MARKOV-MODULATED SELF-SIMILAR PROCESSES: MPEG CODED VIDEO TRAFFIC MODELER AND SYNTHESIZER

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ABSTRACT

Markov modulated self-similar processes are proposed to model MPEG video sequences that can capture the LRD (Long Range Dependency) characteristics of video ACF (Auto-Correlation Function). An MPEG compressed video sequence is decomposed into three parts according to different motion/change complexity such that each part can individually be described by a self-similar process. Beta distribution is used to characterize the marginal cumulative distribution function (CDF) of each self-similar processes, and Markov chain is used to govern the transition among these three self-similar processes. Network cell loss rate using our proposed synthesized traffic is found to be comparable with that using empirical data as the source traffic.

1. INTRODUCTION

The trend to transmit video over network, especially over ATM, is emerging. Traffic models are important to network design, performance evaluation, bandwidth allocation, and bit-rate control. It has, however, been observed that traditional models fall short in describing the video traffic because video traffic is strongly autocorrelated and bursty [1]. To accurately model video traffic, autocorrelations among data should be taken into consideration. A considerable amount of effort on video modeling has been reported that include MMRP [2], DAR(1) [3], Fluid Models [4], Markov Renewal Modulated TES Models [5], LRD [6], $M/G/\infty$ input process models [7], and GBAR Model [8].

The above models can be categorized into SRD and LRD models. They are used to capture two statistical factors: marginal distribution (first-order statistics) and autocorrelation function (second-order statistics) of traffic arrival times.

All these models can only capture the ACF of JPEG video. The ACFs of MPEG videos are quite different from these of JPEG videos. To overcome some shortcomings of video modeling, we propose to model an MPEG compressed video sequence by Markov modulated self-similar processes, in which the original sequence is decomposed into several sequences that can be modeled by self-similar processes.

A Markov chain is then used to govern the transition among these self-similar processes. It has been found that video traffic possesses self-similarity, and thus it is natural to model video traffic by self-similar processes. In addition, self-similar processes have simple ACF forms, hence allowing us to readily derive an analytical model for our proposed approach. Our proposed model is shown to be able to capture both the long range dependency and marginal cumulative distribution.

2. EMPIRICAL DATA AND ACF

We use MPEG-1 coded data of *Star Wars*[1] as the empirical data. The source contains motions ranging from low complexity/motion scenes to those with high and very high actions.

The data file consists of 174,136 integers, whose values are frame sizes (bits per frame). The ACF of MPEG coded *Star War*, shown in Fig. 1, is quite different from that of JPEG coded sequence. The ACF of the MPEG coded data fluctuates around an envelope, reflecting the fact that, after the use of motion estimation techniques, the dependency between frames is reduced. This characteristic should be taken into consideration in modeling MPEG coded video sequences. We propose to

This work was done in part while N. Ansari was on leave at the Chinese Univ. of HK, Hong Kong, and Yun Q. Shi was on leave at the Nanyang Tech. Univ., Singapore

[1]The MPEG-1 coded data were the courtesy of M.W.Garrett of Bellcore and M.Vetterli of UC Berkeley.

0-7803-5796-5/99/$10.00 © 1999 IEEE

use self-similar processes with different ACFs to reflect the fluctuation of ACFs. The basic idea behind this method is to divide the sequence into three different sequences, each modeled by a separate self-similar process. The transition among these processes is governed by a Markov chain, whose transition matrix can be obtained from empirical data.

3. SRD, LRD, AND SELF SIMILARITY

Consider a stationary process $X = \{X_n : n = 1, 2, \ldots\}$ with mean μ and variance σ^2. The autocorrelation function and the variance of X are denoted as:

$$r(k) = \frac{E[(X_n - \mu)(X_{n+k} - \mu)]}{\sigma^2} \quad (1)$$

and

$$\sigma^2 = E[(X_n - \mu)^2]. \quad (2)$$

X is said to be SRD if $\sum_{k=0}^{K=\infty} r(k)$ is finite; otherwise, the process is said to be LRD [9]. Let X defined above has the following autocorrelation function:

$$r(k) \rightsquigarrow k^{-\beta} L(k), k \to \infty \quad (3)$$

where $0 < \beta < 1$, and L is a slowly varying function as $k \to \infty$, i.e., $\lim_{t\to\infty} L(tx)/L(t) = 1$ for all $x > 0$. Consider the aggregated process

$$X^{(m)} = \{X_t^{(m)}\} = \{X_1^{(m)}, X_2^{(m)}, \ldots\},$$

where

$$X_t^{(m)} = \frac{1}{m}(X_{tm-m+1} + \cdots + X_{tm}), t \in P, m \in P, \quad (4)$$

and P is a positive integer set. X is said to be exactly second-order self-similar [9] if

$$var X^{(m)} = \sigma^2 m^{-\beta} \quad (5)$$

and

$$r^{(m)}(k) = r(k) \quad (6)$$

for all $m \in \{1, 2, 3, \cdots\}$ and $k \in \{0, 1, 2, \cdots\}$. Here $r^{(m)}(k)$ is the autocorrelation function of $X^{(m)}$. In fact, Eq. (5) is sufficient to define a self-similar process since Eq. (3) and (6) can be derived from Eq. (5) [9].

It is apparent that a self-similar process is a kind of LRD process. Since empirical video traffic exhibits self-similarity and long range dependency, it is intuitive to use self-similar processes to model video traffic. It is one of the most often used processes to capture LRD of video traffic.

Hurst parameter $H = 1 - \beta/2 (0 < \beta < 1)$ is used to measure the similarity of a process. It is the only parameter needed to describe a second-order self-similar process. For self-similarity processes, $1/2 < H < 1$.

4. CLASSIFICATION OF MPEG DATA

It is apparent that ACF of MPEG compressed video traffic cannot be approximated by a single function $r(k) = k^{-\beta}$ because this kind of function decreases monotonically, while ACF of a MPEG compressed video traffic fluctuates a lot. We therefore suggest to divide the traffic data into three different parts—inactive part, active part, and the most active part (authors in [2] also pointed out that a video bit rate process has three main components: a slowly changing component, a more quickly changing component, and an impulsive component). Suppose $f(i)$ is the number of bits in the *ith* frame. The video traffic can be classified as follows

1. If $f(i+1)/f(i) > T, i = 2, 3, \cdots$, then $f(i+1)$ belongs to the non-inactive part; otherwise, $f(i+1)$ belongs to the inactive part, where T is a threshold value.
2. Similarly, the non-inactive part can be classified into the active and most active part.

Taking these three data sets as three different random processes, we can calculate their ACFs.

5. MODELING OF CLASSIFIED DATA

The ACF of each process is very different from that of the original sequence. For example, Fig. 2 shows the ACF of the active part [2]. The fluctuation is no longer as big. We have used $k^{-\beta}$, $e^{-\beta k}$ and $e^{-\beta\sqrt{k}}$, corresponding to the ACFs of a self-similar process, a Markov process and an $M/G/\infty$ input process, respectively, to approximate ACFs of these three processes. It becomes evident that $k^{-\beta}$ is a better approximation of ACFs of these classified data, and we therefore use self-similar processes s_1, s_2, and s_3 to model these processes.

Using the least squares method, we obtained $\beta = 0.3321$, 0.3069, and 0.4396 for the active, inactive, and the most active part, respectively. The corresponding Hurst parameters for these processes are $H = 0.8339$, 0.8465, and 0.7802. Beta distribution [11] was used to model the marginal distributions of these processes. The probability density function of a Beta process has the following form:

$$f(x; \gamma, \eta, \mu_0, \mu_1) = \begin{cases} \frac{1}{\mu_1 - \mu_0} \frac{\Gamma(\gamma+\eta)}{\Gamma(\gamma)\Gamma(\eta)} (\frac{x-\mu_0}{\mu_1-\mu_0})^{\gamma-1} (1 - \frac{x-\mu_0}{\mu_1-\mu_0})^{\eta-1} & \mu_0 \le x \le \mu_1, 0 < \gamma, 0 < \eta \\ 0 & otherwise \end{cases} \quad (7)$$

[2] Owing to the limited space, the ACFs of the other two parts, not shown here, can be found in [10].

where γ and η are shape parameters, and $[\mu_0, \mu_1]$ is the domain where the distribution is defined. Beta distribution is quite versatile and can be used to model random processes with quite different shapes of marginal distributions. The following formulae [11] are used to derive the parameters of Beta distribution:

$$\hat{\eta} = \frac{1-\bar{x}}{s^2}[\bar{x}(1-\bar{x}) - s^2] \tag{8}$$

$$\hat{\gamma} = \frac{\bar{x}\hat{\eta}}{1-\bar{x}} \tag{9}$$

where

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i, \tag{10}$$

$$s^2 = \frac{N\sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2}{N(N-1)}, \tag{11}$$

and N is the number of data in the data set. Using the classified data sets, $\hat{\gamma} = 1.6179$, $\hat{\eta} = 13.7810$ for the inactive process, $\hat{\gamma} = 1.7977$, $\hat{\eta} = 12.1980$ for the active process, and $\hat{\gamma} = 5.3550$, $\hat{\eta} = 11.4134$ for the most active process. The marginal distributions of active part and its corresponding Beta distributions are shown in Fig. 3.

6. MODELING THE MPEG DATA

To model the whole data set, we need a process to govern the transition among the processes s_1, s_2, and s_3 obtained above. Markov chain is often used owing to its simplicity.

Using Markov chain as the dominating process, our model for MPEG video traffic can be described by the state diagram shown in Fig. 4, where state S_1, S_2, and S_3 correspond to the three respective self-similar processes. At state S_i, bit rates are generated by process s_i. The transition probability from S_i to S_j can be estimated from the empirical data as follows:

$$p_{ij} = \frac{N_{ij}}{N_i}, \tag{12}$$

where N_i is the total number of times that the system goes through state S_i, and N_{ij} is the number of times that the system make transition to state S_j from state S_i. For the *Star Wars* video, the following transition matrix

$$\hat{P} = \begin{bmatrix} 0.0002 & 0.9998 & 0 \\ 0.1174 & 0.5232 & 0.3594 \\ 0.0209 & 0.9791 & 0 \end{bmatrix}$$

is obtained. This matrix is useful for the synthesis of video traffic.

7. VIDEO TRAFFIC SYNTHESIS

To synthesize video traffic using our model requires self-similar traffic generator. Some methods are available to generate approximate self-similar traffic. Two of the most frequently used methods are exactly self-similar fractional Gaussian noise(FGN) [12] and asymptotically self-similar fractional autoregressive integrated moving average (F-ARIMA) process [12]. F-ARIMA can be used to match any kind of ACF. It takes a long time to generate the video traffic since F-ARIMA is an iterative process. The F-ARIMA process can be generated by the following algorithm [6]:

1. Generate X_0 from a Gaussian distribution $N(0, \nu_0)$. Set initial values $N_0 = 0, D_0 = 1$
2. For $k = 1, 2, \cdots, N-1$, calculate $\phi_{kj}, j = 1, 2, \cdots, k$ iteratively using the following formulae

$$\begin{aligned}
N_k &= r(k) - \sum_{j=1}^{k-1} \phi_{k-1,j} r(k-j) && (13)\\
D_k &= D_{k-1} - N_{k-1}^2 / D_{k-1} && (14)\\
\phi_{kk} &= N_k / D_k && (15)\\
\phi_{kj} &= \phi_{k-1,j} - \phi_{kk}\phi_{k-1,k-j}, \\
& \quad j = 1, \cdots, k-1 && (16)\\
m_k &= \sum_{j=1}^{k} \phi_{kj} X_{kj} && (17)\\
\nu_k &= (1 - \phi_{kk}^2)\nu_{k-1} && (18)
\end{aligned}$$

Finally, each X_k is chosen from $N(m_k, \nu_k)$. In this way, we obtain a process X with ACF approximating to $r(k)$.

To generate a self-similar process approximately, autocorrelation function can be calculated in a recursive way as

$$r(0) = 1, r(k+1) = \frac{k+d}{k+1} r(k) \tag{19}$$

where $d = H - 0.5$.

ACFs of F-ARIMA and FGN generated traffic are less than $k^{-\beta}$ for small k. To compensate for the under-estimation of ACFs of a self-similar process, Eq. (19) used to generate F-ARIMA traffic can be enlarged for small k. New self-similar traffic generators need to be devised so that more exact self-similar traffic can be generated.

Distribution of these data is Gaussian. For data to be Beta distributed, the following mapping can be used

$$Y_k = F_\beta^{-1}\left(F_N(X_k)\right), k > 0 \tag{20}$$

where X_k is a self-similar Gaussian process, F_N is the normal cumulative distribution function, and F_β^{-1} is the inverse of the cumulative distribution function of the Beta model.

Video traffic can be synthesized by a combination of the three obtained self-similar processes via a Markov process, whose transition matrix was given in the last section.

8. NETWORK CELL LOSS RATE

Cell Loss Rate (CLR) is an important queuing performance of an ATM network. To justify the queuing performance of our model, our synthetic traffic was used as the source traffic to an ATM switch with a limited buffer size. The performance is compared to the same system using empirical data as the source traffic. A single arrival process is assumed in our simulation, and its service rate is assumed to be constant. To simplify the simulation process, the time is sliced. Every slice is used to transmit one cell (48 bytes of payload per cell). We also assume that cells in a frame must arrive at the switch during the period of this frame. This corresponds to the case that no traffic shaping is applied. Cells are dropped when the switch buffer overflows.

Based on the switch model, performance at different service rates and buffer sizes is examined. Simulation results using empirical data and traffic model are shown in Table 1. The results show that the CLRs obtained using video trace and our proposed model are very close for both high and low service rates.

9. CONCLUSIONS

In this paper, we have proposed a Markov modulated self-similar process for modeling MPEG compressed video sequences. Compared with other methods, the proposed model is easy to analyze, and it is able to capture the LRD of video ACF. An analytical solution may be obtained for this model because of its simple ACF form. Queuing performance for small and large buffers under different traffic intensity using our proposed model is compatible with that using empirical data.

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Table 1: CLRs for different service rate and buffer size

Buffer size(cells)	4000cells/s		6000cells/s		9000cells/s	
	Trace	Model	Trace	Model	Trace	Model
20	2.24E-2	9.95E-2	2.09E-3	2.30E-2	1.50E-4	4.25E-4
40	1.24E-2	7.43E-2	1.36E-3	1.34E-2	8.07E-5	1.34E-4
60	6.81E-3	5.44E-2	9.72E-4	9.25E-3	7.19E-6	2.28E-5
100	2.30E-3	2.72E-2	4.57E-4	2.91E-3	0	0
200	3.55E-4	4.22E-3	1.00E-5	3.40E-5	0	0
400	6.14E-5	6.40E-4	0	0	0	0

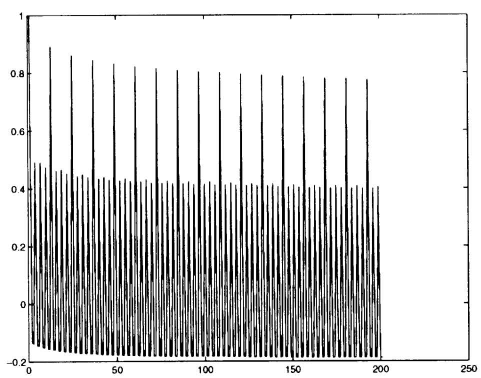

Figure 1: ACF of MPEG compressed video *Star Wars*

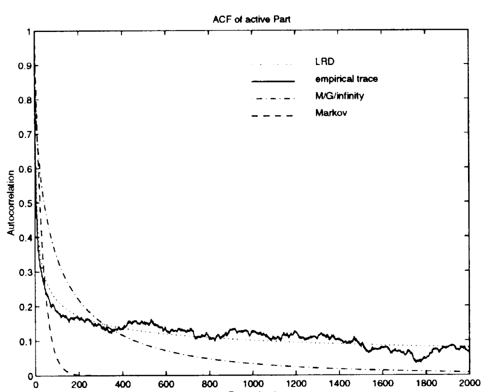

Figure 2: ACF of the active part of *Star Wars*

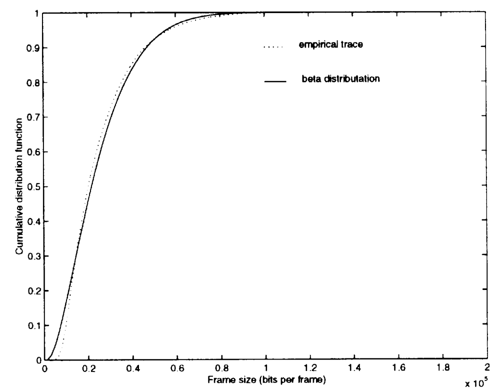

Figure 3: CDF of the active part and the corresponding beta distribution

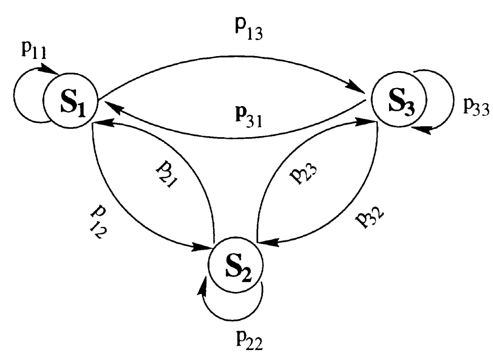

Figure 4: A Markov modulated self-similar process model for MPEG video