Searching for Optimal Frame Patterns in an Integrated TDMA Communication System Using Mean Field Annealing

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***Abstract*— In an integrated time-division multiple access (TDMA) communication system, voice and data are multiplexed in time to share a common transmission link in a frame format in which time is divided into slots. A certain number of time slots in a frame are allocated to voice and the rest are used to transmit data. Maximum data throughput can be achieved by searching for the optimal configuration(s) of relative positions of voice and data transmissions in a frame (frame pattern). When the problem size becomes large, the computational complexity in searching for the optimal patterns becomes intractable. In this paper, mean field annealing (MFA), which provides near-optimal solutions with reasonable complexity, is proposed to solve this problem. The determination of the related parameters are addressed. Comparison with the random search and simulated annealing algorithm is made in terms of solution optimality and computational complexity. Simulation results show that the MFA approach exhibits a good tradeoff between performance and computational complexity.**

***Index Terms*— Combinatorial optimization, energy function, frame pattern, mean field annealing, neural networks, simulated annealing, time division multiple access.**

I. Introduction

THE integration of data and voice in an integrated services data network (ISDN) has received extensive attention in recent years in order to efficiently share the system resources such as transmission, switching and control functions. Many research works have been directed to the time-division multiple access (TDMA) strategy. In a TDMA system, the time axis is divided into frames, and each frame consists of a certain number of fixed-length slots. A certain portion of the time slots in a frame is assigned to voice, and the remaining portion is reserved for data. Many studies in the literature model the voice traffic as a lossy system and data as a queuing system. Therefore a voice traffic will be blocked with no transmission if it cannot find an available time slot at the instant of its arrival. For data traffic, arrivals form a queue and are transmitted in any available data time slots based on first-come first-serve order. Hence the objective of the system design is to minimize the blocking probability of the voice traffic as well as the queuing delay of data traffic. Two approaches, namely, fixed-length boundary (FB) and movable boundary (MB), are generally used as shown in Fig. 1. In the FB scheme, a TDMA frame is partitioned into two regions consisting of a certain number of time slots, one for voice traffic and the other for data traffic. The idle time slots assigned to voice cannot be used to transmit data. Obviously, this scheme does not fully utilize system facilities. In contrast, the MB scheme can utilize any residual voice time slots to transmit data. As a result, the queuing delay is expected to decrease. Many multiplexing strategies have been proposed in order to fully utilize the integrated system resources [1]–[5].

In [5], a slotted ALOHA random access protocol is employed for data transmission in a TDMA mobile communication system. Instead of forming a long queue, data are retransmitted when they collide, i.e., when two or more data packets are transmitted at the same time slot. It has been shown [5] that the arrangement of relative positions of voice and data within a frame (called frame pattern) may affect data throughput. Searching for the optimal frame pattern to obtain maximum data throughput is a combinatorial optimization problem. Conventional methods for solving such an optimization problem usually get stuck in local optima which may be far from the globally optimal solution. Moreover, as the problem size grows, the computational complexity for searching global optima using these methods becomes incredibly complicated. In [5], the stochastic simulated annealing (SA) algorithm is applied in order to search for the optimal frame pattern. SA is a powerful stochastic optimization method in searching for global optimal solutions [6], but very time-consuming. It has been shown that finite-time approximation cannot guarantee convergence to an optimal solution [6]. Furthermore, for a large size problem, the state space is too large that finding the exact global minimum becomes intractable.

In this paper, a mean field annealing (MFA) algorithm is presented to search for the optimal frame patterns. Instead of the stochastic search process of SA, MFA performs thermal averaging operations, which leads to an overall decrease in computational effort. This paper is organized as follows. In the next section, the problem is defined, and the corresponding expression for data throughput is obtained. In Section III, MFA is elaborated. In Section IV, the MFA algorithm to search for the optimal frame patterns is derived. The energy function reflecting both data throughput to be maximized and the constraints are formulated. The determination of the related

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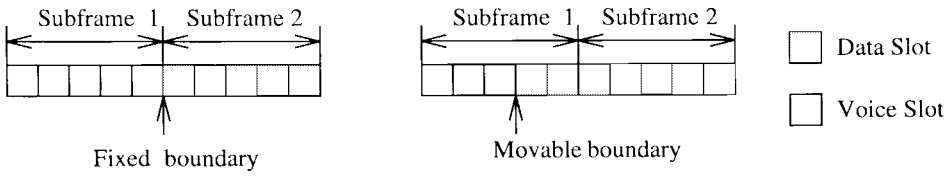

Fig. 1. Integrated voice and data TDMA frame format: (a) FB scheme. (b) MB scheme.

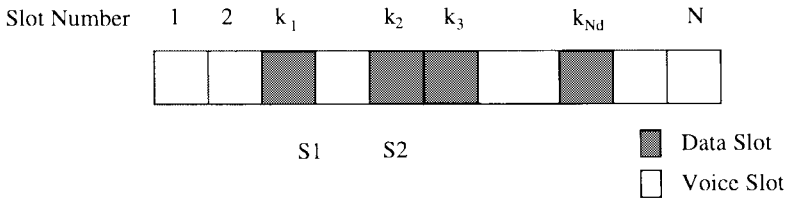

Fig. 2. A frame format.

parameters are discussed. In Section V, numerical examples and simulation results are given, and conclusions are made.

II. The Multiplexing Scheme and Data Throughput

The same multiplexing strategy and assumption discussed in [5] are adopted in this paper for the sake of comparison. Within the TDMA link, a synchronous transmission is provided for voice, and a contention scheme is used for data transmission. The frame format is shown in Fig. 2. Each frame consists of N fixed-length time slots.

Denote the following.

- N_d: Number of time slots that can be used for data packets at a given frame.
- k_i: Slot number of the ith available data slot, where $1 \le k_i \le N$, $1 \le i \le N_d$.
- s_i: Interdistance between the ith data slot and its first successor $(i+1)$th data slot, where

$$s_i = \begin{cases} k_{i+1} - k_i, & \text{if } i = 1, 2, \cdots, N_d - 1, \\ k_1 + N - k_{N_d} & \text{if } i = N_d. \end{cases}$$

$\mathbf{s} = (s_1, s_2, \cdots, s_{N_d})$ is called a *frame pattern* in the integrated communication system.

Obviously

$$s_1 + s_2 + \cdots + s_{N_d} = N. \tag{1}$$

The same assumptions are made as in [5], i.e.,

1) The holding time of the voice call is much longer than the frame time so that the queuing behavior of data for a given frame pattern can reach steady state.
2) The slotted ALOHA random access protocol is assumed for data transmission. The total data traffic, new and retransmitted, constitutes a *Poisson process* with mean arrival rate G packets/slot.

The probability of only one data packet is transmitted in the interval s_i is the one that no Poisson data packets are generated during the time interval s_i

$$\Pr(\text{a packet transmitted in the time interval } s_i) = e^{-G \cdot s_i} \tag{2}$$

The average number of packets successfully transmitted in the time interval s_i is

$$G \cdot s_i \cdot e^{-G \cdot s_i}. \tag{3}$$

The average data throughput is

$$\gamma_{\mathbf{s}} = \frac{1}{N_d} \sum_{i=1}^{N_d} G \cdot s_i e^{-G \cdot s_i}. \tag{4}$$

It can be seen that the relative positions of voice and data will decide data throughput. For a given N and N_d, there are $C_{N_d}^{N}$ frame patterns. For example, assuming $N = 40$ and $N_d = 10$, the total number of feasible frame patterns is $C_{10}^{40} = 8.4 \times 10^8$. The computational complexity using exhaustive search for finding the optimal frame pattern among the set of all frame patterns becomes intractable as the problem size increases. In the next section, an MFA algorithm is introduced to obtain the optimal frame pattern while reducing the computational complexity.

III. Simulated Annealing Versus Mean Field Annealing

A. Statistical Mechanics

In statistical mechanics, a physical process called *annealing* is often performed in order to relax the system to the state with the minimum free energy. In the annealing process, a solid in a *heat bath* is heated up by increasing the temperature of the bath until the solid is melted into liquid, then the temperature is lowered down slowly. At each temperature, all particles randomly arrange themselves until thermal equilibrium is reached. If the cooling is slow enough to allow the solid to reach thermal equilibrium at each temperature, the low energy crystalline solid would be formed when the system is frozen $(T \to 0)$. However, if the annealing is too fast, the solid may become glass with noncrystalline structure or the defected crystal with meta-stable amorphous structures. If a state is defined by the set of particle positions, then, at thermal equilibrium, the probability of the system being in state i is

represented by the *Gibbs distribution* [6], [7]

$$\pi_i = \Pr\{\mathbf{s} = i\} = \frac{\exp\left(-\frac{E(i)}{k_b T}\right)}{\mathcal{Z}} \tag{5}$$

where $\mathcal{Z} = \sum_{i \in S} \exp(-\frac{E(i)}{k_b T})$ is called the *partition function*, k_b is the *Boltzmann constant*, T is the temperature, and $E(i)$ is the energy of state i, and $\mathcal{S}$ is the state space. It is easy to find that [6]

$$\lim_{T \to \infty} \pi_i = \lim_{T \to \infty} \frac{\exp\left(-\frac{E(i)}{k_b T}\right)}{\sum_{j \in \mathcal{S}} \exp\left(-\frac{E(j)}{k_b T}\right)} = \frac{1}{|\mathcal{S}|} \tag{6}$$

implying that, at a very high temperature, all states are equally probable. On the other hand, we have

$$\begin{aligned} \lim_{T \to 0} \pi_i &= \lim_{T \to 0} \frac{\exp\left(-\frac{E(i) - E_{\min}}{k_b T}\right)}{\sum_{j \in \mathcal{S}} \exp\left(-\frac{E(j) - E_{\min}}{k_b T}\right)} \\ &= \begin{cases} \frac{1}{|\mathcal{S}_{\min}|} & \text{if } i \in \mathcal{S}_{\min} \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{7}$$

where $\mathcal{S}_{\min} = \{i : E(i) = E_{\min}\}$ and $E_{\min} = \min_{j \in \mathcal{S}} E(j)$. From this equation, we can see that, as the temperature approaches zero, the system will converge to the states with the minimum energy, i.e., the states with the minimum energy are reached at lower temperature.

B. Simulated Annealing

Based on the annealing process in the statistical mechanics, Kirkpatrick et al. [8] proposed an algorithm, namely *simulated annealing* (SA) for solving complicated combinatorial optimization problems. In the SA algorithm, the simulation of the annealing process is performed. The cost function and configuration in optimization correspond to the energy function and state of statistical physics, respectively. The temperature is introduced as a control parameter.

Suppose that a cost function $f : \mathcal{S} \to \mathbf{R}^+$, $s \in \mathcal{S}$, to be minimized is defined on some finite set $\mathcal{S}$. For each configuration $s \in \mathcal{S}$, there is a neighboring set $\mathcal{N}(s) \subseteq \mathcal{S}$, which is generated by a small perturbation of s.

In SA, given the current state $s(k)$, a neighboring state $s'(k)$ is randomly selected from $\mathcal{N}(s)$, where k is the kth trial. The transition probability from state $s(k)$ to $s'(k)$ is given by the *Metropolis criterion* [6], [9]

$$\begin{aligned} P[s(k), s'(k)] &= \Pr\{s(k) \to s'(k)\} \\ &= \exp\left[-\frac{[f(s'(k)) - f(s(k))]^+}{T}\right] \end{aligned} \tag{8}$$

where

$$[x]^+ = \max\{0, x\}. \tag{9}$$

From (8), it can be seen that the Metropolis criterion while performing the local search for the minimum cost at a fixed temperature T allows occasional transition from a lower cost configuration to a higher cost configuration with certain probability, thus preventing the system from getting stuck in local minima. The random process $\chi = (s(k) : k \geq 0)$ generated in SA can be characterized by a discrete time homogeneous Markov chain [6]. The one-step transition matrix is shown at the bottom of the page, where $G(x, y)$ is the probability of generating configuration y from x. If $G(x, y)$ is symmetric, the generation probability of any configuration x is uniformly distributed in its neighboring configuration set $\mathcal{N}(x)$ and the configuration transition is based on (8). The resulting Markov chains are irreducible, aperiodic, and recurrent [6]. Under these conditions, the stationary equilibrium distribution π_i for configuration i exists after an infinite number of transitions

$$\begin{aligned} \pi_i(T) &= \lim_{k \to \infty} \Pr\{s(k) = i \mid T\} \\ &= \lim_{k \to \infty} \Pr\{s(k) = i \mid s(0) = s_0, T\} \\ &= \frac{\exp\left(-\frac{f(i)}{T}\right)}{\sum_{j \in \mathcal{S}} \exp\left(-\frac{f(j)}{T}\right)}. \end{aligned} \tag{10}$$

From (7), we know that

$$\pi_i^* = \lim_{T \to 0} \pi_i(T) = \begin{cases} \frac{1}{|\mathcal{S}_{\min}|} & \text{if } i \in \mathcal{S}_{\min}, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

Therefore

$$\begin{aligned} \lim_{T \to 0} \left[\lim_{k \to \infty} P(s(k) \in S_{\min})\right] &= \lim_{T \to 0} \sum_{i \in S} \pi_i(T) \\ &= \sum_{i \in S_{\min}} \pi_i^* = 1. \end{aligned} \tag{12}$$

Equation (12) states that the SA algorithm asymptotically converges to configurations with the minimum cost, i.e., if the temperature is slowly lowered and at each temperature the system performs a sufficient number of transitions, the configurations (solutions) with the global minimum cost can be found with probability one.

C. Mean Field Annealing

Even though SA is proven to be able to reach the global optima asymptotically, it is time consuming to reach thermal equilibrium at each temperature. Finite number of transitions at each temperature cannot guarantee convergence to the global optima. In statistical physics, *mean field* approximation is often used. MFA uses a set of deterministic equations to replace the stochastic process in SA. It uses saddle point approximation

$$\begin{aligned} P(x, y) &= \Pr[s(k+1) = y | s(k) = x] \\ &= \begin{cases} 0 & \text{if } y \notin \mathcal{N}(x) \text{ and } y \neq x, \\ G(x, y) \min\left\{1, \exp\left(-\frac{[f(y) - f(x)]}{T}\right)\right\} & \text{if } y \in \mathcal{N}(x) \text{ and } y \neq x, \\ 1 - \sum_{x' \neq x} G(x, x') \min\left\{1, \exp\left(-\frac{[f(x') - f(x)]}{T}\right)\right\} & \text{if } y = x \end{cases} \end{aligned}$$

in the calculation of the stationary probability distribution at equilibrium, and reaches equilibrium at each temperature faster than SA. Even though this approximation method may not guarantee convergence to global minima, it does provide a good approximation in finding near-optimal solutions with less computing effort.

As shown in the previous section, the stationary probability distribution at equilibrium for configuration $\mathbf{s}'$ is given by

$$\pi_{\mathbf{s}'}(T) = \frac{\exp\left(-\frac{f(\mathbf{s}')}{T}\right)}{\mathcal{Z}}$$

$$\mathcal{Z} = \sum_{\mathbf{s}} \exp\left(-\frac{f(\mathbf{s})}{T}\right)$$

where $\mathbf{s}, \mathbf{s}' \in \mathbf{Z}^n$ are configurations and $\mathbf{Z}$ is the set of integer. For a large optimization problem, the direct calculation of the partition function $\mathcal{Z}$ is prohibitive, and thus an approximation method such as the saddle point approximation [11] is used. Note that the Dirac delta function $\delta(\cdot)$ can be expressed as

$$\delta(x) = \frac{1}{2\pi i}\int_{\mathbf{I}} e^{xy}\, dy \tag{13}$$

where the integral is taken along the imaginary axis. Hence,

$$\begin{aligned}\mathcal{Z} &= \sum_{\mathbf{s}} \exp\left(-\frac{f(\mathbf{s})}{T}\right) \\ &= C\sum_{\mathbf{s}} \int_{\mathbf{R}} d\mathbf{v} \int_{\mathbf{I}} e^{-\frac{f(\mathbf{v})}{T}} \cdot e^{\mathbf{u}(\mathbf{s}-\mathbf{v})}\, d\mathbf{u} \\ &= C\int_{\mathbf{R}} d\mathbf{v} \int_{\mathbf{I}} e^{-f_e(\mathbf{u},\mathbf{v})}\, d\mathbf{u}\end{aligned} \tag{14}$$

where

$$f_e(\mathbf{u},\mathbf{v}) = \frac{f(\mathbf{v})}{T} + \mathbf{u}\mathbf{v} - \ln\sum_{\mathbf{s}} e^{\mathbf{u}\cdot\mathbf{s}} \tag{15}$$

C is a complex constant, and f_e is called the *effective energy* in statistical mechanics. At saddle points,

$$\frac{\partial f_e}{\partial \mathbf{u}} = \mathbf{v} - \frac{\sum_{\mathbf{s}} \mathbf{s}\cdot e^{\mathbf{u}\cdot\mathbf{s}}}{\sum_{\mathbf{s}} e^{\mathbf{u}\cdot\mathbf{s}}} = 0$$

and (16)

$$\frac{\partial f_e}{\partial \mathbf{v}} = \frac{1}{T}\frac{\partial f(\mathbf{v})}{\partial \mathbf{v}} + \mathbf{u} = 0.$$

Therefore

$$\begin{aligned}\mathbf{v} = \bar{\mathbf{s}}_T &= \frac{\sum_{\mathbf{s}} \mathbf{s}\cdot e^{\mathbf{u}\cdot\mathbf{s}}}{\sum_{\mathbf{s}} e^{\mathbf{u}\cdot\mathbf{s}}} \\ \mathbf{u} &= -\frac{1}{T}\frac{\partial f(\mathbf{v})}{\partial \mathbf{v}}\end{aligned} \tag{17}$$

where $\bar{\mathbf{s}}_T$ is the thermal average of $\mathbf{s}$ at temperature T,

In statistical physics, $\mathbf{h} = -\frac{\partial f(\mathbf{v})}{\partial \mathbf{v}}$ is called the *mean field*. If a configuration $\mathbf{s} = [s_1, s_2, \cdots, s_n]^T$ is represented by a sequence of binary values, i.e., $\mathbf{s} \in \{0,1\}^n$, then we have $\mathbf{v} = [v_1, v_2, \cdots, v_n]^T$ and

$$v_i = \frac{\sum_{s_i=0}^{1} s_i \cdot e^{u_i s_i}}{\sum_{s_i=0}^{1} e^{u_i s_i}} = \frac{e^{u_i}}{1+e^{u_i}} = \frac{1}{2}\left[1 + \tanh\left(\frac{u_i}{2}\right)\right] \tag{18}$$

where $\mathbf{u} = [u_1, u_2, \cdots, u_n]^T$ and $u_i = -\frac{1}{T}\cdot\frac{\partial f(\mathbf{v})}{\partial v_i}$.

For the binary system, we have the following MFA equations:

$$v_i = \frac{1}{2}\left[1 + \tanh\left(\frac{h_i}{2T}\right)\right] \tag{19}$$

$$h_i = -\frac{\partial f(\mathbf{v})}{\partial v_i}. \tag{20}$$

In 1982, Hopfield [12] defined the following energy function of the Hopfield net for optimization:

$$f_h(\mathbf{s}) = -\frac{1}{2}\sum_i\sum_j T_{ij}s_i s_j - \sum_i s_i I_i \tag{21}$$

where $s_i \in \{0,1\}$. In the Hopfield model, the system is represented by a network composed of n neurons. Each neuron i can be represented by an operational amplifier, s_i is the output of neuron i, and T_{ij}, which is symmetric ($T_{ij} = T_{ji}$ and $T_{ii} = 0$), represents the synaptic connection between neuron i and j. I_i is the input current to amplifier i. The stable states of the network correspond to the 2^n corners of the hypercube $\{0,1\}^n$, the local minima of the energy function defined in (21). For the MFA approximation, if the energy function is formulated as (21), the mean field h_i and the thermal average v_i become

$$h_i = -\frac{\partial f_h(\mathbf{v})}{\partial v_i} = \sum_j T_{ij}v_j + I_i \tag{22}$$

$$v_i = \bar{s}_i = \frac{1}{2}\left[1 + \tanh\left(\frac{h_i}{2T}\right)\right]. \tag{23}$$

In MFA, the iterative procedure to reach thermal equilibrium at each temperature is called *relaxation*, in which the mean field is updated by

$$h_i(t+\Delta t) = h_i(t) + \Delta t\left[-\frac{\partial f_h(\mathbf{v})}{\partial v_i} - h_i(t)\right] \tag{24}$$

Taking the limit, we have

$$\frac{dh_i}{dt} = \lim_{\Delta t\to 0}\frac{h_i(t+\Delta t) - h_i(t)}{\Delta t} \tag{25}$$

or

$$\frac{dh_i}{dt} = -\frac{\partial f_h(\mathbf{v})}{\partial v_i} - h_i(t) = \sum_j T_{ij}v_j + I_i - h_i. \tag{26}$$

The MFA relaxation operation at each temperature should lead the system to stable equilibrium. The MFA procedure can be summarized in the flow chart shown in Fig. 3.

IV. Searching for the Optimal Patterns by MFA

As shown in Section II, maximizing data throughput is equivalent to finding a specific frame pattern $\mathbf{s}^{\text{opt}} = \{s_1^{\text{opt}}, s_2^{\text{opt}}, \cdots, s_{N_d}^{\text{opt}}\}$ such that

$$\gamma_{\mathbf{s}^{\text{opt}}} = \max_{\mathbf{s}\in S}\gamma_{\mathbf{s}} = \max_{\mathbf{s}\in S}\frac{1}{N_d}\sum_{i=1}^{N_d} G\cdot s_i \cdot e^{-G\cdot s_i} \tag{27}$$

subject to

$$\sum_{i=1}^{N_d} s_i = N, \quad \text{where } 1 \le s_i \le N - N_d + 1. \tag{28}$$

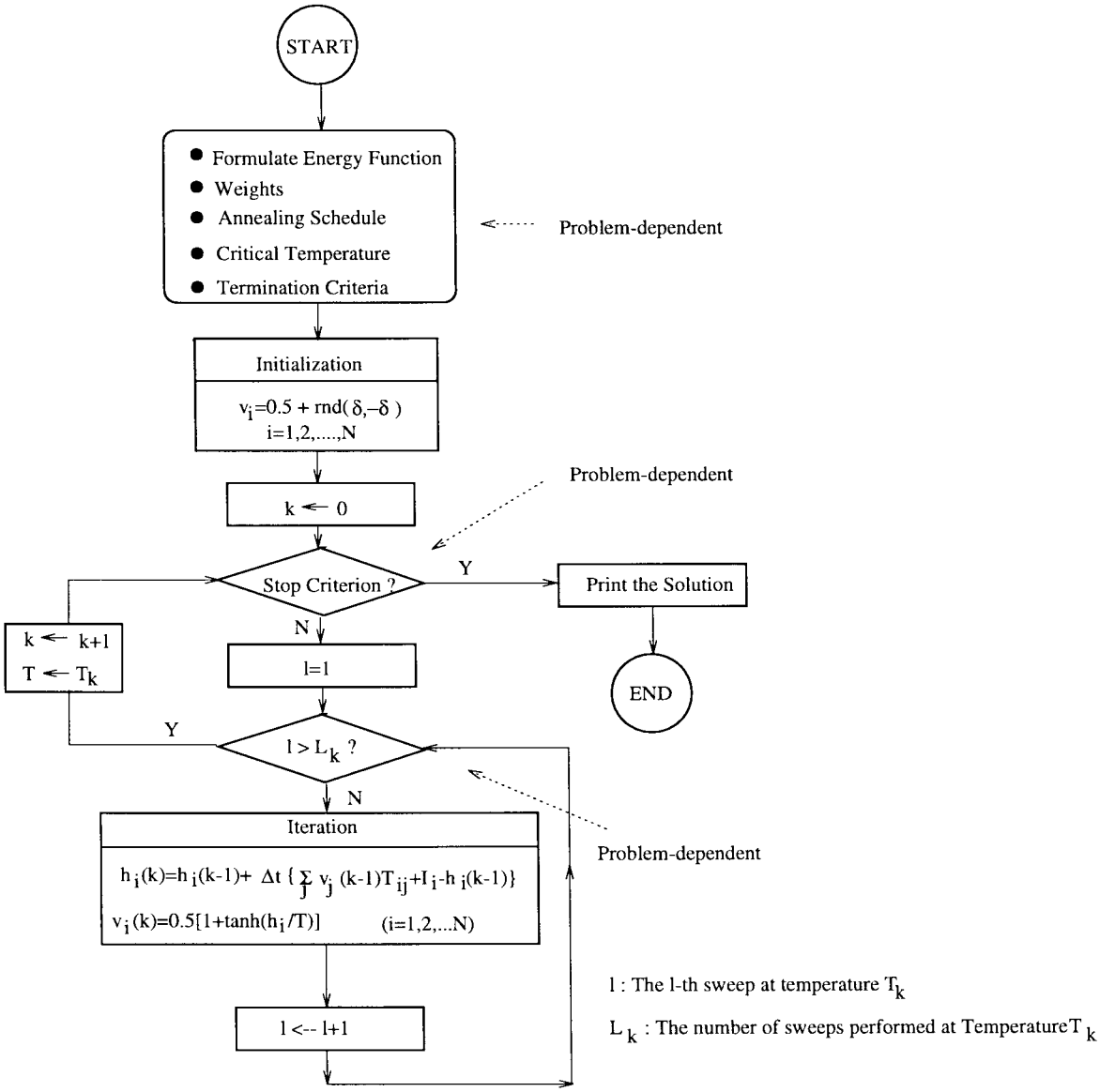

Fig. 3. The MFA iteration procedure.

and

$$s_i \geq 1 \quad \forall i. \tag{29}$$

where S is the set of feasible frame patterns. To map this optimization problem onto the MFA framework, we need to determine the following.

A. Energy Function

$$\begin{aligned} E(\mathbf{s}) &= -\frac{w_1}{N_d}\sum_{i=1}^{N_d} G \cdot s_i \cdot e^{-G \cdot s_i} + \frac{w_2}{2}\left(\sum_{i=1}^{N_d} s_i - N\right)^2 \\ &\quad + w_3 \sum_{i=1}^{N_d}\sum_{j=1}^{m} s_{ij}(1 - s_{ij}) \\ &= -w_1 \cdot E_1 + w_2 \cdot E_2 + w_3 \cdot E_3 \end{aligned} \tag{30}$$

and

$$s_i = \sum_{j=1}^{m} s_{ij} \cdot 2^j \tag{31}$$

where $s_{ij} \in \{0,1\}\ \forall i,j$ are binary neurons, $m = \lceil \log_2(N - N_d + 1) \rceil + 1$ and $\lceil x \rceil$ is the ceiling of x. For example, for $N = 40$, if $N_d = 9$, then $m = 5$, and if $N_d = 5$, then $m = 6$. $w_l > 0$, $l = 1, 2, 3$, are called the *weights*.

The interdistance between the ith data slot and its first successive data slot is denoted by s_i. Since s_i is an integer and $1 \leq s_i \leq N - N_d + 1$, s_i can be expressed in terms of m binary neurons defined by (31). The first term in (30) is the negatively weighed data throughput, and therefore maximizing data throughput is equivalent to minimizing the negative throughput. The second term introduces penalty for constraint violation. If the constraint in (28) is satisfied, the energy introduced by the second term is zero. The third term equals to zero only if all neurons converge to either zero

or one. If the weights (w_1, w_2, w_3) are properly chosen, the annealing procedure will lead the system to a configuration with the minimum energy corresponding to the optimal frame pattern.

B. Determination of the Weights

Taking the derivative of E_1 and setting it equal to zero

$$\frac{\partial E_1}{\partial s_i} = (1 - G \cdot s_i) \exp(-G \cdot s_i) = 0.$$

Therefore, the frame pattern $\mathbf{s} = \{s_i = \frac{1}{G} \ \forall i\}$ provides the maximum throughput $\gamma_{\max} = e^{-1}$. However, this frame pattern may not satisfy the constraint of (28) and (29). On the other hand, the minimum value of E_2 is zero when $\sum_{i=1}^{N_d} s_i = N$, but not any valid combination of s_i would achieve the maximum data throughput. Therefore, there exists a profound relationship between w_1 and w_2.

The annealing procedure attempts to relax the system into a state with the minimum energy, and simultaneously with all constraints satisfied. A state (frame pattern) $\mathbf{s}'$ that violates the constraint should yield higher energy than a state $\mathbf{s}$ which satisfies the constraints. Consider the situation that a state $\mathbf{s}$ satisfies the constraints (28), (29), and each neuron has converged to either zero or one. If a neighboring state $\mathbf{s}'$ of $\mathbf{s}$ violates the constraint such that

$$s_i' = \begin{cases} s_k - 1 & \text{if } i = k, \text{ for a certain } k, \\ s_i & \forall i \text{ except } i = k, \end{cases} \tag{32}$$

then

$$\sum_{i=1}^{N_d} s_i' = N - 1 \neq N$$

and

$$E(\mathbf{s}') = -\frac{w_1}{N_d} \sum_{i=1}^{N_d} G \cdot s_i' \cdot e^{-G \cdot s_i'} + \frac{w_2}{2} \tag{33}$$

$$E(\mathbf{s}) = -\frac{w_1}{N_d} \sum_{i=1}^{N_d} G \cdot s_i \cdot e^{-G \cdot s_i}. \tag{34}$$

According to the above statement, $E(\mathbf{s}') > E(\mathbf{s})$. Therefore, from (33) and (34), we have

$$\begin{aligned} &E(\mathbf{s}') - E(\mathbf{s}) \\ &= -\frac{w_1}{N_d} \sum_{i=1}^{N_d} G \cdot s_i' \cdot e^{-G \cdot s_i'} + \frac{w_2}{2} + \frac{w_1}{N_d} \sum_{i=1}^{N_d} G \cdot s_i \cdot e^{-G \cdot s_i} \\ &= \frac{w_2}{2} - \frac{w_1 \cdot G}{N_d} [s_k' \cdot e^{-G \cdot s_k'} - s_k \cdot e^{-G \cdot s_k}] \\ &= \frac{w_2}{2} - \frac{w_1 \cdot G}{N_d} \cdot e^{-G \cdot s_k} [(s_k - 1) \cdot e^G - s_k] \\ &> \frac{w_2}{2} - \frac{w_1 \cdot G}{N_d} \cdot e^{-G \cdot s_k} [(s_k - 1) \cdot e^G - (s_k - 1)] \\ &= \frac{w_2}{2} - \frac{w_1 \cdot G}{N_d} \cdot e^{-G \cdot (s_k - 1)} \cdot (s_k - 1) \cdot [1 - e^{-G}] \end{aligned}$$

Using the fact that $x \cdot e^{-x} \leq e^{-1}$, we have

$$\begin{aligned} E(\mathbf{s}') - E(\mathbf{s}) &> \frac{w_2}{2} - \frac{w_1}{N_d} \cdot (1 - e^{-G}) \cdot e^{-1} \geq 0. \\ \Rightarrow w_2 &\geq \frac{2w_1}{N_d} \cdot (1 - e^{-G}) \cdot e^{-1}. \end{aligned} \tag{35}$$

Here only one specific case is considered, indicating that the selection of w_1 and w_2 is related to G and N_d. The adjustment of the weights according to G and N_d is required to obtain better solutions. The term, w_3, is a weak constraint, and $w_3 = 1$ in this paper.

C. Critical Temperature

According to (19) and (20), each neuron is updated as follows:

$$v_{ij}(t + \delta t) = \frac{1}{2} + \frac{1}{2} \tanh\left[-\frac{1}{T} \frac{\partial E(\mathbf{v}, t)}{\partial v_{ij}}\right] \quad \forall i, j. \tag{36}$$

where

$$v_{ij} = \overline{s_{ij}} \quad 1 < i < N_d, \quad 1 \leq j \leq m$$

$$\mathbf{v} = \{v_1, v_2, \cdots, v_{N_d}\}, \quad v_i = \{v_{i1}, v_{i2}, \cdots, v_{im}\}.$$

The critical temperature is defined as the temperature at which the sharp state transition starts. That is, each neuron is likely pushed toward the "0" or "1" state. In (36), a very large value of T leads each v_{ij} to fluctuate around $\frac{1}{2}$, and the state transition is very slow. Therefore there must exist a critical temperature at which quick state transitions are expected to start. In this paper, the critical temperature is obtained by *trial-and-error*. That is, temperature is slowly decreased from a very high value. At each temperature, only one sweep is taken, where one complete updating of all neurons at a fixed temperature is referred to as a *sweep*. At the end of each sweep, compute the average absolute error

$$\epsilon = \frac{1}{N_d \cdot m} \sum_{i=1}^{N_d} \sum_{j=1}^{m} |v_{ij}(t + \delta t) - v_{ij}(t)| \tag{37}$$

where t stands for the time a sweep starts and $t + \delta t$ for the time a sweep ends. When $\epsilon \geq 0.1$, the above procedure stops, and the corresponding temperature is the critical one.

D. Annealing Schedule

The following annealing schedule is employed:

$$T_{n+1} = \frac{T_n}{1 + \alpha \cdot n} \tag{38}$$

where α is a small positive value, and n is the iteration index. Other schedules [13] may be adopted.

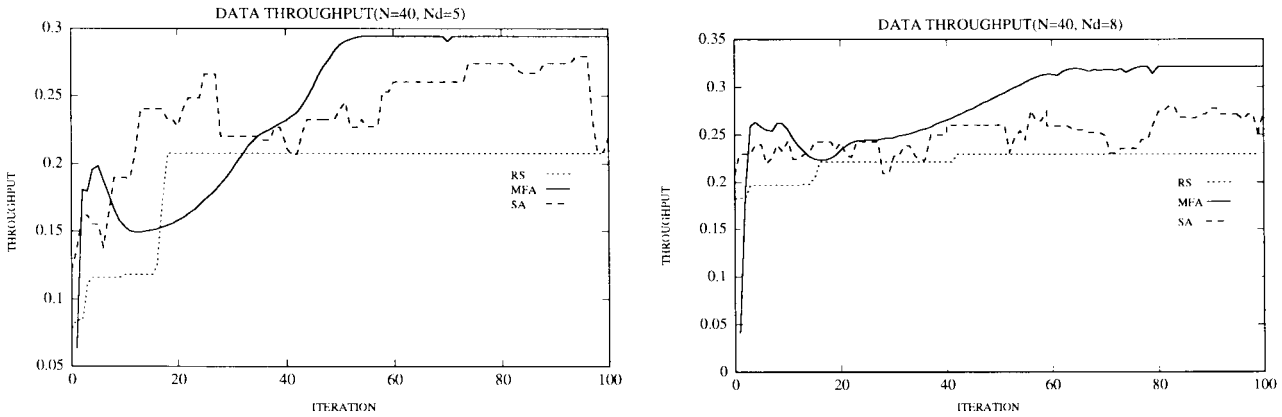

Fig. 4. One run and 100 iterations $(G = 0.5)$.

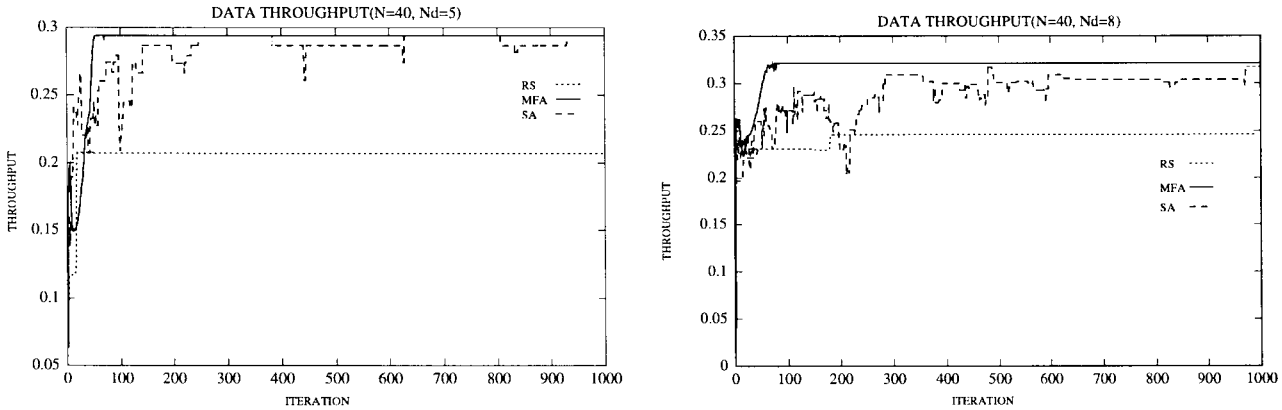

Fig. 5. One run and 1000 iterations $(G = 0.5)$.

E. Convergence Criterion

1) Termination of Sweeps: At each temperature, each neuron is updated according to (36). The sweep is terminated when $\epsilon \leq \delta_1$, where δ_1 is a small positive value. On the other hand, at some temperature, the condition may not be satisfied after a large number of sweeps. To avoid infinite sweeps, the sweep procedure is forced to end after a fixed number of sweeps, n_{sweep}. Then the temperature is further decreased, and a new updating process begins.

2) Convergence Criterion: All v_{ij} should converge to either zero or one after the last iteration. Therefore, we define the convergence criterion as

$$\frac{1}{N_d \times m} \sum_{i=1}^{N_d} \sum_{j=1}^{m} v_{ij}(1 - v_{ij}) < \delta_2 \tag{39}$$

where δ_2 is a small positive value. When the criterion is satisfied, all neurons are clamped, and the interdistances for the optimal frame pattern are found to be

$$s_i = \sum_{j=1}^{m} 2^j \cdot U\left(v_{ij} - \frac{1}{2}\right) \quad \forall i. \tag{40}$$

where $U(\cdot)$ is a step function.

V. Simulation Results

Four instances with $N_d = 5, 8, 10, 15$, and $N = 40$ are tested by using the MFA algorithm. To demonstrate the advantage of the MFA scheme, a comparison with the random search and the SA approach [5] is made in terms of computational complexity and throughput optimality.

1) Random Search (RS): In random search, a frame pattern is randomly selected from the frame pattern space, and the one that yields the largest throughput is kept until termination. There is no fixed rule for terminating the procedure. Usually, the procedure terminates after a certain number of iterations. Here an *iteration* consists of a frame pattern generation and a throughput comparison.
2) Simulated Annealing: To make the comparison fair, an *iteration* in [5] consists of a pattern generation and transition test based on the Metropolis criterion.
3) Mean Field Annealing: In MFA, to exploit the parallelism of neural networks, synchronous updating is adopted, i.e., the current value for each neuron is updated by using the previous neurons' values. Therefore, neurons in the neural network operate in parallel, and an *iteration* implies that the whole network is updated once, i.e., all neurons are updated once.

The three algorithms are implemented and compared. Each algorithm is executed for 1000 times, and the throughput

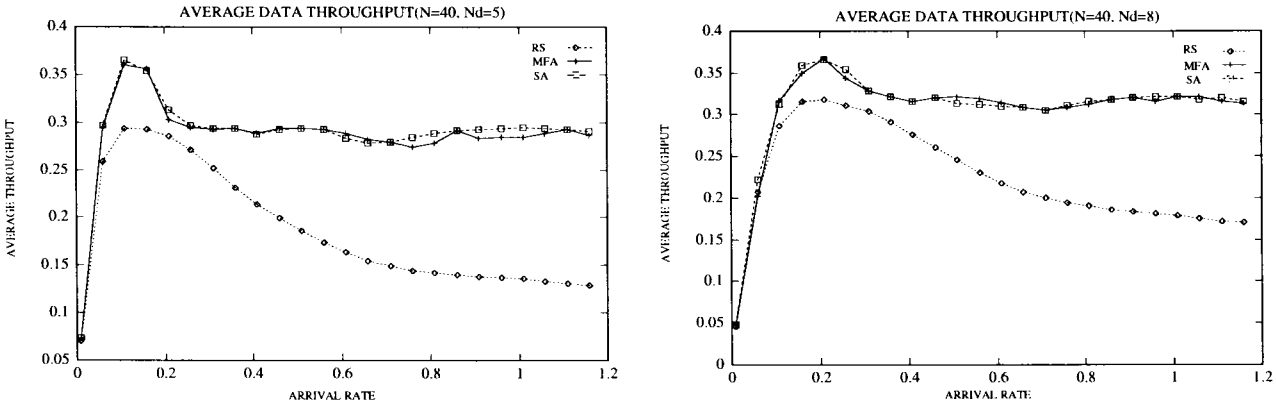

Fig. 6. Average throughputs for different N_d (1000 runs and 1000 iterations for each run).

is averaged over the 1000 runs. Each *run* is defined as an execution of an algorithm. Fig. 4 shows the first 100 iterations of a run at the arrival rate $G = 0.5$. It can be seen that the MFA approach reaches its steady throughput within the first 100 iterations. Fig. 5 shows that, after 1000 iterations, all of the algorithms reach the steady throughput. Fig. 6 shows the average throughput achieved by the three algorithms over an ensemble of 1000 runs. It can be seen that the average throughput achieved by both MFA and SA are very close. At some arrival rates, MFA can achieve even better average throughput than its counterpart because finite-time implementation (1000 iterations) of SA does not guarantee convergence to global optima. The RS algorithm is not effective in searching for the optimal pattern. It can be seen from these results that MFA achieves faster convergence in terms of the number of required iterations than SA while achieving suboptimal performance. In our simulations, $w_1 = 750$, $w_3 = 1$, and $w_2 = 750$ when $G \geq 0.4$, and $w_2 = 6.5$ when $G < 0.4$. $\delta_1 = 0.05$, $\delta_2 = 0.01$. $\alpha = 0.01$. The critical temperature $T_0 = 5$.

VI. Conclusion

Searching for the optimal patterns in an integrated TDMA communication system is a combinatorial optimization problem. As the problem size gets large, the computational complexity becomes intractable. SA is a good algorithm in finding global optimal solutions, but it is usually time-consuming. MFA, which uses saddle point approximation, is proposed to solve for the optimal patterns. It is computationally efficient, and is able to acquire suboptimal solutions comparable to those obtained by SA. The determination of related parameters are addressed, and comparisons with the RS and SA approaches are presented. Numerical results have shown that MFA needs one order of magnitude less iterations than SA and at the same time achieves comparable solutions.

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