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Citation:
Mohammad Yousefvand, Nirwan Ansari, and Siavash Khorsandi, "Maximizing Network Capacity of Cognitive Radio Networks by Capacity-aware Spectrum Allocation," *IEEE Trans. on Wireless Communications*, DOI: 10.1109/TWC.2015.2431691, vol. 14, no. 9, pp. 5058-5067, Sep. 2015.

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Maximizing Network Capacity of Cognitive Radio Networks by Capacity-aware Spectrum Allocation

M. Yousefvand[1], *Member*, *IEEE*, N. Ansari, *Fellow*, *IEEE*, and S. Khorsandi, *Member*, *IEEE*

***Abstract*—In this paper, we present a novel capacity-aware spectrum allocation model for cognitive radio networks. First, we model interference constraints based on the interference temperature model, and let the secondary users (SUs) increase their transmission power until the interference temperature on one of their neighbors exceeds its interference temperature threshold. Then, knowing the SINR and bandwidth of potential links, we calculate the link capacity based on the Shannon formula, and model the co-channel interference between potential links on each channel by using an interference graph. Next, we formulate the spectrum assignment problem in the form of a binary integer linear programming (BILP) to find the optimal feasible set of simultaneously active links among all the potential links in the sense of maximizing the overall network capacity. We also propose a new radix tree based algorithm that, by removing the sparse areas in the search space, leads to a considerable decrease in time complexity of solving the spectrum allocation problem as compared to the BILP algorithm. The simulation results have shown that this proposed model leads to a considerable improvement in overall network capacity as compared to genetic algorithm, and leads to a considerable decrease in time duration needed to find the optimal solution as compared to the BILP algorithm.**

***Index Terms*—Cognitive Radio; Spectrum Allocation; Network Capacity; Interference Constraints; Cognitive Cycle.**

I. Introduction And Related Works

BY the advent of cognitive radio (CR) technology and transition from traditional fixed spectrum assignment paradigms toward new spectrum access techniques [1]-[8], numerous algorithms for spectrum access in CR networks have been proposed. All of these works aim to improve spectrum efficiency by provisioning dynamic and opportunistic spectrum access for secondary users (SUs). Attributed to the two important capabilities and features of CR nodes, namely, cognition and reconfigurability, most of these spectrum allocation techniques comprise two general phases. During the first phase, through sensing the spectral environment, CR users capture information of spectrum bands and classify available channels in terms of their quality level. The second phase of these methods usually comprises a spectrum allocation algorithm that, apart from satisfying interference constraints, allocates available channels to SUs such that spectrum efficiency is maximized. Some of these works like [3][9], disregarding QoS parameters of available channels such as their SINR and capacity, simply try to maximize the number of active links between SUs as their objective function by using a binary integer linear programming (BILP) formulation. These types of works usually rely on an unrealistic assumption that all the available channels are homogeneous in terms of their QoS parameters. For example, Rezagah et al. [10] showed that under some conditions, provisioning a smaller number of simultaneous communications links among SUs but each with a higher rate can result in a higher total capacity. In some other works, the heterogeneity of potential links in terms of their QoS parameters has been considered in a coarse manner. For example, Chen et al. [4] introduced a three-step method for identifying the available spectrum, in which after computing the maximum allowable received power of receiving nodes and the actual transmission power at transmitting nodes, the identified available spectrum between SUs will be classified into two categories for high-power and low-power transmissions. Although the heterogeneity of available channels has been taken into account in this work, it has just classified them into two general categories according to their allowed transmission power, and does not capture the exact capacity of links in its context. Moreover, in some other

[1] Manuscript received September 10, 2014; revised February 4 and April 15, 2015; accepted April 27, 2015. This work was supported in part by the National Science Foundation under Grant CNS-1320468. The associate editor coordinating the review of this paper and approving it for publication was H. Hassanein.

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works [11][12][13], interference models based on interference range have been used, but they are not accurate enough. These models usually define an interference range around the receiver in which no other transmission is allowed. As will be discussed further in the next section, the conservative nature of these models will degrade the level of spectrum reuse in the CR network, and may lead to a considerable degradation in spectrum utilization.

In this work, unlike the conventional spectrum allocation models, we propose a new link capacity aware spectrum allocation algorithm to maximize the overall network capacity that not only satisfies interference constraints based on the more realistic interference temperature model [14], but also takes channels' QoS parameters, like their SINR and bandwidth, into consideration, and calculates channel capacities according to the Shannon model. Then, after modeling co-channel interference among all the potential links on each channel by using an interference graph, we formulate the spectrum allocation problem in the form of the BILP formulation to find the optimal feasible set of simultaneously active links, among all the potential links in the interference graph, such that the overall network capacity is maximized. To alleviate the high time complexity of the BILP problem, we propose to reduce the search space of this problem by mapping the solution space into a radix tree (trie[2]) structure in which the sparse parts of the solution space have been removed. Then, we formulate this problem in the form of a radix tree search to find the capacity-optimized set of non-interfering potential links in the interference graph.

The rest of the paper is organized as follows. Section II includes a brief introduction to interference modeling techniques in CR networks. Then, a short review of the radix-tree structure and its compression methods is presented in Section III. Next, we describe our proposed model for spectrum allocation in CR networks in Section IV. After presenting the simulation results and verifying the impact of the new proposed model on the overall network capacity in Section V, we draw the conclusion in Section VI.

II. Choosing a Proper Interference Model

Usually, interference models represent different abstractions at various details with respect to their primary goals of a specific application [15][16][17][18]. The first step in designing a suitable interference model in CR networks is to determine a channel propagation model based on the propagation environment. Three important propagation effects that are usually considered in channel models are deterministic path loss, large-scale fading and smallscale fading. In this work, assuming that there are not many obstacles in the environment and to make the problem tractable, we disregard the fading effects and only consider deterministic path loss effects in our channel model. The second step of designing a suitable interference model is to determine a transmission channel model, which demonstrates how interference disturbs the reception of a desired signal at the reciever. Early works within this context based on the collision channel model [19] assume that if two or more terminals transmit to a receiver simultaneously, all of their signals would be lost at the reciever due to the collision, regardless of their transmission powers. In a more realistic class of models such as the recently proposed capture channel model, if one of the contending received signals is sufficiently stronger than other signals, that signal would be successfully decoded by the reciever. Two models in this category readily found in literature are the vulnerability circle capture model and power capture model. Based on the vulnerability circle capture model, the condition for successful reception of the ith transmitter signal at a given receiver is that the received power level of this signal at the receiver, $P_{r,i}$, must be larger than the power of any other received signal by a ratio of β_ν with $1 \leq \beta_\nu < \infty$:

$$P_{r,i} / P_{r,j} > \beta_\nu \qquad for\ j = 1, 2, \ldots, n;\ j \neq i \tag{1}$$

Considering a simple deterministic path-loss channel model and the same transmit power level for all transmitters, the above condition leads to the definition of the vulnerability circle of radius $r_\nu = \beta_\nu{}^{1/\eta}.\, r$ centered at the receiver.

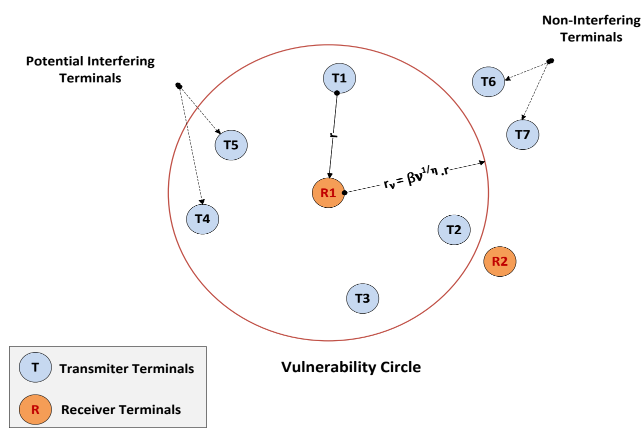


Fig. 1. Vulnerability circle model. [20]

As illustrated in Fig. 1 [20], transmission from a transmitter which is located within the vulnerability circle around receiver R_1 is considered to interfere with the successful reception of the transmitted signal from the active transmitter T_1 located at a distance r from this receiver according to the vulnerability circle capturre model. So, this model is a rather conservative model that incurs huge restrictions on transmitters around a reciever and results in spectrum efficiency degradation. For example, in the presented scenario in Fig. 1, if we assume that transmitters have the capability of power control, the transmitter node T_2 can convey its signal to its intended receiver R_2 without imposing any significant interference on the receiver R_1. However, in accordance with more realistic power capture models, a signal can be successflly received if its received power at the receiver exceeds the power of aggregated signal from all other received signals by a given threshold, here denoted as β_P [21]. In other words, the signal-

[2] Radix tree and trie are used interchangeably in this paper.

to-interference ratio of the intended signal shoud meet the condition below:

$$SIR = (P_{r,i} / \sum_{j \neq i} P_{r,j}) > \beta_P \quad (2)$$

where $P_{r,i}$ is the received power of transmitted signal i at the receiver. Assuming the deterministic path-loss channel model and considering the additive noise effect on the receiver, here denoted as σ^2, we can rewrite the above condition as follows:

$$SINR = \left(\left(P_{t,i} / \left| x_i - x_{R(i)} \right|^2 \right) / \left(\sigma^2 + \sum_{j \neq i, j \in N} \left(P_{t,j} / \left| x_j - x_{R(i)} \right|^2 \right) \right) \right) > \beta_P \quad (3)$$

where $|x_i - x_{R(i)}|$ represents the distance between transmitter i and its receiver, here denoted as R(i), and $P_{t,i}$ denotes the transmission power of this transmitter. Based on this model, the bandwidth of a given link (T_i, R_i) is equal to a constant rate W_i if the SINR of that link exceeds a given threshold for reliable transmission; otherwise, it equals to zero. In accordance with the Shannon formula for link capacity, the highest data rate W_i on a given link (T_i, R_i) is a function of SINR and the bandwidth of the channel that is used for that link:

$$W_i = B \log_2 \left(1 + \left(\left(P_{t,i} / \left| x_i - x_{R(i)} \right|^2 \right) / \left(N_0 B + \sum_{j \neq i, j \in N} \left(P_{t,j} / \left| x_j - x_{R(i)} \right|^2 \right) \right) \right) \right) \quad (4)$$

where B denotes the channel bandwidth and N_0 is the noise spectral density.

III. Radix Tree Structure and its Compression Methods

Radix tree is a kind of retrieval tree, in which an edge is associated with a bit and a node denotes a string (bitmap) that represents the bits of the path from the root to that node [22]. The left child of each node adds a '0' to its string, and the right child adds a '1'. Fig. 2(a) shows a simple trie with four prefixes: P1: 00*, P2: 01*, P3: 10* and P4: 11*, in which P1:00* stands for all binary strings started with 00 and likewise for the other prefixes [23]. In the trie structure, for w bit strings, the worst case time complexity for search and update is $O(w)$, and the memory complexity is $O(nw)$, in which n represents the number of nodes in the tree. Although it is simple and easy to implement a radix tree, the memory access complexity of $O(w)$ for search and update is relatively high. After the introduction of trie, some improvements like level compressed trie (LC-trie) have been proposed to reduce its search and update complexity. LC-trie is a kind of multi-bit tries, in which the necessity for processing one bit on each level has been removed; as a result, we can process more than one bit at each level of a trie. For example, if we transform a trie with the height of w into a multi-bit trie, in which we

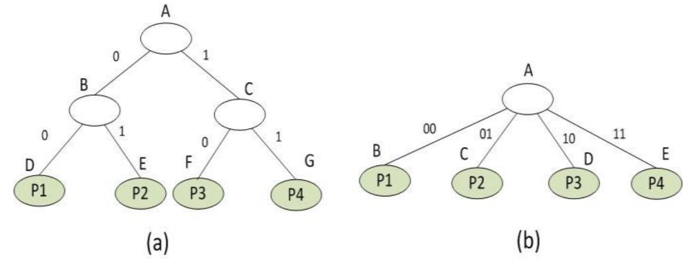


Fig. 2. Level compressed trie: (a) a simple trie with four prefixes, and (b) the corresponding level compressed trie [23].

process k bits on each stride, the height of the new trie will be w/k, and as a result the search and update complexity will be reduced to $O(w/k)$. In fact, each stride comprises 2^k different possibilities, for each of which there is a child representing its related string. Fig. 2(b) represents the LC-trie derived from the simple trie in Fig. 2(a) [23]. Moreover, null pointers of the intermediate nodes of the trie that have only one child can lead to a waste in memory space, especially when they are not representing any potential solution, and they are just intermediate to those nodes which are representing potential solutions at the lower levels of the trie. Path compression is a proposed technique for solving this problem. In a path compressed trie, by removing the constraint of sequential processing of bits of a string, the intermediate nodes that are not representing any potential solution will be removed from the path, and, in their parents, the index of the next bit that should be processed on each of its left and right children will be saved. In fact, in this situation, each node should comprise these informational components: $(s, p, next)$, in which s dentoes the string associated with the path from the root to this current node, p represents the prefix ID, and next determines the next bit that should be processed on this node to continue the path toward the lower levels of the trie. Unlike the trie presented in Fig. 2(a), the path compression techniques are particularly important when many nodes have only one child in the trie. For example, Fig. 3(b) represents the path compressed trie derived by applying the path compression technique on the simple trie in Fig. 3(a), in which the nodes with one child have been removed, and each node represents the currently processed bits, the prefix ID included in that node, and the index of the next bit of the key string which should be processed [23].

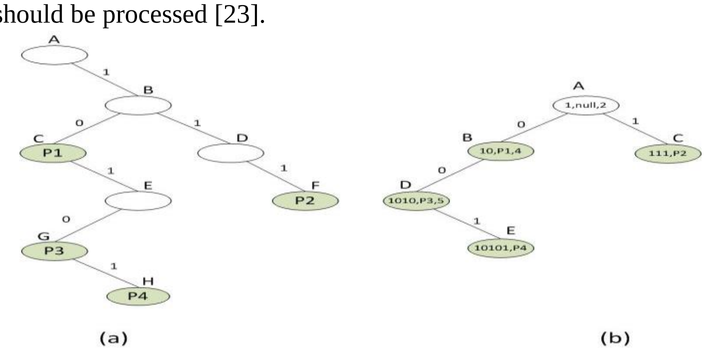


Fig. 3. Path compressed trie: (a) a simple trie with four prefixes: 10*, 111*, 1010*, 10101*, and (b) the corresponding path compressed trie. [23]

IV. PROPOSED MODEL FOR SPECTRUM ALLOCATION

A. Cognitive Cycle

We propose an extended cognitive cycle, as shown in Fig. 4 [23], for spectrum assignment, based on a basic model presented in [24]; it comprises five phases. We use this extended version to distinguish between different functions included in the basic Cognitive Cycle. It is called Cognitive Cycle to highlight the cognition capability of CR nodes that enables them to capture the information of their spectral environment and dynamically adapt themselves to the current state of the environment. Based on this cycle, in the spectrum sensing phase, SUs monitor their operational frequency channels to capture the information related to the physical layer of the channels, like their noise and interference level. Then, SUs, knowing the position and interference temperature threshold of PUs, determine their maximum allowable transmission power on each channel in a way that all PUs interference constraints still remain satisfied. In fact, in the second phase, we let SUs raise their transmission power on each channel provided that the interference temperature on all affected PUs on that channel does not exceed their acceptable threshold. In the next step, regarding the transmission power of SUs on each channel and their positions, we can identify all the potential links between SUs on each channel and calculate their capacities based on their SINR and bandwidth according to the Shannon model. Then, in the third phase, since all the potential links between SUs on each channel cannot be activated simultaneously, we first capture the co-channel interference between them by using an interference graph, and then by applying an optimization technique, in the network capacity optimization phase, we identify a feasible set of simultaneously active links among all the potential links such that the total network capacity is maximized. Three different algorithms for executing the network capacity optimization phase are presented in Subsections IV.C, IV.D and IV.E, respectively. Finally, in the channel assignment phase, after having selected a set of non-interfering links that maximizes the network capacity, the available channels will be assigned to SUs accordingly.

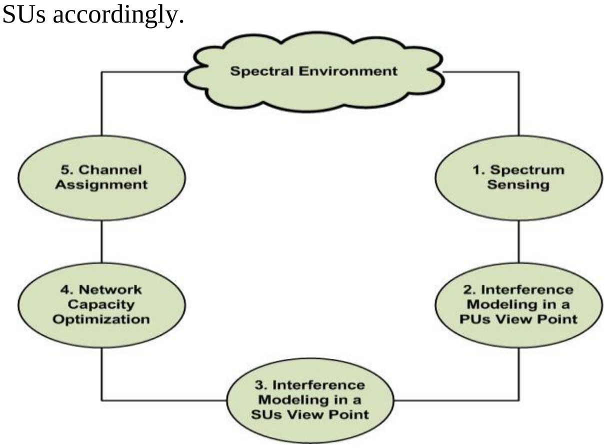


Fig. 4. Cognitive cycle for CR users. [23]

Note that we have two phases for interference control in our cognitive cycle. In the second phase of this cycle, knowing the PUs activity, we determine the maximum allowable transmission power for SUs to ensure that interference temperature on PUs will remain below their acceptable threshold for interference. Then, in the third phase, we model the co-channel interferences between secondary users to ensure that interfering links between SUs on each channel will not be activated simultaneously. Also, note that the maximum allowable transmission power is just an upper bound of the transmission power of SUs, and SUs may be assigned a power level less than this threshold after having performed the optimization as long as the SINRs of their links are higher than the minimum SINR needed to ensure reliable transmission. In fact, we will do the joint channel assignment and power allocation for SUs by using the BILP algorithm presented in Section IV.C, and the actual transmission power for each SU will be determined after having performed the optimization.

B. System Model and Problem Statement

Here, we present an extended model for spectrum allocation based on the general spectrum allocation model presented in [25]. We have assumed that we have M primary users in the environment transmitting to each other through C channels, and N secondary users competing for opportunistic access to these channels. The link availability matrix on the given channel c, $\mathrm{F_c}$, is a binary matrix representing the potential links between SUs on this channel:

$$F_c = \{ f_{i,j,c} \mid f_{i,j,c} \in \{0,1\}\} \tag{5}$$

where $\mathrm{f_{i,j,c}} = 1$ if both the secondary users i and j sense the channel c as a free channel; otherwise, $\mathrm{f_{i,j,c}} = 0$. To determine whether the channel c is free or not for any SU, each SU compares its perceived interference on this channel with the interference temperature threshold for this channel, $\bar{\mathrm{I}}_\mathrm{c}$, which is the maximum acceptable interference level for this channel. Indeed, the given SU i senses the channel c as a free channel if the following condition is met:

$$\sum_{k=1,k\neq i}^{M} P_{ri,k,c} + N_c \leq \bar{\mathrm{I}}_\mathrm{c} \qquad 1 \leq i \leq N, 1 \leq \mathrm{c} \leq \mathrm{C} \tag{6}$$

where $\mathrm{P_{r_{i,k,c}}}$ is the received power at SU i of a signal transmitted from PU k on channel c, and $\mathrm{N_c}$ represents the noise on channel c. After determining the potential links for each user on each channel, we assign a weight to each of these links; since our goal is to maximize the total network capacity, we assign to each link its link capacity as its weight. In order to compute the link capacity, we first determine the maximum allowable transmission power on each channel that SUs may use. We let the given secondary user i raise its transmission power on the given channel c without violating interference constraints on receivers of its surrounding primary users. In fact, the maximum allowable transmission power of a given secondary user i on channel c, $\mathrm{P_{max_{i,c}}}$, is the value such that if a secondary user i transmits its signal with a power level higher than this value, then there will be at least one primary user receiver, r, around it such that its received interference on channel c, $\mathrm{I_{r,c}}$, exceeds the interference temperature threshold for this channel, $\bar{I}_c$. In other words, for a given small ε, $P_{max_{i,c}}$ is the maximum positive value which satisfies the following constraint:

$$\begin{cases} \text{if } P_{i,c} \le p_{max_{i,c}} & \text{then } \forall r: I_{r,c} \le \bar{I}_c \quad 1 \le r \le M \\ \text{if } P_{i,c} = (p_{max_{i,c}} + \varepsilon) & \text{then } \exists r: I_{r,c} > \bar{I}_c \quad \varepsilon > 0 \end{cases} \tag{7}$$

Also, there is a minimum bound for each SU's transmission power on each channel to guarantee that its transmission will satisfy the minimum acceptable SINR needed to have a reliable transmission. For each SU i, if $P_{max_{i,c}}$ and $P_{min_{i,c}}$ denote its maximum and minimum allowable transmission powers on channel c, respectively, then the transmission power of SU i on channel c, $P_{i,c}$, can just be chosen from the set $\{P_1,..., P_{max}\} \in [P_{min_{i,c}}, P_{max_{i,c}}]$, unless we choose to keep it silent on this channel, i.e., $P_{i,c} = 0$.

After determining the transmission power of any secondary user i on its available channels, we calculate the weight of each potential link on these channels, i.e., its link capacity. We use a matrix $R_c = \{ r_{i,j,c} | r_{i,j,c} \ge 0 \}$ to denote the weight matrix for channel c in which the weight of the link between transmitter i and receiver j on channel c can be calculated as follows:

$$r_{i,j,c} = W \times \log_2(1 + SINR_{i,j,c}) \quad 1 \le i, j \le N, 1 \le c \le C \tag{8}$$

where W denotes the bandwidth of channel c and $SINR_{i,j,c}$ represents the SINR of the link between transmitter i and receiver j on channel c. According to the above formula, to calculate the reward value of potential links, we should first calculate the SINR of all potential links between SUs on each channel. The SINR of a link between transmitter i and receiver j on channel c is:

$$SINR_{i,j,c} = \frac{Pr_{j,i,c}}{N_c + \sum_{m=1, m \ne i}^{K} Pr_{j,m,c}} \tag{9}$$

where $Pr_{j,i,c}$ is the power received by receiver j from transmitter i on channel c, N_c is the level of background noise on channel c, and K is the total number of active transmitters on channel c. To control the co-channel interference between the potential links on each channel, we use the conflict (C) matrix for each channel:

$$C_c = \{ C_{c_{i,j,m,n}} | C_{c_{i,j,m,n}} \in \{0,1\} \} \tag{10}$$

where $C_{c_{i,j,m,n}} = 1$ if and only if the link between transmitter i and receiver j interferes with the link between transmitter m and receiver n on channel c; otherwise, $C_{c_{i,j,m,n}} = 0$. Two links are considered to be interfering with each other if concurrent transmissions on that two links make the SINR of at least one of them to drop below a minimum SINR needed to have a reliable transmission. In the conflict-free channel assignment matrix for channel c, $A_c = \{a_{i,j,c} | a_{i,j,c} \in \{0,1\}\}$, there are no link conflicts. In this binary matrix, $a_{i,j,c} = 1$ if the link between transmitter i and receiver j on channel c exists in the final simultaneously active set of links for this channel; otherwise, $a_{i,j,c} = 0$. To avoid all of the link conflicts on the given channel c, the following condition should be satisfied in a conflict-free channel assignment matrix for this channel:

$$a_{i,j,c} + a_{m,n,c} \le 1 \;\; if \;\; C_{c_{i,j,m,n}} = 1, \quad 1 \le i, j, m, n \le N \tag{11}$$

Having the conflict-free spectrum assignment matrix A_c for all channels, we can calculate the network capacity as the total reward of each feasible solution based on the following objective function:

$$S = Sum\left(\sum_{c=1}^{C} A_c \times R_c \right) \tag{12}$$

where R_c is the reward matrix for the channel c , $R_c = \{r_{i,j,c} | r_{i,j,c} \ge 0 \}$, which contains the capacity of each of the potential links on this channel, Sum is the operator that returns the summation of all entries of a matrix, and S represents the total network capacity of the conflict-free spectrum assignment solution. Note that A_c and R_c are two matrices with the same dimension, and the result of $\sum_{c=1}^{C} A_c \times R_c$ is another matrix with the same dimension on which we apply the SUM operator to find the total network capacity. Now, the above can be formulated as an optimization problem from which the optimal or suboptimal solution can be derived.

C. BILP Formulation

Note that there are many possible solutions to the above spectrum allocation problem, each of which leads to a different network capacity; we shall explore all the possible solutions in the solution space to find the optimal one. This problem can be formulated as a binary integer linear programming (BILP) problem as follows:

$$\max_{A_c, R_c} \left(Sum\left(\sum_{c=1}^{C} A_c \times R_c \right) \right) \tag{13}$$

Subject to:

$$a_{i,j,c} \in \{0,1\} \; , r_{i,j,c} \ge 0 \qquad (1 \le i, j \le N, 1 \le c \le C) \tag{14}$$

$$A_c \le F_c \qquad (1 \le c \le C) \tag{15}$$

$$a_{i,j,c} + a_{m,n,c} \le 1 \; if \; C_{c_{i,j,m,n}} = 1 \quad (1 \le i, j, m, n \le N) \tag{16}$$

$$P_{min_{k,c}} \le P_{k,c} \le P_{max_{k,c}} \quad if \; P_{k,c} \ne 0 \qquad (1 \le k \le N) \tag{17}$$

$$\sum_{k=1}^{N} P_{i,k,c} \le \bar{P}_{i,c} \qquad (1 \le i \le M) \tag{18}$$

where $a_{i,j,c}$ and $r_{i,j,c}$ represent an entry of matrices A_c and R_c, respectively. Also, $P_{i,k,c}$ denotes the power received by primary user i from secondary user k on channel c, and $\bar{P}_{i,c}$ is the maximum acceptable interference power for primary user i on channel c. The last constraint guarantees that for any primary user i, the total received power of the aggregated interference signal on any given channel c will remain below its maximum acceptable threshold for interference on this channel. Note that in this optimization problem, aside from A_c which is the channel assignment variabe, R_c is also an optimization variable representing the potential link capacities whose values depend on the level of available noise and interference on the SUs' operational frequencies and the transmission power of each SU on these channels. The Max operator here is over A_c and R_c, and ($Sum\left(\sum_{c=1}^{C} A_c \times R_c\right)$) is the total network capacity that can be achieved by using A_c as the channel assignment matrix and R_c as the matrix which includes the potetial link capacities. The channels assigned to each SU and the optimal transmission power of each SU on each channel will be known after having executed the optimization. Here, our goal is to find the values of A_c and R_c that maximize the network capacity, which is our objective function. This can be solved by using the AIMMS software [26].

D. Genetic Algorithm Formulation

Owing to the high complexity of the BILP algorithm on one hand, and high dynamics and instability of available spectrum in CR networks on the other hand, many researchers have resorted to evolutionary algorithms to solve the spectrum allocation problem in cognitive radio networks [27][28][29]. The main purpose is to approximate BILP and radix tree algorithms by a heuristic algorithm with lower complexity to find a sub-optimal solution. For this purpose, among various heuristic algorithms, Genetic Algorithm has been proven effective in solving optimization problems [30][30][31] and is thus applied here. Instead of drilling into the details of GA, we only highlight some important parameters in the GA formulation; readers are referred to [31] for the fundamentals of GA. In our GA formulation, each chromosome represents a valid solution in the search space. Indeed, the chromosomes are formed by sequentially placing the rows of the assignment matrices (A_c) of different channels next to each other to form a single binary string. For example, if we have N secondary users with C operating frequency channels, then each chromosome will be a binary string with $C.N^2$ bits of length in which its first N^2 bits are associated with frequency channel 1 and its last N^2 bits are associated with frequency channel C. Let L1, L2 and L3 matrixes represent the available links on channels 1, 2 and 3, respectively:

$$L1 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, L2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, L3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

where $Lc_{ij} = 1$ if and only if there is a link between secondary users i and j on channel c. So, we can sequentially map these availability matrices into a binary string with 27 bits of length to form the first chromosome as:

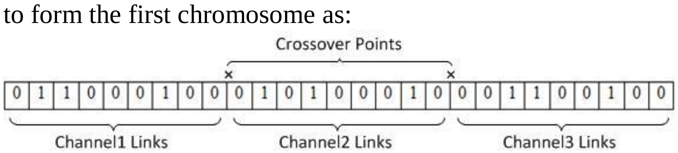


Fig. 5. Sample chromosome in GA formulation.

However, if the links $L1_{12}$ and $L1_{13}$ are assumed to conflict on channel 1, the chromosome represented in Fig. 5 will not be a valid chromosome as it includes both of these conflicting links. To remove these link conflicts in the next generation chromosomes, we should apply a function on them to guarantee that each chromosome does not include more than one link of each set of conflicting links. Also, because our goal is to maximize the network capacity, the fitness value of each chromosome reflects the capacity of a viable network configuration, i.e., each chromosome is assigned the fitness value of $\mathrm{Sum}\left(\sum_{c=1}^{C} A_c \times R_c\right)$ corresponding to the mapped network configuration of the chromosome. In fact, in each of the iterations of GA, we will sort the chromosomes in terms of their fitness values, and select those with better fitness to produce the next generation. Also, as shown in Fig. 5, we put the crossover points on the boundaries between the links which are associated with different channels. Putting the crossover points at these boundaries will ensure that the removed co-channel link conflicts in previous chromosomes will not appear (will remain “removed”) in the next generation.

E. Radix Tree Based Formulation

Experiments have proven that in the environments with relatively high level of interference, the link availability matrix for secondary users is a sparse matrix, in which many of the entries have a value of 0. So, on one hand, a BILP formulation needs to explore all the binary search space involving these sparse areas to find the optimal solution, leading to a considerable amount of search in time. On the other hand, the GA formulation does not explore all the search space; it just explores the limited areas of the search space using the mutation and crossover operators, and it usually finds a solution that may not be optimal or accurate enough owing to the limited number of iterations. To mitigate this problem, we propose a novel radix-tree based formulation for the spectrum allocation problem. In this method, we remove the sparse areas of the search space and just map the solution space into a radix tree structure. Here, we define a solution space as the areas of the search space that comprise valid solutions only. Then, using a path and level compression techniques as described in Section III, we compress this trie to find the optimal solution in less time as compared to the BILP formulation. To achieve this, we construct a trie for each channel, and associate each level of a trie with a potential link on its related channel. Then, starting from the root, if we pass the right child in each level of trie, we add its associated link to the set of active links, and remove its conflicting links from the list of potential links for that channel. Note that the set of active links is empty in the trie root, and the set of potential links comprises all the potential links on the trie related channel. This way, each node on the trie is associated with a set of active links such that the path from the root to that node has passed from a right child on their associated level in the trie. Therefore, we can define the node fitness as the sum of the capacities of the links in the set of active links. We explore all the possible paths from the root to the lower level nodes until all potential links in the set of potential links have been exhausted. Consequently, we have a complete solution associated with each leaf of the trie; by comparing their equivalent network capacities, we can find the optimal solution for the spectrum allocation problem. Considering the example provided in the last section, Fig. 6 represents the radix tree associated with channel 1 links included in the L1 matrix in which we have three links: L_{12}, L_{13} and L_{31}. As we can see in this figure, each node is represented by a table with two rows. The first row of this table represents the available links for this node, and the second row includes a binary string which represents the selected links of the path from the root to this node. Note that each path from the root to the leaf nodes should not include any conflicting links. That is why we have pruned the right child of node C, because the links L_{12} and L_{13} are conflicting on channel 1, and should not be simultaneously active on this channel. After constructing a radix tree for each channel, and calculating the fitness of each node which is the summation of the capacities of its selected links, the best solution will be extracted. In this example, if we assume that L_{12}, L_{13} and L_{31} link capacities are 32 Kb, 64 Kb and 128 Kb, respectively, the node L represents the optimal solution which includes L_{13} and L_{31} links which are not conflicting, and have a total capacity of 192 Kb. In this example, owing to the fact that all the potential solutions are in the last row of the tree, we can easily do the level compression by letting nodes G, H, K, L, M and N to be the direct children of the root node with 000, 001, 010, 011, 100, and 101 labels, respectively. Also, by applying the path compression, the intermediate node C can be removed as it only has one child and does not include any potential solution.

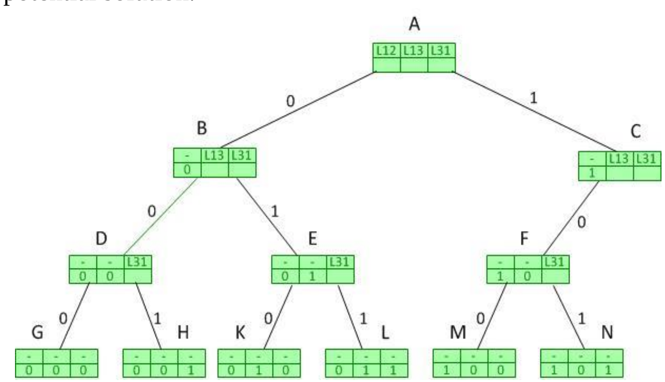


Fig. 6. Sample radix tree for channel 1.

Suppose we have N secondary users with C operating frequency channels and L potential links on each channel, then the time complexity of BILP will be $O(2^{N^2}.C)$ while the time complexity of the proposed radix tree based solution will be $O(2^{L}.C)$, which is considerably smaller. In fact, supposing that α fraction of frequency channels are not available, and β fraction of available channels do not satisfy the minimum requirements needed for a reliable transmission $(0 \leq m, n \leq 1)$, then the number of potential links on each channel will be $(1-\alpha).(1-\beta).N^2$, resulting in the time complexity of BILP being $2^{(1-(1-\alpha)(1-\beta))N^2}$ times as high as that of the proposed radix tree based formulation.

V. Simulation Results

We consider a scenario in which 16 SUs and 4 PUs have been normally distributed in the area of $32\,m \times 32\,m$ as shown in Fig. 7(a). Next, we apply the cognitive cycle phases to this scenario assuming that the SUs have 6 operational channels, the interference temperature threshold on each channel is 8 dB, and the minimum SINR for a reliable transmission is 1. After having identified all the potential links between SUs and calculated their SINRs and capacities, we ignore the links with SINR less than 1 in the list of potential links. Then, we construct an interference graph for each channel. Fig. 7(b) represents the interference graph for channel 1 in which each pair of interfering links on channel 1 has been connected through an edge. Next, we apply both capacity aware BILP and GA algorithms to find the optimal and sub-optimal solutions for this scenario, respectively. We also implement two other algorithms to compare the results. The greedy algorithm is a recursive algorithm in which each iteration simply selects the link with the maximum capacity and removes its interfering links in the interference graph; this procedure continues until no other link remains in the interference graph. Also, we implement the BILP algorithm in which the objective function is to maximize the number of active links between SUs regardless of their capacities as in [3]. We compare the results of these algorithms as shown in Fig. 8. Note that both capacity aware GA and BILP algorithms lead to a higher network capacity than greedy and capacity unaware BILP algorithms. Fig. 8 shows that the capacity aware GA algorithm is converging to the optimal solution resulted from the capacity-aware BILP algorithm as the number of iterations increases.

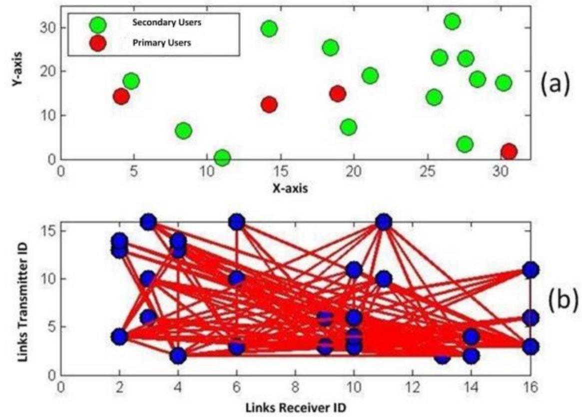


Fig. 7. The scenario with 16 SUs and 4 PUs: (a) spatial distribution of SUs and PUs; (b) interference graph for channel 1.

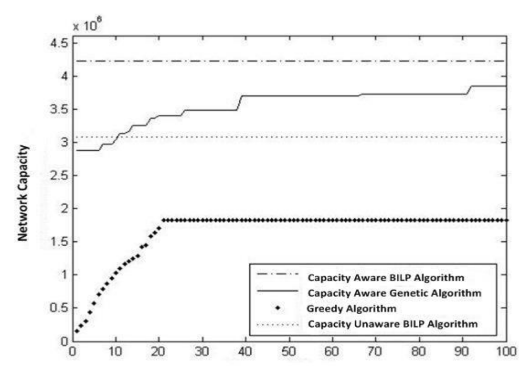
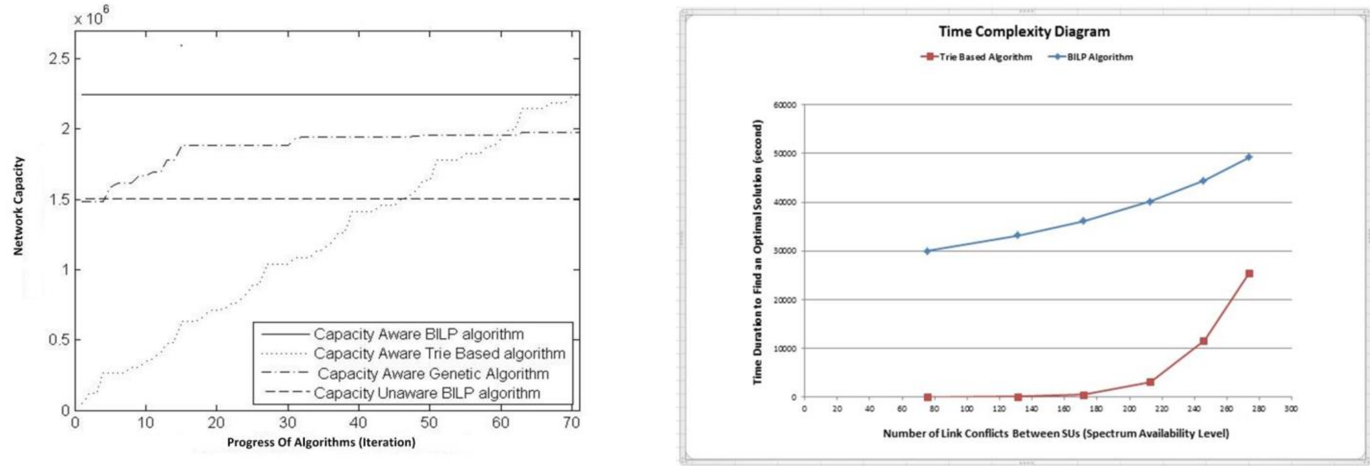


Fig. 8. Comparing the results of different algorithms.



(a) (b)

Fig. 9. Performance comparison: (a) network capacities; (b) time complexities.

Also, we can see that maximizing the spectrum efficiency in terms of the number of active links, regardless of their capacities, leads to a considerable degradation in the overall network capacity. In fact, our simulation results show that maximizing the number of active links between SUs does not necessarily maximize the network capacity unless we assume that the capacities of all links are equal; this is not realistic.

In another scenario in which 25 SUs and 5 PUs have been normally distributed in the area of $50\,m \times 50\,m$, we apply both capacity-aware BILP and radix tree based algorithms to find the optimal solution for this scenario. Again, for better comparison of the results, we implement the capacity-aware GA algorithm to find a suboptimal solution for this scenario in less time, and the capacity-unaware BILP algorithm which purely aims to maximize the number of active links between SUs regardless of their heterogeneity in QoS parameters. We compare the results of these algorithms in Fig. 9(a). First, by comparing the results of the capacity-aware and capacity-unaware BILP algorithms, we can see again that maximizing the number of active links as our objective function does not necessarily optimize the network capacity due to the heterogeneity of potential links' QoS parameters like their capacities. Moreover, as expected, Fig. 9(a) shows that the network capacity provisioned by the capacity-aware trie based algorithm is surpassing that provisioned by the GA algorithm as the number of iterations increases, and converges to the optimal solution, resulted from the capacity-aware BILP algorithm. Also, by tracing the results of the GA algorithm, we can see that due to the random nature of GA operators, like crossover and mutation, increasing the number of GA iterations does not necessarily guarantee that an optimal solution will be found, and we can only observe some sporadic improvements in its new generations.

In addition, we compare the time complexity of the capacity-aware BILP and radix tree based algorithms. We can see the time duration needed by these two algorithms to reach the optimal solutions for the six different scenarios (with different numbers of link conflicts) in Fig. 9(b). As we can see in this figure, removing the sparse areas of the search space in the proposed radix tree based algorithm leads to a considerable decrease in the time duration needed to find the optimal solution to the spectrum assignment problem in these scenarios; however, by increasing the number of link conflicts in these scenarios, the differences of time duration between these algorithms become smaller because of the decrease in the size of the sparse areas.

We also evaluate the effects of spatial distribution of SUs on the network capacity in our proposed model. As shown in Fig. 10(a), we consider two different scenarios in which the SUs are either positioned in accordance with a grid structure in the environment, or have been randomly distributed in the environment. Fig. 10(b) shows the network capacity resulted from the scenario in which SUs are randomly distributed in the environment; it is much higher than the network capacity of the former scenario. Grid positioning of SUs will decrease the heterogeneity of the physical distribution of SUs as well as the number of link conflicts between SUs, in the case that SUs transmit only with a fixed transmission power. However, in our model when nodes are randomly distributed, due to the power control feature which enables SUs to control their transmission power, decreasing the physical distance between SUs in the dense areas leads to a considerable increase in the link SINR. Moreover, the number of potential links that satisfy the minimum SINR needed for a reliable transmission will be increased, thus enhancing the network capacity considerably.

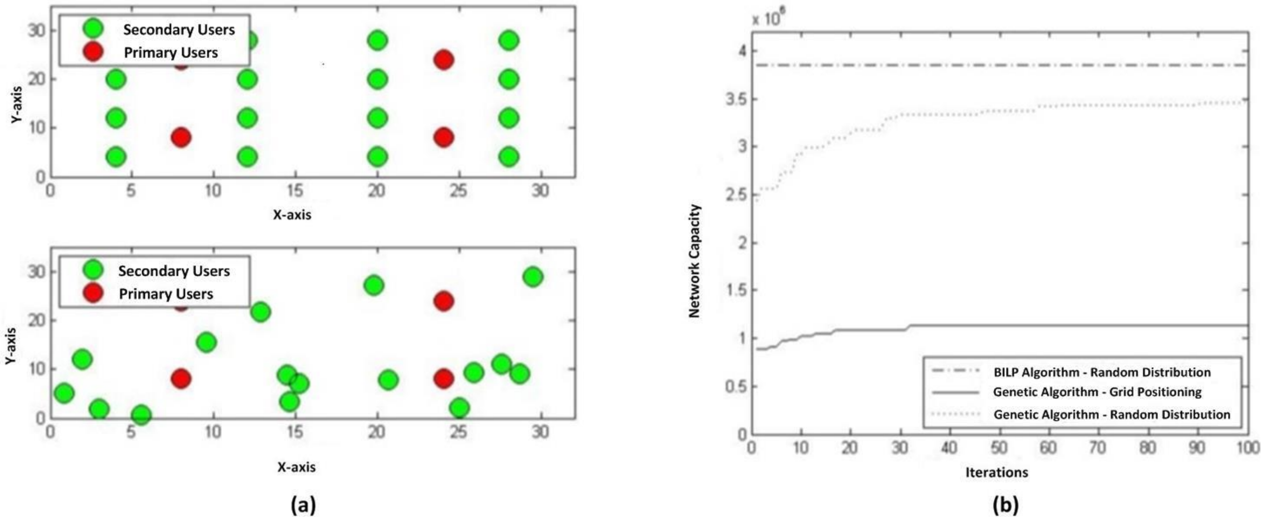


Fig. 10. Effects of spatial distribution of SUs: (a) different spatial distributions; (b) associated network capacities.

VI. Conclusion

Cognitive radio is a promising technology that provides opportunistic access to “free” channels for the SUs, and thus enhances the spectrum efficiency. Spectrum allocation in CR networks usually involves optimization of various parameters. In this work, we have proposed a new interference constraint capacity-aware spectrum allocation model to maximize the network capacity of CR networks. Simulation results have shown that we can achieve a higher network capacity by using the proposed model than that of other approaches for spectrum allocation. Moreover, we have demonstrated that removing the sparse areas in the search space and mapping the solution space into a radix tree in our proposed model leads to a considerable decrease in the time complexity of the spectrum allocation algorithm as compared to the BILP solutions for the same problem. Also, our simulation results show that maximizing the number of active links between SUs does not necessarily maximize the network capacity unless we assume that the capacity of all links are equal, which is not a realistic assumption. Finally, we show that in scenarios in which SUs are equipped with power control capability, we can achieve a higher network capacity in the case that SUs are non-uniformly distributed in the environment as compared to the case that SUs are positioned in accordance with a grid structure.

References

[1] I. F. Akyildiz, W. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: a survey," *Computer Networks*, vol. 50, no. 13, pp. 2127-2159, 2006.

[2] P. Houze, D. Ruiz, S. Ben Jemaa, and P. Cordier, "Dynamic spectrum allocation algorithm for cognitive networks," in *Proc. 3rd Int. Conf. on Wireless and Mobile Communications (ICWMC)*, 2007.

[3] M. Ma and D. H. K. Tsang, "Efficient spectrum sharing and power control in cognitive radio networks," in *Proc. 1st ACM Int. Workshop on Cognitive Wireless Networks*, 2007.

[4] J. Chen, C. Liao, C. Han, and S. Li, "Extending available spectrum in cognitive radio: Hierarchical spectrum sharing network," *Computer Networks*, vol. 52, no. 4, pp. 850-863, 2008.

[5] X. Li and S. A. R. Zekavat, "Distributed channel assignment in cognitive radio networks," in *Proc. 2009 Int. Conf. Wireless Communications and Mobile Computing: Connecting the World Wirelessly*, 2009.

[6] S. Tapaswi, R. S. Jadon, and P. Sharma, "Dynamic spectrum allocation technique with reduced noise in cognitive radio networks," *IJCSNS, vol.* 10, no. 1, pp. 274-280, 2010.

[7] X. Huang and N. Ansari, “Joint spectrum and power allocation for multi-node cooperative wireless systems,” *IEEE Transactions on Mobile Computing*, DOI: 10.1109/TMC.2015.2388478, 2015. (Also available in CoRR, abs/1405.5730, 2014)

[8] X. Huang, T. Han, and N. Ansari, “On green energy powered cognitive radio networks,” *IEEE Communications Surveys and Tutorials*, DOI: 10.1109/COMST.2014.2387697, 2015. (Also available in CoRR, abs/1405.5747, 2014)

[9] M. Ma and D. H. K. Tsang, "Impact of channel heterogeneity on spectrum sharing in cognitive radio networks," in *Proc. 2008 IEEE Int. Conf. Communications (ICC'08)*, 2008.

[10] R. E. Rezagah and A. Mohammadi, "Analyzing the capacity of wireless ad hoc networks," *Telecommunication Systems,* vol. 55, no. 1, pp. 159-167, 2014.

[11] C. Ghosh, D. P. Agrawal, M. B. Rao, and C. Cordeiro, "Channel capacity maximization in cooperative cognitive radio networks using game theory," *ACM SIGMOBILE Mobile Computing and Communications*, vol. 13, no. 2, pp. 2-13, 2009.

[12] P. Cardieri, "Modeling interference in wireless ad hoc networks," *IEEE Communications Surveys & Tutorials*, vol.12, no. 4, pp. 551-572, Fourth Quarter 2010.

[13] H. Li and P. D. Mitchell, "Full interference model in wireless sensor network simulation," in *Proc. 2009 6th Int. Symp. Wireless Communication Systems (ISWCS)*, pp.647-651, 7-10 Sept. 2009.

[14] Federal Communications Commission, “Establishment of interference temperature metric to quantify and manage interference and to expand available unlicensed operation in certain fixed mobile and satellite frequency bands,” ET Docket 03-289, Notice of Inquiry and Proposed Rulemaking, 2003.

[15] G. Xing, M. Sha, J. Huang, G. Zhou, X. Wang, and S. Liu, "Multi-channel interference measurement and modeling in low-power wireless networks," in *Proc. 30th IEEE Real-Time Systems Symp. (RTSS)*, pp. 248-257, 2009.

[16] X. Hong, Z. Chen, C. X. Wang, S. A. Vorobyov, and J. S. Thompson, "Cognitive radio networks," *IEEE Vehicular Technology Magazine*, vol. 4, no. 4, pp.76-84, December 2009.

[17] K. Gulati, A. Chopra, B. L. Evans, and K. R. Tinsley, "Statistical modeling of co-channel interference," in *Proc. 2009 IEEE Global Telecommunications Conference (GLOBECOM)*, pp.1-6, Nov. 30 2009-Dec. 4 2009.

[18] R. S. Dhillon and T. X. Brown, "Models for analyzing cognitive radio interference to wireless microphones in TV bands," in *Proc. 3rd IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks*, pp. 1-10, 2008.

[19] J. Tang, S. Misra, and G. Xue, "Joint spectrum allocation and scheduling for fair spectrum sharing in cognitive radio wireless networks," *Computer Networks*, vol. 52, no. 11, pp. 2148-2158, 2008.
[20] M. Yousefvand, S. Khorsandi, and A. Mohammadi, "Interference-constraint spectrum allocation model for cognitive radio networks," in *Proc. 2012 6th IEEE Intelligent Systems (IS) Conf.*, pp. 357-362, 6-8 Sep. 2012.
[21] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388-404, Mar. 2000.
[22] M. Behdadfar, H. Saidi, H. Alaei, and B. Samari, "Scalar prefix search: a new route lookup algorithm for next generation internet," in *Proc. 2009 IEEE INFOCOM*, pp. 2509-2517, 19-25 April 2009.
[23] M. Yousefvand, S. Khorsandi, and A. Mohammadi, "Radix-tree based spectrum allocation model for cognitive radio networks: Maximizing network capacity," in *Proc. 14th IEEE Int. Conf. Communication Technology (ICCT)*, pp. 16-21, 9-11 Nov. 2012.
[24] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201-220, Feb. 2005.
[25] C. Peng, H. Zheng, and B. Y. Zhao, "Utilization and fairness in spectrum assignment for opportunistic spectrum access." *Mobile Networks and Applications*, vol. 11, no. 4, pp. 555-576, 2006.
[26] *AIMMS Optimization Modeling*, Paragon Decision Technology, Mar. 2012.
[27] Z. Zhao, Z. Peng, S. Zheng, and J. Shang, "Cognitive radio spectrum allocation using evolutionary algorithms," *IEEE Trans. Wireless Commun*, vol. 8, no. 9, pp. 4421-4425, September 2009.
[28] A. S. Hamza and M. M. Elghoneimy, "On the effectiveness of using genetic algorithm for spectrum allocation in cognitive radio networks," in *Proc. 2010 High-Capacity Optical Networks and Enabling Technologies (HONET)*, pp. 183-189, 19-21 Dec. 2010.
[29] M. Y. El Nainay, D. H. Friend, and A. B. MacKenzie, "Channel allocation and power control for dynamic spectrum cognitive networks using a localized island genetic algorithm," in *Proc. 2008 3rd IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, pp. 1-5, 14-17 Oct. 2008.
[30] E.S.H. Hou, N. Ansari and R. Hong, “A genetic algorithm for multiprocessor scheduling,” IEEE Trans. on Parallel and Distributed Systems, vol. 5, no. 2, pp. 113-120, Feb. 1994.
[31] N. Ansari and E. S. H. Hou, *Computational Intelligence for Optimization*, Springer Publishing Company, 1997.

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