1,N} + \frac{\overline{P_{1,N}} - P_{1,N}}{\beta\gamma} \end{cases}$$

Since $P_{1,N} \leq P_{1,N}^* \leq P_{1,N-1}^* < P_{1,N-1}$, the new allocation will have better sum throughput. This contradicts with the assumption. □

A. $P_{1,0} \geq P_{2,0}\frac{N-(N-1)\beta\gamma}{N\gamma}$

According to *Remark 1*, Eq. (14) satisfies EC_N, and thus the relaxed optimal solution is feasible.

B. $P_{1,0} < P_{2,0}\frac{N-(N-1)\beta\gamma}{N\gamma}$

Let $p_c = p_j$, $j \in \{2, \cdots, N-1\}$, then from *Proposal 6* and *Remark 4*, the constraints in Eq. (4) become

$$\begin{cases} p_1 + \alpha \geq p_c \geq \max\{p_1, p_N\} \\ p_1, p_N \geq 0 \\ EC_j, C_1, C_2, j \in \{1, \cdots, N\} \end{cases} \tag{18}$$

Meanwhile, the optimal solution falls into one of the following four cases.

1) $p_1 = P_{1,0}\gamma \leq p_c$, $p_N = p_c$: In this case, *Remark 1* indicates EC_N is sufficient to represent EC_j, $j \in \{2, \cdots, N\}$, and the constraints in Eq. (18) are simplified as

$$\begin{cases} P_{1,0}\gamma + \alpha \geq p_c \geq P_{1,0}\gamma \\ EC_N, C_1 \end{cases} \tag{19}$$

2) $p_1 = p_c \leq P_{1,0}\gamma$, $p_N = p_c$: Similarly, the constraints become Eq. (20)

$$\begin{cases} P_{1,0}\gamma \geq p_c \geq 0 \\ P_{2,0} \geq \alpha \geq 0 \\ EC_N, C_1 \end{cases} \tag{20}$$

3) $p_1 = P_{1,0}\gamma \le p_c$, $p_N = \overline{P_{1,N}}\gamma$: The constraints are:

$$\begin{cases} P_{1,0}\gamma + \alpha \ge p_c \ge \max\{P_{1,0}\gamma, p_N\} \\ EC_{N-1}, EC_N, C_1 \end{cases} \tag{21}$$

where EC_N is satisfied with equality. EC_{N-1} is satisfied if and only if $p_N \ge \beta\gamma p_c$.

4) $p_1 = p_c \le P_{1,0}\gamma$, $p_N = \overline{P_{1,N}}\gamma$: Similar to Eq. (21), EC_{N-1} requires $p_N \ge \beta\gamma p_c$, and the corresponding constraints become

$$\begin{cases} P_{1,0}\gamma \ge p_c \ge p_N \ge \beta\gamma p_c \\ p_c, \alpha \ge 0 \\ EC_N \text{ satisfied with equality}, C_1 \end{cases} \tag{22}$$

With different constraints, the conversions of Eq. (4) are given in Table II, and the corresponding solutions are as follows:

$$\begin{cases} P_{2,1} = p_1^* + \alpha_k^*, k \in \{1, \cdots, 4\} \\ P_{2,N} = p_N^*, P_{2,j} = p_j^*, j \in \{2, \cdots, N-1\} \\ P_{1,1} = p_1^*/\gamma, P_{1,j} = P_{2,j}/\gamma, j \in \{2, \cdots, N\} \end{cases} \tag{23}$$

where α_k^*, $k \in \{1, 2\}$, is equal to Eq. (28) and Eq. (29), respectively, if they are feasible. Otherwise, α_k^* does not exist. When $k \in \{3, 4\}$, α_k^* is equal to Eq. (30) or Eq. (31), if it falls within $[\alpha_{min}, \alpha_{max}]$; otherwise, $\alpha^* = \alpha_{min}$ or $\alpha^* = \alpha_{max}$. For each k, p_j^*, $j \in \{1, \cdots, N\}$, are given in the corresponding constraints.

VI. System Model with Direct Link between SN and DN

To this end, we have solved the throughput maximization problem for the relay system where there is no direct link between SN and DN, due to severe channel attenuation. Here, we will discuss the scenario with direct link between SN and DN, $\gamma_1 > \gamma_1' > 0$, as illustrated in Fig. 1.

First, the objective function in Eq. (1) becomes

$$\max_{P_{i,j}} C = \frac{B}{2}\sum_{j=1}^{N} \log(1 + \min\{P_{1,j}\gamma_1, P_{1,j}\gamma_1' + P_{2,j}\gamma_2\}) \tag{24}$$

Similarly to *Proposition 1* and Eq. (2), it is reasonable to assume

$$P_{1,j}(\gamma_1 - \gamma_1') \le P_{2,j}\gamma_2$$

Then, it is obvious that *Proposition 2* holds for the direct link case.

By classifying the residual power of phase N into: 1) $\overline{P_{2,N}}\gamma_2 \ge \overline{P_{1,N}}(\gamma_1 - \gamma_1')$ and 2) $\overline{P_{2,N}}\gamma_2 < \overline{P_{1,N}}(\gamma_1 - \gamma_1')$, we can prove *Proposition 3* also holds for the scenario with direct link between SN and DN.

Applying *Propositions 2-3*, the optimization problem corresponding to Eq. (4) becomes

$$\begin{aligned} C^* = \max_{\{p_j, \alpha\}} C = \tfrac{B}{2}\sum_{j=1}^{N} \log(1 + p_j \tfrac{\gamma_1}{\gamma^*}) \\ s.t. \quad EC_j, C_1, C_2, j \in \{1, \cdots, N\} \end{aligned} \tag{25}$$

where $\gamma^* = (\gamma_1 - \gamma_1')/\gamma_2$.

Consequently, simply by substituting γ with γ^*, the analysis in Section II - V is still applicable here.

VII. Numerical Results

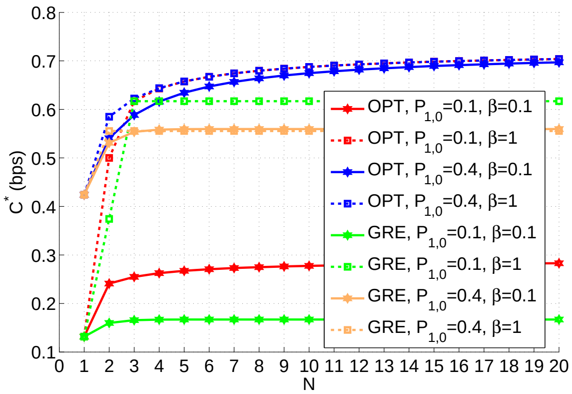
Based on the theoretical analysis, the optimal power allocation algorithm (OPT) to maximize the system throughput is given below.

Algorithm OPT algorithm with $\mathcal{O}(N)$ complexity

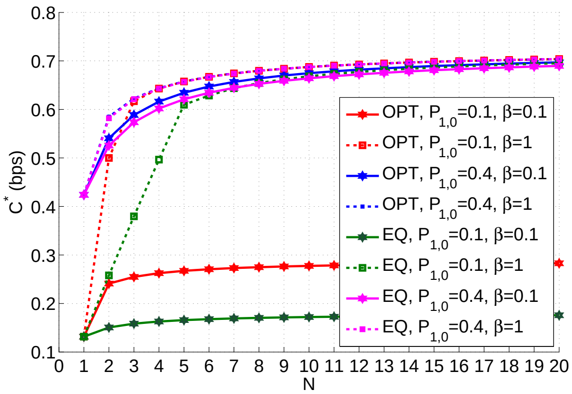
$C^* = 0$
if $\beta\gamma \ge 1$ **then**
  if $P_{1,0} \ge P_{2,0}\frac{1}{N\gamma}$ **then**
    Calculate C^* according to Eq. (14)
  else
    Calculate k, such that $P_{1,0} \in \left(P_{k-1}^{th}, P_k^{th}\right]$
    for $l = 1$ to k **do**
      Calculate C according to Eq. (17)
      $C^* = \max\{C^*, C\}$
    end for
  end if
else
  if $P_{1,0} \ge P_{2,0}\frac{N-(N-1)\beta\gamma}{N\gamma}$ **then**
    Calculate C^* according to Eq. (14)
  else
    for $k = 1$ to 4 **do**
      Calculate C^* according to Eq. (23)
      $C^* = \max\{C^*, C\}$
    end for
  end if
end if
return C^*

To verify the performance of the proposed OPT power allocation scheme, we assume the system has unit bandwidth $B = 1$. The budget of reliable power supply for DF-RN is $P_{2,0} = 1$, and SNR for the RN-DN link is $\gamma_2 = 1$. Since one of the key parameters of the OPT algorithm is the product of SNR ratio $\gamma = \gamma_1/\gamma_2$ and energy harvesting efficiency β, we assume SNR for the RN-DN link γ_1 is greater than 1, so that with $\beta \in [0, 1]$, the value of $\beta\gamma$ could cover both scenarios given in Sections IV and V. These assumptions are sufficient to demonstrate the basic principles of the proposed schemes as well as facilitate tractable theoretical derivations. The consideration of a more sophisticated channel model does not affect the key ideas of the proposed schemes and is left for our future work.

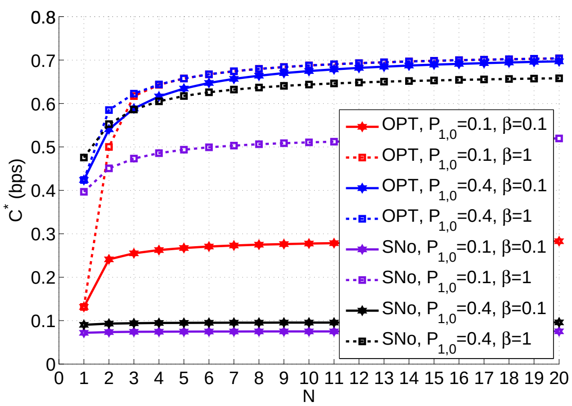
The greedy (GRE), equal (EQ) and SN-only (SNo) power allocation algorithms are used to provide performance reference for our proposed optimal power allocation algorithm. In each phase of the GRE algorithm, at least SN or RN will transmit with all of the residual



(a)

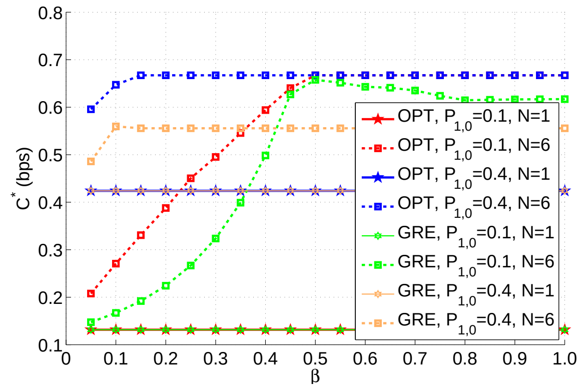


(b)

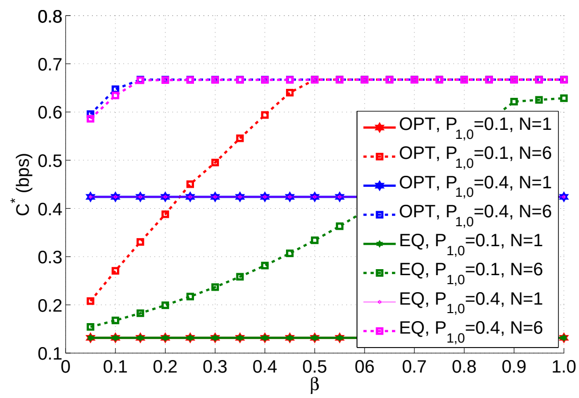


(c)

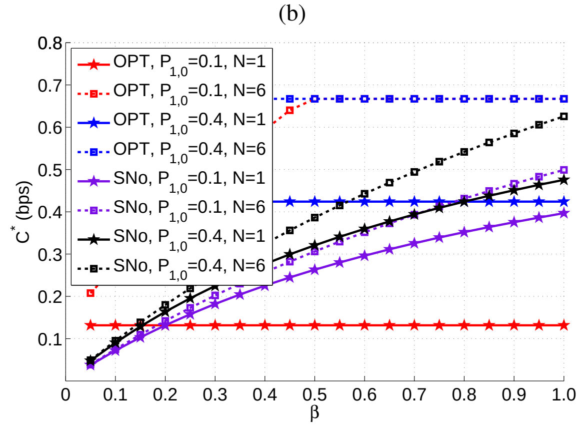
Fig. 2: Comparison of throughput vs. N.



(a)



(b)



(c)

Fig. 3: Comparison of throughput vs. β.

power, depending on the value of $\overline{P_{1,j}}\gamma_1 - \overline{P_{2,j}}\gamma_2$. If it is negative, SN will transmit with $\overline{P_{1,j}}$ in phase j, and RN will transmit with $\overline{P_{1,j}}\gamma$. If it is positive, $P_{2,j} = \overline{P_{2,j}}$, $P_{1,j} = \overline{P_{2,j}}/\gamma$. Similarly, the EQ algorithm states that at least SN or RN will transmit with $\overline{P_{i,j}}/(N-j+1)$, depending on whether $\overline{P_{1,j}}\gamma_1 \leq \overline{P_{2,j}}\gamma_2$ or not. Finally, the SNo algorithm is designed for a system where SN has total power supply of $P_{1,0} + P_{2,0}$, and RN with co-located data and energy reception components splits the received signals between data detector and energy harvester. To maximize the throughput, SN will distribute them equally among the N phases and RN has a power split ratio of γ/β, where $\gamma/(\gamma+\beta)$ percent of the received signal is used for energy harvesting, and the rest is used for data detection.

Through numerical results shown in Figs. 2 and 3, the theoretical results are validated. In the OPT algorithm, the system throughput will increase with N. When

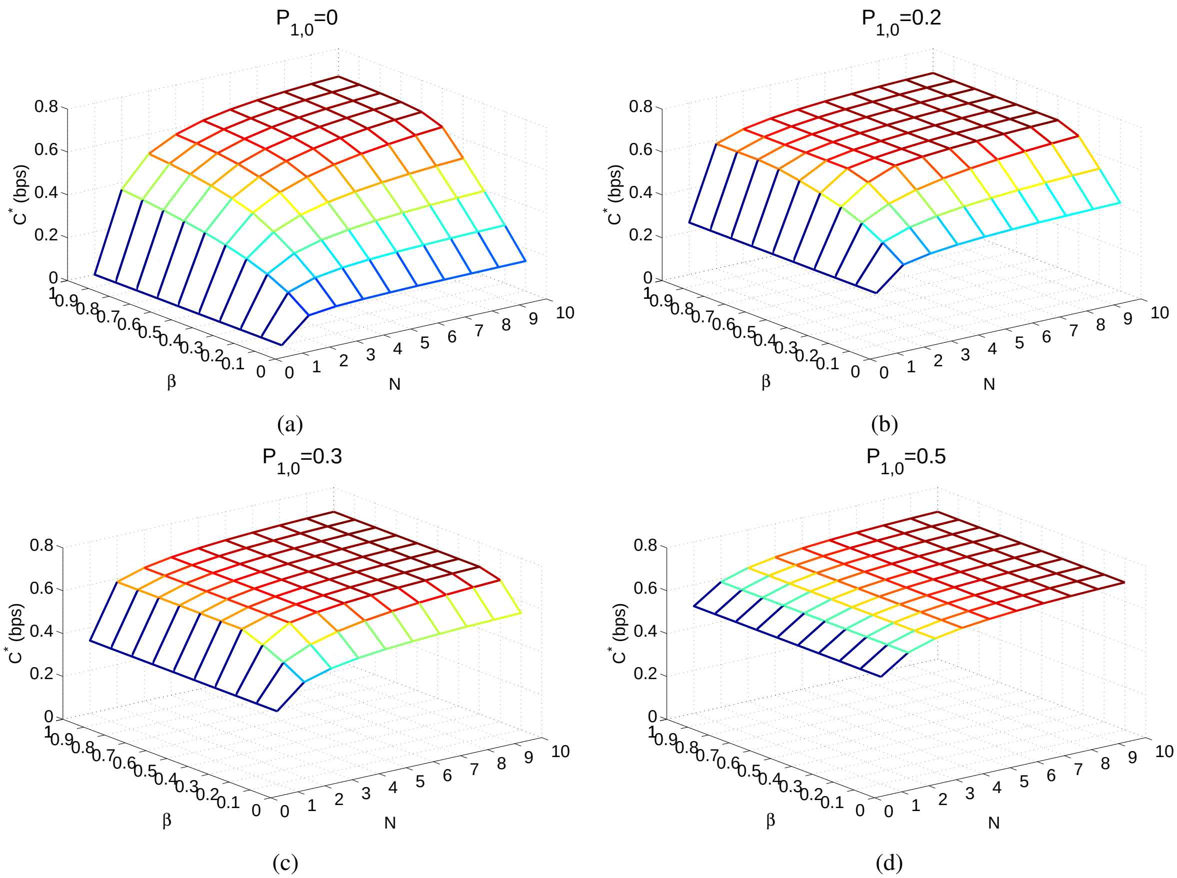


Fig. 4: Optimal throughput with $\gamma_1 = 2$.

$N = 1$, the system throughput will only increase with $P_{1,0}$ because the harvested energy cannot be utilized in the first phase. Meanwhile, the total transmission power budget of RN is limited, and $P_{2,0}$ will be the leading deciding factor of the overall throughput as N grows. More specifically, as illustrated in the OPT algorithm, when N increases to a certain level, Eq. (14) will always be the optimal solution. Since $C^* = \frac{NB}{2}\log(1+\frac{P_{2,0}\gamma_2}{N})$ is a monotonically increasing convex function of N, the increment in the overall throughput is less obvious with higher value of N. Similarly, the system performance will increase with the harvesting efficiency β, and the performance improvement will be less obvious as β increases to a certain point where the power resource of the relay node is more stringent.

Our OPT algorithm always outperforms the GRE algorithm. The performance of OPT and EQ will converge when β or N increases. This is because high β or N will relax the demand for $P_{1,0}$ (*Remark 1*), and equal power allocation will become the feasible optimal solution. As compared with the SNo algorithm, the OPT algorithm will have lower throughput with $N = 1$. This is because the energy harvested by SN cannot be used to improve the throughput in the first phase, while the energy harvesting RN (EH-RN) can transmit data using the harnessed energy in the second time slot of the first phase. As we can see, when N increases, the performance difference between the OPT and SNo algorithms indicates that in the half duplex relay system, when RN is equipped with EH, SN will not gain anything in the even time slots, while with EH-SN, it can always harvest energy from the signals received in the even TSs.

As illustrated in Fig. 4, the proposed OPT algorithm provides a tradeoff between SN's initial power storage $P_{1,0}$ and $\{N, \beta\}$. For $P_{1,0} \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5\}$, where $\{0.1, 0.4\}$ are given in Figs. 2 and 3, and $\{0, 0.2.0.3, 0.5\}$ are given in Fig. 4, we can see that as $P_{1,0}$ increases, the effect of N and β over the overall throughput C^* is less obvious. The reason is that with high $P_{1,0}$, Eq. (6) in *Remark 1* is more likely to be satisfied, and the relaxed optimal solution in Eq. (5) is feasible even for small N and β. As a matter of fact, the throughput performance with $P_{1,0} > 0.5$ is similar to the one with $P_{1,0} = 0.5$, and is therefore not presented here due to the page limit.

Consequently, with a certain amount of throughput requirement, instead of demanding high available power from either on-grid power supply or green energy source, the system can improve energy harvesting efficiency, or utilize more time to transmit delay tolerant traffic by using the harvested energy.

VIII. Conclusion

Radio frequency energy harvesting provides a new approach for wireless devices to share each other's energy storage, either on-grid power or green power. With simultaneous data and energy transmission, it can also decrease the total power consumption of the wireless system. This is of particular interest to sensor networks where nodes have limited storage capacity, and to cellular networks where handsets try to maximize the throughput within time limits. However, simultaneous data and energy transmission is hardware demanding since it requires two RF chains in the RF-EH device. In this paper, we have studied the throughput maximization problem for the orthogonal relay channel, where EH-source node only needs to harvest energy from the forwarding signals transmitted by the regular relay node. Assuming a deterministic EH model, for both cases with and without direct link between SN and DN, we have derived the closed form optimal solutions for the joint source and relay power allocation problem. The developed algorithm can achieve the optimal solution for each system setting with linear complexity.

Appendix

As shown in Fig. 5 (a), when both SN and DF-RN can harvest energy from each other's signals, DF-RN has two RF chains, one for data and one for energy harvesting. Assume the energy harvesting efficiency factor η determined by the hardware of SN and RN are the same, then the overall energy harvesting efficiencies of the SN-RN and RN-SN links are the same, shown as $\beta = \eta|h_1|^2$ in Fig. 5 (b).

The corresponding power allocation problem is given as follows:

$$\begin{aligned} C^* = \max_{\{P_{i,j}\}} C &= \frac{B}{2}\sum_{j=1}^{N} \min_{i=1,2}\{\log(1+P_{i,j}\gamma_i)\} \\ s.t. \quad EC_{1,j}: &\quad \sum_{k=1}^{j} P_{1,k} \le P_{1,0} + \beta\sum_{k=1}^{j-1} P_{2,k} \\ EC_{2,j}: &\quad \sum_{k=1}^{j} P_{2,k} \le P_{2,0} + \beta\sum_{k=1}^{j} P_{1,k} \\ NC: &\quad P_{i,j} \ge 0, i \in \{1,2\}, j \in \mathcal{N} \end{aligned} \tag{26}$$

Similar to the analysis in Section III, we can divide the transmission power of SN and RN into two parts.

$$\begin{cases} P_{i,j} = p_{i,j} + \alpha_{i,j}, & i \in \{1,2\}, j \in \mathcal{N} \\ p_{1,j}\gamma_1 = p_{2,j}\gamma_2, & j \in \mathcal{N} \end{cases} \tag{27}$$

where $p_{1,j}$ is for data transmission, $p_{2,j}$ is for data forwarding, and $\alpha_{i,j}$ is the power supplement provided by SN ($i=1$) and RN ($i=2$), respectively.

By categorizing energy harvesting efficiency β and SNR ratio γ into different scenarios, we can transform the joint power allocation problem into various forms and get the corresponding solutions. Details can be found in [19].

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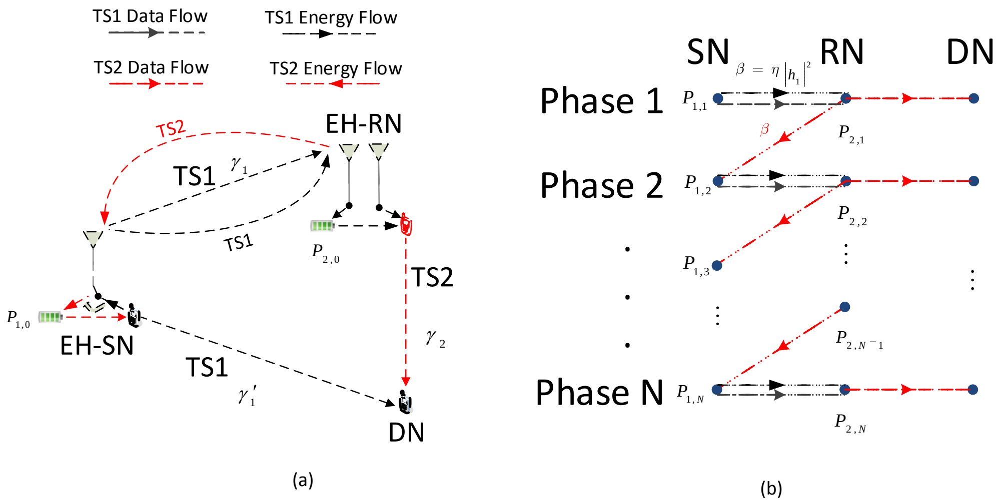

Fig. 5: Energy and data flows in the DF-relay enhanced system with RF-EH SN and RF-DF-RN.

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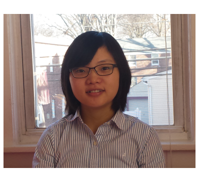

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TABLE II: Optimal power allocation with $\beta\gamma < 1$

1) Problem	$C_1^* = \max_{\{\alpha\}} C = \frac{B}{2}\log(1+p_1\gamma_2)(1+p_c\gamma_2)^{N-2}(1+p_N\gamma_2)$ $s.t. \quad \alpha \geq \alpha_{min} = max\{\frac{P_{2,0}-NP_{1,0}\gamma}{N}, P_{2,0}-P_{1,0}\gamma-\frac{(N-1)\beta\gamma P_{2,0}}{N-1+\beta\gamma}\}$ $\alpha \leq \alpha_{max} = P_{2,0}-NP_{1,0}\gamma$ $p_1 = P_{1,0}\gamma, p_N = p_c = \frac{P_{2,0}-(P_{1,0}\gamma+\alpha)}{N-1}$
1) Solution	$\alpha_1^* = max\{\frac{P_{2,0}-NP_{1,0}\gamma}{N}, P_{2,0}-P_{1,0}\gamma-\frac{(N-1)\beta\gamma P_{2,0}}{N-1+\beta\gamma}\} \quad (28)$
2) Problem	$C_2^* = \max_{\{\alpha\}} C = \frac{B}{2}\log(1+p_1\gamma_2)(1+p_c\gamma_2)^{N-2}(1+p_N\gamma_2)$ $s.t. \quad \alpha \geq \alpha_{min} = max\{0, P_{2,0}-NP_{1,0}\gamma, P_{2,0}-P_{1,0}\gamma-\frac{(NP_{2,0}-P_{1,0}\gamma)\beta\gamma}{N+\beta\gamma}\}$ $\alpha \leq \alpha_{max} = P_{2,0}$ $p_1 = p_N = p_c = \frac{P_{2,0}-\alpha}{N}$
2) Solution	$\alpha_2^* = max\{0, P_{2,0}-NP_{1,0}\gamma, P_{2,0}-P_{1,0}\gamma-\frac{(NP_{2,0}-P_{1,0}\gamma)\beta\gamma}{N+\beta\gamma}\} \quad (29)$
3) Problem	$C_3^* = \max_{\{\alpha\}} C = \frac{B}{2}\log(1+p_1\gamma_2)(1+p_c\gamma_2)^{N-2}(1+p_N\gamma_2)$ $s.t. \quad \alpha \geq \alpha_{min} = max\{P_{2,0}-P_{1,0}\gamma-\frac{P_{2,0}(N-2+\beta\gamma)\beta\gamma}{(1+\beta\gamma)\beta\gamma+N-2}, \frac{P_{2,0}}{1+(N-1)\beta\gamma}-P_{1,0}\gamma\}$ $\alpha \leq \alpha_{max} = min\{\frac{P_{2,0}-P_{1,0}\gamma[1+(N-1)\beta\gamma]}{1+\beta\gamma}, P_{2,0}-P_{1,0}\gamma-\frac{(N-1)\beta\gamma}{N-1+\beta\gamma}P_{2,0}\}$ $p_1 = P_{1,0}\gamma, p_N = P_{2,0}+\frac{P_{1,0}\gamma+\alpha-P_{2,0}}{\beta\gamma}, p_c = \begin{cases} \frac{P_{2,0}-(P_{1,0}\gamma+\alpha)(1+\beta\gamma)}{(N-2)\beta\gamma}, N>2 \\ 0, N=2 \end{cases}$
3) Solution	$\alpha_3^* = \frac{P_{2,0}}{(N-1)(1+\beta\gamma)} + \frac{P_{2,0}(1-\beta\gamma)(N-2)}{N-1} - \frac{(\beta\gamma)^2(N-2)}{(1+\beta\gamma)(N-1)\gamma_2} - P_{1,0}\gamma \quad (30)$
4) Problem	$C_4^* = \max_{\{\alpha\}} C = \frac{B}{2}\log(1+p_1\gamma_2)(1+p_c\gamma_2)^{N-2}(1+p_N\gamma_2)$ $s.t. \quad \alpha \geq \alpha_{min} = max\{0, \frac{P_{2,0}-P_{1,0}\gamma[1+(N-1)\beta\gamma]}{1+\beta\gamma}, \frac{P_{2,0}[N-1-(N-2)\beta\gamma]-P_{1,0}\gamma(\beta\gamma+N-1)}{N-1+(1+\beta\gamma)\beta\gamma}\}$ $\alpha \leq \alpha_{max} = min\{\frac{P_{2,0}-P_{1,0}\gamma}{1+\beta\gamma}, P_{2,0}-P_{1,0}\gamma-\frac{\beta\gamma(NP_{2,0}-P_{1,0}\gamma)}{N+\beta\gamma}\}$ $p_1 = p_c = \frac{P_{2,0}-P_{1,0}\gamma-\alpha(1+\beta\gamma)}{(N-1)\beta\gamma}, p_N = P_{2,0}+\frac{P_{1,0}\gamma+\alpha-P_{2,0}}{\beta\gamma}$
4) Solution	$\alpha_4^* = \frac{\gamma_2(P_{2,0}-P_{1,0}\gamma)[(1+\beta\gamma)N-\beta\gamma]-P_{2,0}(N-1)(1+\beta\gamma)\beta\gamma\gamma_2-(\beta\gamma)^2(N-1)}{N(1+\beta\gamma)\gamma_2} \quad (31)$