Coherent Decorrelating Detector with Imperfect Channel Estimates for CDMA Rayleigh Fading Channels

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Abstract

A coherent multiuser decorrelating detector for an asynchronous CDMA, time-varying Rayleigh fading channel is proposed and analyzed. The detector employs fractionally sampled correlators' outputs at time instants corresponding to users' relative delays to simultaneously achieve two goals: the novel realization of a one-shot decorrelator with lower computational complexity; and to exploit a form of the time diversity for improved error performance compared to symbol spaced sampling. The decision statistics are formed from the decorrelator output according to the maximal ratio combining rule. The proposed detector performance is compared to that of a conventional symbol-spaced receiver in a single-user environment, and the impact of the channel estimation error is illustrated.

1. Introduction

Because present second-generation, conventional CDMA systems offer a rather limited range of services and information rates, the appeal of the CDMA concept for third-generation systems that can carry high-rate integrated traffic has been enhanced by research results in multiuser detection. A large body of literature on multiuser detectors in the AWGN channel exists, a rather comprehensive survey of which can be found in [1]. Realistic wireless multiple access channels exhibit multipath fading, which is an additional cause of the detrimental near-far effect. Multiuser interference in fading channels can significantly degrade the system performance, thereby imposing serious demands on receiver design. The performance limitations of receivers in fading channels are due to inadequate receiver design for such an environment, rather than the channel itself. Multiuser detection, by alleviating the near-far problem, provides means for improving system performance in mobile channels. An overview on multiuser design for fading channels is provided in [2].

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Coherent multiuser detector for an asynchronous CDMA flat Rayleigh channel was proposed in [3]. An adaptive multiuser detector that considers the effect of channel estimation error was analyzed in [4], and in [5], where Kalman filter was used to estimate the fading channel parameters necessary for coherent detection. Carrier recovery in single-user transmission through channel quadratic amplitude estimation was proposed in [6], and optimal channel estimation was given in [7]. For the Markov fading channel model, Kalman filter was shown to be an optimal channel estimator [7]. When the channel Doppler effect can not be neglected, even with optimal channel estimation, an error floor for coherent detection occurs, becoming more pronounced as the fading rate increases.

This paper proposes and analyzes a coherent multiuser detector for an asynchronous CDMA single-path time varying Rayleigh fading channel. The detector utilizes fractionally sampled correlators' outputs for the design of a decorrelator with lower complexity. Additionally, fractional samples are utilized for maximal ratio combining to provide effective diversity for improved performance. Effect of imperfect channel estimates on the error performance is analyzed for two different fading channel models.

2. Detector

In the K-user flat Rayleigh fading CDMA channel the received baseband equivalent complex signal $r(t)$ is expressed as

$$r(t) = \sum_{k=1}^{K} \sum_{i} b_k(i)\sqrt{a_k} c_k(t) s_k(t - iT - \tau_k) + n_w(t), \quad (1)$$

where $b_k(i) \in \{-1, 1\}$, a_k, τ_k, and $n_w(t)$ denote the information bit of duration T, the bit energy, user's relative delay, and an additive, zero-mean, complex white Gaussian noise with the one-sided power spectral density N_0, respectively. The channel gain $c_k(t)$ is modeled as an independent, zero-mean complex-valued wide-sense stationary

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Gaussian process with the normalized spaced-time correlation function $\Phi_k(\Delta t)$ defined as

$$\Phi_k(\Delta t) = E\{c_k(t)c_k^*(t+\Delta t)\}, \quad k = 1, \ldots, K.$$

The channel for each user is assumed to fade independently and at the same rate.

The front end of the proposed detector has a bank of K correlators each correlating the received signal $r(t)$ with the corresponding unit energy, rectangular signature waveform $s_k(t)$ of duration T. Fractionally sampled correlators outputs, with relative spacing of the sampling instants coinciding with the users' relative delays, are used for the realization of the decorrelator. Without loss of generality, we assume that $0 = \tau_1 < \tau_2 < \ldots < \tau_K < \tau_{K+1} = T$, and we focus on bit i of user 1 that we, as shown in Fig. 1, position in the interval $[(i-1)T, iT]$. We can view each block of time $[(i-1)T+\tau_j, (i-1)T+\tau_{j+1}]$ $j = 1, \ldots, K$ as a K-user synchronous channel with unit-energy waveforms $\overline{s}_k^{(bj)}(t) = s_k^{(bj)}(t)/\sqrt{\epsilon^{(bj)}}$ $k, j = 1, \ldots, K$, where[1]

$$s_k^{(bj)}(t) = \begin{cases} s_k(t + \beta_{kj}T - \tau_k) & \tau_j < t < \tau_{j+1} \\ 0 & \text{otherwise,} \end{cases}$$

$$\beta_{kj} = \begin{cases} 1 & j < k \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\begin{aligned} \epsilon^{(bj)} &= \int_{\tau_j}^{\tau_{j+1}} s_k^2(t + \beta_{kj}T - \tau_k)dt \\ &= \int_0^T \left[s_k^{(bj)}(t)\right]^2 dt, \quad k = 1, \ldots, K. \end{aligned}$$

In all notations the superscript $^{(bj)}$ denotes block j while the subscript usually refers to a particular user.

By introducing matrix $\boldsymbol{P}^{(bj)}$, whose $(m,l)^{th}$ element, $p_{ml}^{(bj)}$, is defined as

$$p_{ml}^{(bj)} = \frac{1}{\epsilon^{(bj)}} \int_0^T s_m^{(bj)}(t)s_l^{(bj)}(t)dt \qquad m, l \in (1, \ldots, K),$$

the fractionally sampled correlator output for the j^{th} block in the i^{th} bit interval is[2]

$$\boldsymbol{x}^{(bj)} = \sqrt{\epsilon^{(bj)}}\boldsymbol{P}^{(bj)}\boldsymbol{A}\boldsymbol{C}^{(bj)}\boldsymbol{b}^{(bj)} + \boldsymbol{n}^{(bj)}, \qquad (2)$$

where

$$\begin{aligned} \boldsymbol{x}^{(bj)} &= [x_1^{(bj)}, x_2^{(bj)}, \ldots, x_K^{(bj)}]^T, \\ \boldsymbol{b}^{(bj)} &= [b_1(i), \ldots, b_j(i), b_{j+1}(i-1), \ldots, b_K(i-1)]^T, \\ \boldsymbol{A} &= \mathrm{diag}[\sqrt{a_1}, \ldots, \sqrt{a_K}]. \end{aligned}$$

(To simplify notations, the time index is omitted whenever possible). The additive noise vector in each block for all users can be written as

$$\boldsymbol{n}^{(bj)} = [n_1^{(bj)}, \ldots, n_K^{(bj)}]^T,$$

and has the covariance matrix

$$E\{\boldsymbol{n}^{(bj)}\boldsymbol{n}^{(bj)H}\} = N_0\boldsymbol{P}^{(bj)},$$

where ^{T} and ^{H} denote transpose and Hermitian transpose, respectively. The matrix of the channel fading coefficients in (2) is

$$\begin{aligned} &\boldsymbol{C}^{(bj)} \\ &= \mathrm{diag}[c_1((i-1)T + \tau_{j+1}), \ldots, c_K((i-1)T + \tau_{j+1})] \\ &= \mathrm{diag}[c_1^{(bj)}, \ldots, c_K^{(bj)}]. \end{aligned}$$

After performing a decorrelating operation on K equivalent synchronous users in the j^{th} block, the resulting multiuser interference-free vector is

$$\begin{aligned} \boldsymbol{z}^{(bj)} &= \left(\boldsymbol{P}^{(bj)}\right)^{-1}\boldsymbol{x}^{(bj)} \\ &= \sqrt{\epsilon^{(bj)}}\boldsymbol{A}\boldsymbol{C}^{(bj)}\boldsymbol{b}^{(bj)} + \boldsymbol{\xi}^{(bj)}, \end{aligned} \qquad (3)$$

where

$$\begin{aligned} \boldsymbol{z}^{(bj)} &= [z_1^{(bj)}, \ldots, z_K^{(bj)}]^T, \\ \boldsymbol{\xi}^{(bj)} &= [\xi_1^{(bj)}, \ldots, \xi_K^{(bj)}]^T \\ &= \left(\boldsymbol{P}^{(bj)}\right)^{-1}\boldsymbol{n}^{(bj)}, \end{aligned} \qquad (4)$$

and the covariance matrix of $\boldsymbol{\xi}$ is

$$E\{\boldsymbol{\xi}^{(bj)}\boldsymbol{\xi}^{(bj)H}\} = N_0\left(\boldsymbol{P}^{(bj)}\right)^{-1}.$$

The K-sample snapshot of decorrelated outputs for the desired user[3] over one bit interval can then be written in vector form as

$$\begin{aligned} \boldsymbol{z}_1 &= [z_1^{(b1)}(i), \ldots, z_1^{(bK)}(i)]^T \\ &= \sqrt{a_1}\boldsymbol{C}_1 b_1 + \boldsymbol{\xi}_1, \end{aligned} \qquad (5)$$

where

$$\begin{aligned} \boldsymbol{C}_1 &= [\sqrt{\epsilon^{(b1)}}c_1^{(b1)}(i), \ldots, \sqrt{\epsilon^{(bK)}}c_1^{(bK)}(i)]^T, \\ \boldsymbol{\xi}_1 &= [\xi_1^{(b1)}(i), \ldots, \xi_1^{(bK)}(i)]^T. \end{aligned}$$

The covariance matrix of $\boldsymbol{\xi}_1$ is

$$\boldsymbol{R}_{\xi_1} = E\{\boldsymbol{\xi}_1\boldsymbol{\xi}_1^H\} = N_0\boldsymbol{D}_1, \qquad (6)$$

where matrix $\boldsymbol{D}_1$ accounts for noise enhancement in each block over one bit interval of user 1, and can be obtained as

$$\begin{aligned} \boldsymbol{D}_1 &= \mathrm{diag}[D_1^{(b1)}, \ldots, D_1^{(bK)}] \\ &= \mathrm{diag}[(\boldsymbol{P}^{(b1)})_{(1,1)}^{-1} \cdots, (\boldsymbol{P}^{(bK)})_{(1,1)}^{-1}], \end{aligned}$$

[1] Because $s_k^{(bj)}(t)$, $\epsilon^{(bj)}$, and $p_{ml}^{(bj)}$ are bit index invariant, for simplicity $i = 1$ was chosen in their definitions below.

[2] In the discrete-time channel formulation adopted here, it is assumed that the channel gain is constant during each block interval but varies from block to block.

[3] The decorrelated outputs can be obtained for all K users, but performance analysis of user 1, without loss of generality, will only be considered.

in which $(P^{(bj)})^{-1}_{(1,1)}$, $j = 1, \ldots, K$ stands for the $(1,1)^{st}$ element of matrix $(P^{(bj)})^{-1}$. Defining

$$\begin{aligned} y &= (D_1^{-1})^{1/2} z_1 \\ &= \sqrt{a_1}(D_1^{-1})^{1/2} C_1 b_1 + (D_1^{-1})^{1/2} \xi_1 \\ &= \sqrt{a_1} w b_1 + \zeta, \end{aligned} \tag{7}$$

where w stands for the channel vector adjusted by noise enhancement due to decorrelation in one bit interval of user 1, it is easy to show that

$$E\{\zeta\zeta^H\} = N_0 I, \tag{8}$$

where I is the $K \times K$ identity matrix. The noise components in (7) are mutually independent, and assuming the availability of $\tilde{w}$, the estimates of w, the detection can be done by using the maximum ratio combining (MRC) on the elements of y [8] [9]. The decision variable can then be written as

$$d_1 = Re\{\tilde{w}^H y\}. \tag{9}$$

3. Error Performance Analysis

The analysis is similar to that of the quadratic receivers [10]. Introducing a $2K \times 1$ vector

$$v = \begin{bmatrix} \tilde{w} \\ y \end{bmatrix},$$

the decision variable in (9) can be written as

$$\begin{aligned} d_1 &= \begin{bmatrix} \tilde{w} \\ y \end{bmatrix}^H \begin{bmatrix} \mathbf{o} & 0.5I \\ 0.5I & \mathbf{o} \end{bmatrix} \begin{bmatrix} \tilde{w} \\ y \end{bmatrix} \\ &= v^H Q v, \end{aligned} \tag{10}$$

where $\mathbf{o}$ is a $K \times K$ matrix with all elements equal to zero. The $2K \times 2K$ covariance matrix of v can be obtained as

$$\begin{aligned} V &= E\{vv^H\} \\ &= E\left\{ \begin{bmatrix} \tilde{w} \\ y \end{bmatrix} [\tilde{w}^H y^H] \right\} \\ &= E\left\{ \begin{matrix} \tilde{w}\tilde{w}^H & \tilde{w}y^H \\ y\tilde{w}^H & yy^H \end{matrix} \right\}, \end{aligned} \tag{11}$$

and the $2K \times 2K$ matrix VQ can be written as

$$VQ = 0.5E\left\{ \begin{matrix} \tilde{w}y^H & \tilde{w}\tilde{w}^H \\ yy^H & y\tilde{w}^H \end{matrix} \right\}.$$

In order to evaluate the impact of the channel mismatch, we assume that $\tilde{w}$, a minimum mean square error estimate of vector w, is available and that it can be expressed as[4]

$$E\{\tilde{w}\tilde{w}^H\} = E\{\tilde{w}w^H\} = E\{w\tilde{w}^H\} = R_w - G,$$

[4] This assumption can be justified by our recent results on a maximum a priori (MAP) channel estimator for a fractionally spaced coherent detector in a time-varying fading channel [11].

where all elements of G are positive. VQ can now be expressed as

$$VQ = 0.5 \begin{bmatrix} \sqrt{a_1}(R_w - G) & R_w - G \\ a_1 R_w + N_0 I & \sqrt{a_1}(R_w - G) \end{bmatrix}. \tag{12}$$

The probability of error of the quadratic decision variable is the one given in [10]

$$P_e = P\{d_1 < 0\} = \sum_{\gamma_l < 0} \prod_{\substack{m=1 \\ m \neq l}}^{2K} \frac{\gamma_l}{\gamma_l - \gamma_m}, \tag{13}$$

in which $\gamma_1, \ldots, \gamma_{2K}$ are the eigenvalues of the matrix VQ.

4. Numerical Examples and Disscussion

The piece-wise constant representation of the fading process in (3) exhibits nonuniformity along the time axis. For convenience, in our numerical examples the blocks of time were chosen as integer multiples of the chip interval. A three-user scenario was considered with signature waveforms chosen from Gold sequences of length 31. In the examples presented the relative delays were set at $\tau_1 = 0$, $\tau_2 = \frac{10}{31}T$, and $\tau_3 = \frac{22}{31}T$. With the previous assumption of $E\{|c_k|^2\} = 1$, the signal to noise ratio for user 1 is defined as $SNR_1 = a_1/N_0$.

The error performance of the proposed detector is compared to that of the conventional single-user PSK receiver in a single-user environment that utilizes *one sample per symbol*. Fig. 2 provides the performance comparison for the perfectly estimated channel modeled by the first-order Markov process with

$$\Phi_k(\Delta t) = e^{-2\pi f_d T |\Delta t / T|}, \tag{14}$$

where f_d represents the Doppler shift. The fading rates $f_d T = 0.04$ and $f_d T = 0.08$ were chosen, and for both fading rates the error probability of the proposed detector is below that of the conventional single-user PSK receiver. In Fig. 3 the Jakes model with the correlation function

$$\Phi_k(\Delta t) = J_0(2\pi f_d \Delta t), \tag{15}$$

and $f_d T = 0.08$ and $f_d T = 0.16$ was employed, and with perfect channel estimates similar performance improvement is observed. As expected, in both models the error performance, under the assumption of perfect channel estimates, improves with the increased fading rate. The performance improvement of the proposed detector is due to the utilization of a form of the time diversity through multiple samples within the same information bit interval. By analyzing the eigenvalues of $R_w = E\{ww^H\}$, we found that when fading becomes extremely slow, all but one eigenvalue become zero, so the proposed detector performance is the same as that of the conventional single user detector with the SNR that includes the effect of the noise enhancement induced by decorrelation.

Higher fading rates, however, make accurate estimation of the fading coefficients more difficult. Figures 4 and 5 show the effect of the channel estimation error on the performance for both the proposed detector and for the single user transmission. The numerical values for the relative estimation error, defined as $\boldsymbol{G}_{(m,n)}/\boldsymbol{R}_{w(m,n)}$, $m, n = 1, \ldots, K$, where subscript (m, n) stands for $(m, n)^{th}$ element of the corresponding matrix, were chosen to be 1% and 3%. The channel estimation error affects the error performance of both symbol-spaced single user transmission and the proposed detector. Even with a small relative estimation error variance, we can observe the leveling trend of the error curves. But the performance of the proposed detector is still better than that of the symbol-spaced single user transmission.

Finally, it should be noted that with the proposed detector, fractionally sampled correlators' outputs of the received multiuser signal are utilized for a dual benefit. Besides providing an effective diversity, fractional sampling also provides the means for a novel realization of the decorrelator, which requires inversion of $K \times K$ matrices only. The number of samples per symbol is implied by the number of users. In the case when the relative delays are very close to each other, there is a possibility that the correlation matrix for such a short block will become singular. Such a block can be excluded from consideration, for the remaining $K-1$ blocks carrying the same information will be sufficient for performing a proper detection.

5. Conclusion

A coherent decorrelating type of multiuser detector for frequency nonselective time-varying Rayleigh fading asynchronous CDMA channel that employs fractionally sampled correlators' outputs is proposed. This detector needs a priori knowledge of spaced-time correlation function of the fading channel process and all users relative delays. Performance under perfect channel estimates is analyzed, and impact of channel mismatch is given. Because of the channel's time-varying nature, a form of the time diversity is utilized for the improved error performance. Additionally, this detector provides a real time processing and much simpler decorrelator structure when compared to the full length decorrelator. To perform a K-user detection, K inversions of $K \times K$ matrices are needed, which can be done in parallel.

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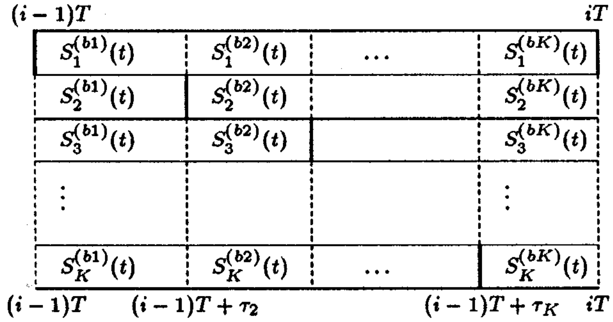

Figure 1: Partition of the i^{th} bit interval of user 1 in a K-user asynchronous system into K blocks according to relative delays of each user

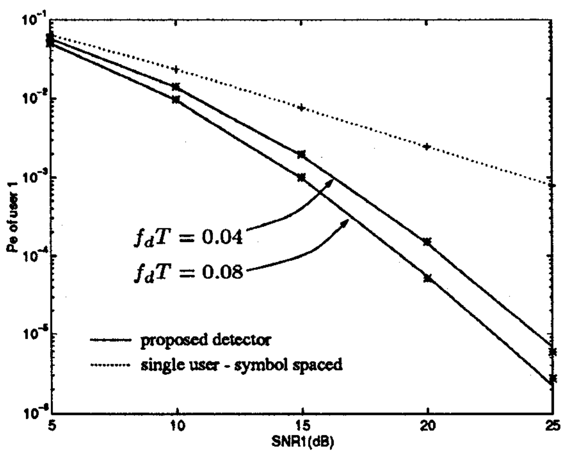

Figure 2: Analytical and simulated (∗ ∗ ∗) error performance with first-order Markov model and perfect channel estimates

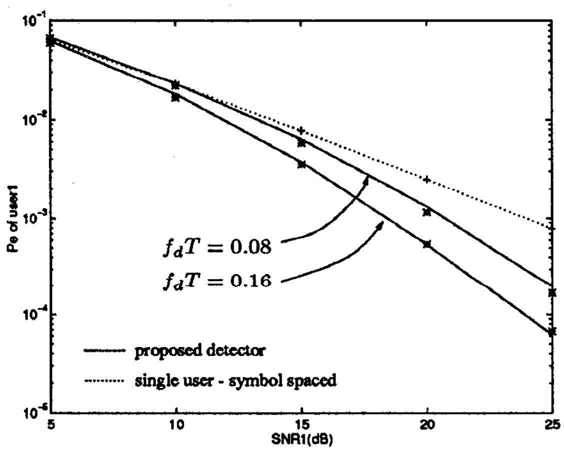

Figure 3: Analytical and simulated (∗ ∗ ∗) error performance with Jakes model and perfect channel estimates

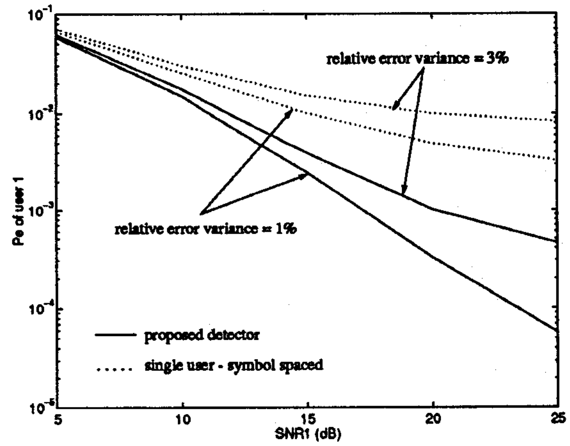

Figure 4: Analytical error performance with first-order Markov model in the presence of channel estimation error ($f_dT = 0.04$)

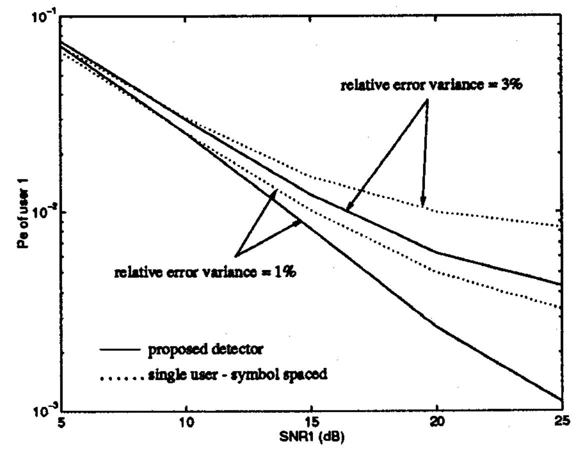

Figure 5: Analytical error performance with Jakes model in the presence of channel estimation error ($f_dT = 0.08$)