Adaptive Fusion by Reinforcement Learning for Distributed Detection Systems

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Chair and Varshney have derived an optimal rule for fusing decisions based on the Bayesian criterion. To implement the rule, the probability of detection P_D and the probability of false alarm P_F for each detector must be known, but this information is not always available in practice. An adaptive fusion model which estimates the P_D and P_F adaptively by a simple counting process is presented. Since reference signals are not given, the decision of a local detector is arbitrated by the fused decision of all the other local detectors. Furthermore, the fused results of the other local decisions are classified as "reliable" and "unreliable." Only reliable decisions are used to develop the rule. Analysis on classifying the fused decisions in term of reducing the estimation error is given, and simulation results which conform to our analysis are presented.

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I. INTRODUCTION

Distributed detection systems with data fusion have been investigated widely in recent years [1–6]. The problem of decision fusion in a binary hypothesis system was considered by Chair and Varshney [1], and Thomopoulos et al. [6]. Chair and Varshney [1] developed an optimal decision rule by using the minimum probability of error criterion. Thomopoulos et al. [6] proposed the optimal decision rule based on the Neyman-Pearson test. They showed that the optimal fusion rule is obtained by a weighed sum of local decisions through a hard limiter. The weight associated with each local detector indicates the degree of reliability of the detector. Each weight is a function of the probability of detection P_D and the probability of false alarm P_F of the detector. The P_D and P_F can be obtained when either the distribution of the observations at each detector is given, or when some reference signals are provided to estimate the P_D and P_F by an empirical method. However, in practice, neither P_D nor P_F is known. Furthermore, since the sensors are usually exposed to a changing environment, the performance of each individual detector may not always be the same, i.e., the P_D and P_F may vary with time. We propose an adaptive system to estimate the P_D and P_F. Without knowledge of the performance of each detector, the proposed system is capable of approximately estimating the P_D and P_F of the detector in the course of performing the decision fusion.

Consider a binary hypothesis testing system consisting of n local detectors with the probabilities of two hypotheses H_0, H_1 denoted as $P(H_0) = P_0$ and $P(H_1) = P_1$, respectively. Assume that under each hypothesis, the observations at each detector are statistically independent. Let u_i and u denote the decisions made by the ith detector and the fusion center, respectively. When the ith local detector favors the H_1 hypothesis, $u_i = +1$; otherwise $u_i = -1$. The output u is similarly defined. We let P_{Di} and P_{Fi} denote the probability of detection and the probability of false alarm of the ith detector, respectively.

Chair and Varshney [1] showed that the optimal fusion rule for the minimum probability of error criterion is

$$u = \begin{cases} +1, & \text{if } a_0 + \sum_{j=1}^{n} a_j u_j > 0 \\ -1, & \text{otherwise} \end{cases} \tag{1}$$

where

$$a_0 = \log \frac{P_1}{P_0} \tag{2}$$

$$a_i = \begin{cases} \log \dfrac{P_{Dj}}{P_{Fj}}, & \text{if } u_j = +1 \\ \log \dfrac{1-P_{Fj}}{1-P_{Dj}}, & \text{if } u_j = -1. \end{cases} \tag{3}$$

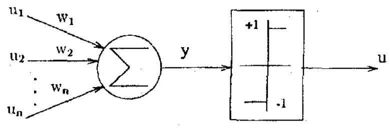

Fig. 1. Structure of the fusion system.

For the case $P_0 = P_1$ and the probability of false alarm P_{Fj} is equal to the probability of miss P_{Mj}, $a_0 = 0$ and the optimal fusion rule can be simplified to

$$u = \begin{cases} +1, & \text{if } \sum_{j=1}^{n} w_j u_j > 0 \\ -1, & \text{otherwise} \end{cases} \tag{4}$$

where

$$w_j = \log \frac{P_{Dj}}{P_{Fj}}, \qquad \text{for each } j. \tag{5}$$

The system structure is shown in Fig. 1, where

$$y = \sum_{j=1}^{n} w_j u_j. \tag{6}$$

The structure shown in Fig. 1 is similar to a single neuron system, in particular, the perceptron [7]. If reference signals are given, they can be used as a "reference" to train the system such that the weights converge to the optimal values defined by (5). However, in practice, such a reference is not readily available and at the same time, the P_D and P_F of a detector may vary with time. Since the fused decisions are usually better than local decisions, they can be considered as the reference. When the ith local decision u_i is equal to the fused decision u, then u_i is considered to be correct; otherwise, u_i is considered to be incorrect. Since $u = \mathrm{sgn}(y) = \mathrm{sgn}(\sum_{j=1}^{n} w_j u_j)$, the fused decision u has already taken into account the decision of the ith detector, u_i. If u is used as a reference for u_i, a bias is established for u_i. Thus, in the proposed system, the decision of the ith local detector u_i is arbitrated by the fused decision of all the other $(n-1)$ local detectors. Denote the fused decision as $\overline{u}_i$, and define

$$y_i = \sum_{j \neq i} w_j u_j \tag{7}$$

i.e., y_i is the weighed sum of all local decisions except u_i, then

$$\overline{u}_i = \mathrm{sgn}(y_i). \tag{8}$$

Note that $\overline{u}_i$ and u_i are conditionally independent given H_j, $j = 0, 1$. The "reference" $\overline{u}_i$ may not always be correct. To reduce the possibility of using incorrect references, the decisions $\overline{u}_i$ are further classified. The decision $\overline{u}_i$ is considered unreliable when the weighed sum defined by (7) is close to the decision threshold 0. Our strategy is to determine an "unreliable range" around the decision threshold such that when the weighed sum y_i falls in this range, the fused decision $\overline{u}_i$ is considered unreliable and will not be used for training the system. The selection of this unreliable range is discussed next.

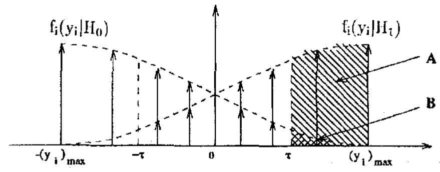

Fig. 2. Conditional probability mass function: $f_i(y_i/H_1)$ and $f_i(-y_i/H_0)$.

II. ADAPTIVE MODEL ANALYSIS

Consider the structure shown in Fig. 1. From (6) and (7), we have

$$y_i = y - w_i u_i. \tag{9}$$

Under the assumptions that $P_0 = P_1$ and $P_{Fj} = P_{Mj}$, the conditional probability mass functions $f_i(y_i/H_1)$ and $f_i(y_i/H_0)$ are symmetric with each other, i.e., $f_i(y_i/H_1) = f_i(-y_i/H_0)$, as shown in Fig. 2.

We establish the above relationship as follows:

$$y_i = \sum_{j \neq i} w_j u_j = \sum_{S_i^+} w_j - \sum_{S_i^-} w_j \tag{10}$$

where $S_i^+ = \{j : j \neq i \text{ and } u_j = 1\}$ and $S_i^- = \{j : j \neq i \text{ and } u_j = -1\}$. By the earlier assumption of independent observations,

$$P(y_i = \xi/H_1) = \sum_{S_i} \prod_{S_i^+} P(u_j = 1/H_1) \prod_{S_i^-} P(u_j = -1/H_1) \tag{11}$$

where $S_i = \{\{S_i^+, S_i^-\}$: combinations of S_i^+ and S_i^- such that $\sum_{S_i^+} w_j - \sum_{S_i^-} w_j = \xi\}$. Since we have assumed that $P_{Mj} = P_{Fj}$, i.e.,

$$P(u_j = -1/H_1) = P(u_j = 1/H_0) = P_{Fj} \tag{12}$$

which also implies that

$$P(u_j = 1/H_1) = P(u_j = -1/H_0) = P_{Dj}, \tag{13}$$

we have

$$P(y_i = \xi/H_1) = \sum_{S_i} \prod_{S_i^+} P_{Dj} \prod_{S_i^-} P_{Fj}. \tag{14}$$

Now, assume that the local detectors, except the ith detector, make opposite decisions as compared with (11) such that y_i becomes $-\xi$. That is, S_i^+ and S_i^- remain the same, but the decisions are reversed. Thus,

$$P(y_i = -\xi/H_1)$$
$$= \sum_{S_i}\prod_{S_i^+} P(u_j = -1/H_1) \prod_{S_i^-} P(u_j = 1/H_1)$$
$$= \sum_{S_i}\prod_{S_i^+} P_{Mj} \prod_{S_i^-} P_{Dj}$$
$$= \sum_{S_i}\prod_{S_i^+} P_{Fj} \prod_{S_i^-} P_{Dj}. \quad (15)$$

We next evaluate $P(y_i = \xi/H_0)$,

$$P(y_i = \xi/H_0) = \sum_{S_i}\prod_{S_i^+} P(u_j = 1/H_0) \prod_{S_i^-} P(u_j = -1/H_0)$$
$$= \sum_{S_i}\prod_{S_i^+} P_{Fj} \prod_{S_i^-} P_{Dj}. \quad (16)$$

Since (15) and (16) are the same,

$$f_i(y_i/H_1) = f_i(-y_i/H_0). \quad (17)$$

Since $f_i(y_i/H_1)$ and $f_i(-y_i/H_0)$ have such a symmetric relation, let the *unreliable range* be symmetric about its decision threshold and denote the upper limit of the range as τ. We call τ the *reliability threshold*. Only the fused decisions $\overline{u}_i$ which satisfy $|y_i| > \tau$ are chosen to adapt the weight w_i. These decisions are considered as *reliable decisions*, denoted as $\overline{u}_i^*$. Other decisions are ignored. Intuitively, the bigger the value τ, the more reliable the decisions $\overline{u}_i^*$, the less the errors are between the estimates and theoretical values. Note that since u_i and $\overline{u}_i$ are conditionally independent and since i is deterministic, u_i and $\overline{u}_i^*$ are also conditionally independent.

Let $\hat{P}_{Di}$, $\hat{P}_{Fi}$ be the estimates of P_{Di}, P_{Fi}. When the local decision u_i agrees with the reliable decision $\overline{u}_i^*$, it is considered a detection of the local detector; otherwise, it is considered a false alarm. Using the conditional independence of u_i and $\overline{u}_i^*$, the assumption of an equiprobable source, and the definition of P_{Di} and P_{Fi},

$$\begin{aligned}\hat{P}_{Di} &= P(u_i = 1, \overline{u}_i^* = 1) + P(u_i = -1, \overline{u}_i^* = -1)\\ &= P(H_0)P(u_i = 1, \overline{u}_i^* = 1/H_0)\\ &\quad + P(H_1)P(u_i = 1, \overline{u}_i^* = 1/H_1)\\ &\quad + P(H_0)P(u_i = -1, \overline{u}_i^* = -1/H_0)\\ &\quad + P(H_1)P(u_i = -1, \overline{u}_i^* = -1/H_1)\\ &= P(H_0)P(u_i = 1/H_0)P(\overline{u}_i^* = 1/H_0)\\ &\quad + P(H_1)P(u_i = 1/H_1)P(\overline{u}_i^* = 1/H_1)\\ &\quad + P(H_0)P(u_i = -1/H_0)P(\overline{u}_i^* = -1/H_0)\\ &\quad + P(H_1)P(u_i = -1/H_1)P(\overline{u}_i^* = -1/H_1)\\ &= \tfrac{1}{2}P_{Fi}P(\overline{u}_i^* = 1/H_0) + \tfrac{1}{2}P_{Di}P(\overline{u}_i^* = 1/H_1)\\ &\quad + \tfrac{1}{2}P_{Di}P(\overline{u}_i^* = -1/H_0) + \tfrac{1}{2}P_{Fi}P(\overline{u}_i^* = -1/H_1).\end{aligned} \quad (18)$$

Because of the symmetry of the conditional probability mass function (see (17)),

$$P(\overline{u}_i^* = 1/H_0) = P(\overline{u}_i^* = -1/H_1) \quad (19)$$
$$P(\overline{u}_i^* = 1/H_1) = P(\overline{u}_i^* = -1/H_0). \quad (20)$$

Thus,

$$\hat{P}_{Di} = P_{Di}P(\overline{u}_i^* = 1/H_1) + P_{Fi}P(\overline{u}_i^* = 1/H_0). \quad (21)$$

By the same reasoning, we have

$$\begin{aligned}\hat{P}_{Fi} &= P(u_i = 1, \overline{u}_i^* = -1) + P(u_i = -1, \overline{u}_i^* = 1)\\ &= P_{Di}P(\overline{u}_i^* = 1/H_0) + P_{Fi}P(\overline{u}_i^* = 1/H_1).\end{aligned} \quad (22)$$

Let $\xi_1 < \xi_2 < \cdots < \xi_N$, where $\xi_N = (y_i)_{\max}$, be the set of values that y_i can attain for the ith local detector. Without loss of generality, let $\xi_1 < \tau < \xi_N$, and $k \in \{1,2,\ldots,N\}$ be the smallest integer such that $\xi_j > \tau$, $\forall j \geq k$. Define

$$A = P(\overline{u}_i^* = 1/H_1) = \sum_{j=k}^{N} P(y_i = \xi_j/H_1) \quad (23)$$

$$B = P(\overline{u}_i^* = 1/H_0) = \sum_{j=k}^{N} P(y_i = \xi_j/H_0). \quad (24)$$

Then, (21) and (23) can be written as

$$\hat{P}_{Di} = P_{Di}A + P_{Fi}B \quad (25)$$

and

$$\hat{P}_{Fi} = P_{Di}B + P_{Fi}A. \quad (26)$$

Let $r_i = P_{Di}/P_{Fi}$, $\hat{r}_i = \hat{P}_{Di}/\hat{P}_{Fi}$, then, $\log r_i = w_i$ is the weight of the ith detector defined by (5) and $\log \hat{r}_i = \hat{w}_i$ is the estimate for w_i

$$\hat{r}_i = \frac{\hat{P}_{Di}}{\hat{P}_{Fi}} = \frac{P_{Di}A + P_{Fi}B}{P_{Di}B + P_{Fi}A} = r_i \frac{1 + \frac{B}{r_i A}}{1 + \frac{r_i B}{A}} \quad (27)$$

$$\hat{w}_i = \log \hat{r}_i = \log r_i + \log \frac{1 + \dfrac{B}{r_i A}}{1 + \dfrac{r_i B}{A}} = w_i + \varepsilon_i. \quad (28)$$

As seen in (28), the estimate for the weight is equal to the correct weight plus an error term ε_i, where,

$$\varepsilon_i = \log \frac{1 + \dfrac{B}{r_i A}}{1 + \dfrac{r_i B}{A}}. \quad (29)$$

Since r_i is fixed, ε_i will approach 0 as B/A is approaching 0. We prove that increasing the *reliability threshold* τ will reduce the fraction B/A, and thus the error.

For notational convenience, let $p_i = P_{Di}$, $q_i = P_{Fi}$. Since $P(y_i = \xi/H_0) = P(y_i = -\xi/H_1)$ (see (17)),

$$\frac{P(y_i = \xi/H_1)}{P(y_i = \xi/H_0)} = \frac{\sum_{S_i} \prod_{S_i^+} p_j \prod_{S_i^-} q_j}{\sum_{S_i} \prod_{S_i^+} q_j \prod_{S_i^-} p_j}. \tag{30}$$

From (10), we have

$$\exp(y_i) = \frac{\exp(\sum_{S_i^+} w_j)}{\exp(\sum_{S_i^-} w_j)} = \frac{\prod_{S_i^+} \exp(w_j)}{\prod_{S_i^-} \exp(w_j)}. \tag{31}$$

Applying (5) to (31) yields

$$\exp(y_i) = \prod_{S_i^+} \frac{p_j}{q_j} \prod_{S_i^-} \frac{q_j}{p_j} = \frac{\prod_{S_i^+} p_j \prod_{S_i^-} q_j}{\prod_{S_i^+} q_j \prod_{S_i^-} p_j}. \tag{32}$$

The above equation holds for any combination of S_i^+ and S_i^- such that

$$y_i = \sum_{j \in S_i^+} W_j - \sum_{j \in S_i^-} W_j = \xi. \tag{33}$$

Thus, using the following equality

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+c}{b+d} = \frac{a}{b}. \tag{34}$$

Equation (30) becomes

$$\frac{P(y_i = \xi/H_1)}{P(y_i = \xi/H_0)} = \frac{\sum \prod_{S_i^+} p_j \prod_{S_i^-} q_j}{\sum \prod_{S_i^+} q_j \prod_{S_i^-} p_j} = \frac{\prod_{S_i^+} p_j \prod_{S_i^-} q_j}{\prod_{S_i^+} q_j \prod_{S_i^-} p_j} = \exp(\xi) \tag{35}$$

and

$$\frac{P(y_i = \xi/H_0)}{P(y_i = \xi/H_1)} = \exp(-\xi). \tag{36}$$

Thus far, we have proved that for each $y_i = \xi$, (36) holds. Using this equation and induction, we shall prove that B/A is monotonically decreasing with respect to τ.

As assumed earlier, $\xi_1 < \xi_2 < \cdots < \xi_N$. From (36), we have

$$\frac{P(y_i = \xi_1/H_0)}{P(y_i = \xi_1/H_1)} > \frac{P(y_i = \xi_2/H_0)}{P(y_i = \xi_2/H_1)} > \cdots > \frac{P(y_i = \xi_{N-1}/H_0)}{P(y_i = \xi_{N-1}/H_1)} > \frac{P(y_i = \xi_N/H_0)}{P(y_i = \xi_N/H_1)}. \tag{37}$$

Repeatedly applying the inequality,

$$\frac{X}{Y} > \frac{a}{b} \Rightarrow \frac{X}{Y} > \frac{X+a}{Y+b} > \frac{a}{b} \tag{38}$$

to (37), and using the definition of A and B in (23) and (24), it is clear that B/A is monotonically decreasing with respect to k, and thus it is also monotonically decreasing with respect to τ. This is consistent with our intuitive reasoning. However, τ cannot go to infinity; the maximum value of τ is $(y_i)_{\max}$. When τ attains its maximum, B/A reaches its minimum value. According to the definition of A and B, the minimum of B/A is

$$\left(\frac{B}{A}\right)_{\min} = \frac{P(y_i = (y_i)_{\max}/H_0)}{P(y_i = (y_i)_{\max}/H_1)}. \tag{39}$$

When P_{Di} is greater than P_{Fi} for each sensor and the learning procedure converges to its steady state, we have

$$(y_i)_{\max} = \sum_{j=1, j\neq i}^{n} \log \frac{P_{Dj}}{P_{Fj}}. \tag{40}$$

Thus,

$$\left(\frac{B}{A}\right)_{\min} = \exp(-(y_i)_{\max}) = \exp\left(-\sum_{j=1, j\neq i}^{n} \log \frac{P_{Dj}}{P_{Fj}}\right) = \prod_{j=1, j\neq i}^{n} \frac{P_{Fj}}{P_{Dj}}. \tag{41}$$

According to (29), the minimum error that can be achieved at steady state for a fixed i is

$$\varepsilon_i = \log \frac{1 + \prod_{j=1}^{n} \frac{P_{Fj}}{P_{Dj}}}{1 + \frac{P_{Di}}{P_{Fi}} \prod_{j=1, j\neq i}^{n} \frac{P_{Fj}}{P_{Dj}}}. \tag{42}$$

Note that $(y_i)_{\max}$ varies from sensor to sensor. In order to let every sensor adjust its weight and achieve the least error, the maximum value of τ is chosen to be the minimum of all $(y_i)_{\max}$:

$$\tau_{\max} = \min\{(y_1)_{\max}, (y_2)_{\max}, \ldots (y_n)_{\max}\}. \tag{43}$$

III. REINFORCEMENT LEARNING ALGORITHM

We assume that the distributed decision system has no knowledge of the probability mass functions of the observations. However, the probabilities of detection and false alarm for the ith detector $\hat{P}_{Di}$ and $\hat{P}_{Fi}$ can be approximated by relative frequencies. That is, in contrast to (18) and (22),

$$\frac{\hat{P}_{Di}}{\hat{P}_{Fi}} \approx \frac{m_i}{n_i} \tag{44}$$

where m_i and n_i are, respectively, the number of decisions made by the ith detector that agree and disagree with the *reliable* fused decisions. Both m_i and n_i are simply obtained by counting in the simulations. We next develop the updating rule for the fusion

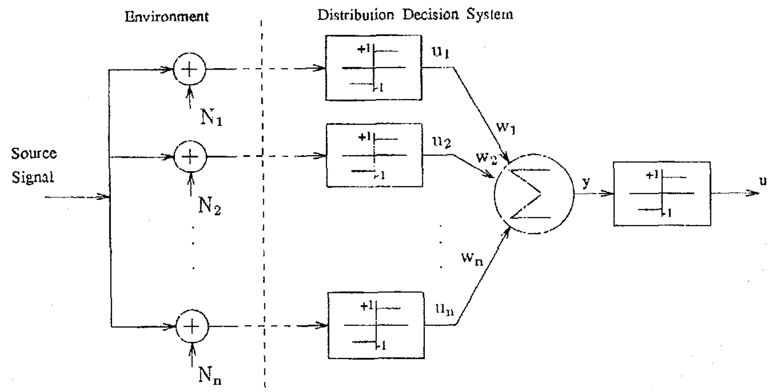

Fig. 3. The structure of the distributed decision system.

center. Similarly,

$$\hat{w}_i = \log\frac{\hat{P}_{Di}}{\hat{P}_{Fi}} \approx \log\frac{m_i}{n_i} \Rightarrow m_i \approx \exp(\hat{w}_i)n_i. \quad (45)$$

Taking the partial derivative with respect to m_i and n_i, respectively,

$$\frac{\partial\hat{w}_i}{\partial m_i} \approx \frac{1}{m_i} \quad \text{and} \quad \frac{\partial\hat{w}_i}{\partial n_i} \approx -\frac{1}{n_i} = -\frac{1}{m_i}\exp(\hat{w}_i). \quad (46)$$

If the decision of the current local detector agrees with the *reliable* fused decision, its weight $\hat{w}_i$ should be increased. In this case,

$$\Delta\hat{w}_i \approx \frac{1}{m_i}\Delta m_i = \frac{1}{m_i}. \quad (47)$$

On the other hand, if the current local decision disagrees with the *reliable* decision, its weight $\hat{w}_i$ should be reduced. That is,

$$\Delta\hat{w}_i \approx -\frac{1}{n_i}\Delta n_i = -\frac{1}{m_i}\exp(\hat{w}_i)\Delta n_i = -\frac{1}{m_i}\exp(\hat{w}_i). \quad (48)$$

Thus, we obtain the following updating rule:

$$\hat{w}_i^+ = \hat{w}_i^- + \Delta\hat{w}_i = \begin{cases} \hat{w}_i^- + \dfrac{1}{m_i}, & \text{if} \quad u_i = \overline{u}_i^* \\ \hat{w}_i^- - \dfrac{1}{m_i}\exp(\hat{w}_i^-), & \text{if} \quad u_i \neq \overline{u}_i^*. \end{cases} \quad (49)$$

LEMMA *Using the updating rule according to* (49), $\hat{w}_i^-$ *will converge to the desired steady state estimate weight* $\hat{w}_i$.

PROOF Using the definition $E[X] = \sum x_i P(x_i)$ and the updating rule according to (49),

$$E[w_i^+ - w_i^-] = E[\Delta w] = \frac{1}{m_i}P(u_i = \overline{u}_i^*) - \frac{1}{m_i}\exp(\hat{w}_i^-)P(u_i \neq \overline{u}_i^*). \quad (50)$$

As the number of iterations increase, m_i approaches infinity. In this case,

$$\frac{1}{m_i}P(u_i = \overline{u}_i^*) - \frac{1}{m_i}\exp(\hat{w}_i^-)P(u_i \neq \overline{u}_i^*) = 0 \quad (51)$$

and from the definition of $\hat{P}_{Di}$ and $\hat{P}_{Fi}$,

$$\hat{P}_{Di} - \hat{P}_{Fi}\exp(\hat{w}_i^-) = 0. \quad (52)$$

Thus,

$$\hat{w}_i^- = \hat{w}_i. \quad (53)$$

Hence, $\hat{w}_i^- \to \hat{w}_i$, for $i = 0,1,\ldots$.

IV. SIMULATION RESULTS

In this section, we present some computer simulation results to demonstrate the validity of our proposed adaptive scheme. Fig. 3 shows the simulation set-up. Here, equally likely binary signals $\{-1,1\}$ are randomly generated as source signals. Additionally, $N_1, N_2, \ldots, N_n$ are assumed to be independent identically distributed (IID) zero mean additive Gaussian random processes. Having selected the random noise process, the theoretical probabilities of detection and false alarm for each detector can be readily evaluated. For the Gaussian case, they can be determined by the standard deviation. They can be calculated according to:

$$P_{Fi} = Q\left(\frac{1}{\sigma_i}\right) = \int_{1/\sigma_i}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\xi^2/2}d\xi \quad (54)$$

where σ_i is the standard deviation of the Gaussian noise fed into the ith sensor,

$$P_{Di} = 1 - P_{Fi}. \quad (55)$$

Note that these theoretical probabilities and weights are calculated for comparison purposes only, and they are not readily available in practice. They are not used in the proposed adaptive fusion system. In the experiment, all the weights are first set to an initial value of 1, and then updated according to (49). The

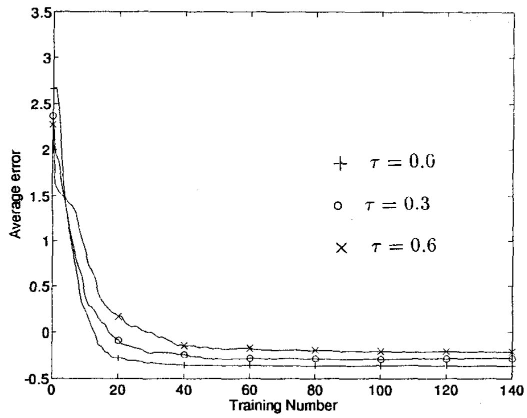

Fig. 4. Simulation results for case with identical detectors.

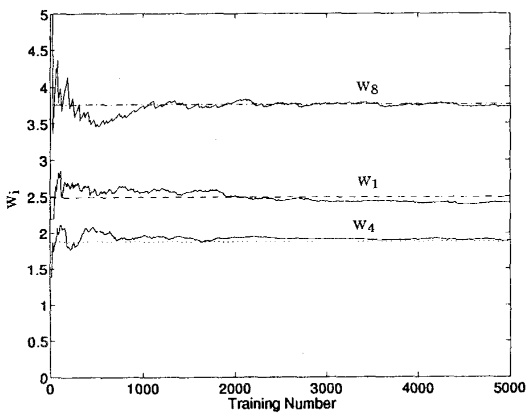

Fig. 5. Simulation results for case with different detectors. Straight lines represent theoretical weight values, curves show transient behavior of weights being updated.

steady state values are obtained after convergence ($\approx$ 1000 iterations).

Figs. 4 and 5 and Table I show the results for two different cases. The first case assumes that each local detector is identical. Here, $P_{Di} = 0.8413$, and $P_{Fi} = 0.1587$, for all $i = 1,2,\ldots,8$, where $w_i = \log P_{Di}/P_{Fi} = 1.6679$. Fig. 4 shows the mean error among 8 sensors between the estimate $\hat{w}_i$ and the actual weight $w_i = 1.6679$ for different values of τ, the *reliability threshold*. The figure conforms to our analytical results. That is, the larger the τ, the smaller the error. On the other hand, larger training time is needed to reach the steady state for a larger τ.

In the second case, the eight local detectors are assumed different, i.e., $P_{Di} = 0.9234$ and $P_{Fi} = 0.0766$, for $i = 1,2,3,5,6,7$; $P_{D4} = 0.8667$ and $P_{F4} = 0.1333$, and $P_{D8} = 0.9772$ and $P_{F8} = 0.0228$. Fig. 5 shows how the estimated weights approach the theoretical values. In the figure, $w_1 = 2.4895$, $w_4 = 1.8721$, $w_8 = 3.7579$. Only three of the eight weights are shown. However, other weights also follow the same trend. Table I summaries the results for this experiment. It is readily seen that the simulation results conform closely to the theoretical results.

Though it has been shown that $\hat{w}_i^-$ converges to $\hat{w}_i$, it does not converge to w_i. The error, (42), depends on the number of sensors, the P_{Di} and P_{Fi}. In the Gaussian noise environment, P_{Di} and P_{Fi} are determined by the Signal-to-Noise ratio (SNR) of the ith sensor. Thus, the error ε_i is totally determined by the number and the SNRs of sensors. Fig. 6 shows, for the case of identical sensors, the error, ε_i, versus n (the number of sensors) for various SNRs. In this case, according to (54) and (55), the error can be simplified to

$$\varepsilon_i = \log \frac{1 + \left(\frac{Q}{1-Q}\right)^n}{1 + \left(\frac{Q}{1-Q}\right)^{n-2}} \tag{56}$$

where Q is the Q-function defined in (54) with the same standard deviation, σ, for all sensors. Note that the error is the same for every sensor.

When the SNR is different from sensor to sensor, the error can be written as

$$\varepsilon_i = \log \frac{1+T}{1 + \left(\frac{1-Q_i}{Q_i}\right)^2 T} \tag{57}$$

where

$$T = \prod_{j=1}^{n} \frac{Q_j}{1-Q_j} \tag{58}$$

and

$$Q_j = Q\left(\frac{1}{\sigma_j}\right). \tag{59}$$

From (57), it is readily seen that the larger the SNR, the smaller the ε_i.

TABLE I
Comparison Between Theoretical and Steady State Values of Weights

Sensor		1st	2nd	3rd	4th	5th	6th	7th	8th
Noise Variance σ^2		0.49	0.49	0.49	0.81	0.49	0.49	0.49	0.25
SNR (dB)		3.098	3.098	3.098	0.915	3.098	3.098	3.098	6.02
Weights	Theoretical	2.4895	2.4895	2.4895	1.8721	2.4895	2.4895	2.4895	3.7579
	Steady state	2.4853	2.4916	2.4889	1.8721	2.4939	2.4971	2.4927	3.7941

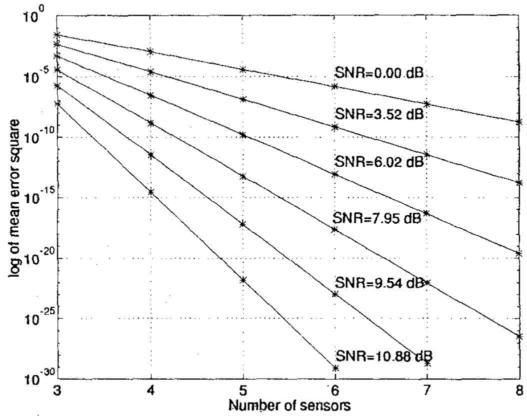

Fig. 6. Error curve versus number of sensors.

V. CONCLUSIONS

In a real-world environment, the probability mass functions of the observations at local detectors may not be known and the performance of the local detectors may not be stationary. Under such circumstances, it is desirable to have a system which can adapt itself during the decision making process. This paper proposes such an adaptive system with the assumption that $P_0 = P_1$ and $P_{Di} = P_{Fi}$. The major advantage of the system is that *a priori* knowledge of the probability mass functions of the observations is not required. The system can acquire the knowledge about the reliability of the local detectors by itself—*it can learn by doing*. A reinforcement learning rule is proposed and adopted, and its convergence is analytically proven. The simulation results conform to our theoretical analysis. If the *reliability threshold* τ can be adjusted adaptively during the process of data fusing, the system may converge faster. Future efforts will focus on adaptively adjusting the *reliability threshold*, and developing a model for unequiprobable sources.

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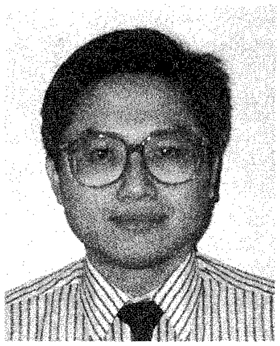

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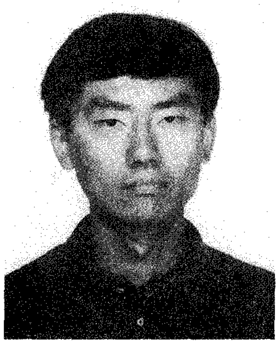

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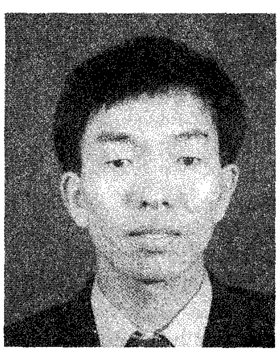

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