PERFORMANCE ANALYSIS AND REALIZATION OF DECISION FUSION FOR MACROSCOPIC DIVERSITY IN CELLULAR WIRELESS SYSTEMS

Jian-Guo Chen

Bell Labs, Lucent Technologies
600 Mountain Ave
Murray Hill, NJ 07974, USA
jianguo11@hdtv.lucent.com

Nirwan Ansari and Zoran Siveski

Center for Communications & Signal Processing
Department of ECE
New Jersey Institute of Technology
Newark, NJ 07102, USA
ang@njit.edu

ABSTRACT

This paper analyzes the performance of the adaptive fusion method for macroscopic diversity combination in the wireless cellular environment when the error probability information from each base station detection is not available. The performance analysis includes the derivation of the minimum achievable error probability. An alternative realization with lower complexity of the optimal fusion scheme by using selection diversity is also proposed. The selection of the information bit in this realization is obtained either from the most reliable base station or through the majority rule from the participating base stations. The performance comparison in Rayleigh fading and log-normal shadowing shows that this method has much better performance over conventional macroscopic selection diversity.

1. INTRODUCTION

Spatial diversity is used to combat fading and shadowing effects in wireless cellular communications. Usually, microscopic spatial diversity is employed to reduce the fading effect by combining signals from different receiver elements of the same base station. Since much larger spatial separation is required to achieve shadowing decorrelation, macroscopic spatial diversity, which is implemented among different base station sites or ports, has been suggested to mitigate the shadowing effect [1]. Several possible combination rules have been proposed to achieve micro diversity [2], such as maximal ratio combining, equal gain combining, and selection diversity. In selection diversity, only the most reliable one is chosen among all the received signals, and all the others are simply ignored. Compared with other combination rules, selection diversity has poor performance, relatively low complexity and bandwidth requirement. Macroscopic diversity is, however, usually realized by selection diversity, because large separation of received signals increases the difficulty of bringing them together for better performance combination. We have proposed an optimal fusion scheme for macroscopic diversity combination based on the minimum error probability criterion for binary signals [3], [4]. There, fusion scheme was shown to have better performance than selection diversity. When the error probability of the local detection in each base station is not available, an adaptive fusion algorithm was also proposed to handle the cellular CDMA handoff problem [5]. No attempt, however, has been made to analyze the performance of the adaptive fusion algorithm. The key contributions of this paper include: 1) the derivation of several properties of the adaptive fusion algorithm such as the minimum error probability that can be achieved by the algorithm, 2) a simplified realization of the optimal fusion scheme by using selection diversity, referred to as the "improved macro selection diversity rule," that has lower complexity and less bandwidth requirement than the direct realization, and 3) performance comparison of the proposed fusion scheme with the conventional macroscopic selection diversity in an environment in which both the Rayleigh fading and log-normal shadowing effects are considered. The above contributions will be described in Section 2, 3 and 4, respectively. Concluding remarks are summarized in Section 5.

This work was supported in part by AT&T/Lucent Technologies Foundation through a special purpose grant.

2. PERFORMANCE ANALYSIS OF THE ADAPTIVE FUSION ALGORITHM

The cell geometry shown in Figure 1 is the same as in [5]. A simple sectored antenna is employed at each site with each antenna sector covering 120° azimuth. The detection is performed at each base station. The detection result is sent through a separate link to a fusion (or switching) center which, as symbolically shown in Figure 1, is shared by three base stations. The final detection is made at the fusion center by optimal fusion [6] based on the detected results from the three base stations covering the same area. Let $U = [u_1, u_2, u_3]$ be the vector of detected bits for the desired user. Here, $u_i \in \{1, -1\}$, $i = 1, 2, 3$, is the local decision made by the ith base station. Synchronization among the base stations is assumed, and thus, u_i for $i = 1, 2, 3$ corresponds to the same information bit transmitted. The final detection result at the fusion center for the same information bit, denoted by u_f, is a function of local decisions. The determination of u_f can be viewed as a two-hypothesis detection problem with individual local decisions being the observations, and the two hypotheses

H_1: The symbol $+1$ is transmitted,
H_0: The symbol -1 is transmitted.

When the minimum probability of error criterion is used, for the binary symmetric channel[1] (BSC) and equiprobable source bits, we have [3]

$$u_f = f(u_1, u_2, u_3) = \begin{cases} +1, & \text{if} \quad \lambda = \sum_{i=1}^{3} a_i u_i > 0, \\ -1, & \text{otherwise,} \end{cases} \tag{1}$$

where the optimal weights are:

$$a_1 = \log \frac{1-P_1}{P_1}, \quad a_2 = \log \frac{1-P_2}{P_2}, \quad a_3 = \log \frac{1-P_3}{P_3}, \tag{2}$$

and P_i, $i = 1, 2, 3$, is the error probability for each base station for the same user. The estimated weights obtained by the adaptive fusion algorithm [3] are:

$$b_1 = \log \frac{1-q_1}{q_1}, \quad b_2 = \log \frac{1-q_2}{q_2}, \quad b_3 = \log \frac{1-q_3}{q_3},$$

where q_i is the estimation of P_i. The errors between the estimated and optimal weights are:

$$\varepsilon_1 = b_1 - a_1, \quad \varepsilon_2 = b_2 - a_2, \quad \varepsilon_3 = b_3 - a_3. \tag{3}$$

According to [7], the minimal weights errors that can be achieved are:

$$\begin{aligned} \varepsilon_1 &= \log \frac{1 + \frac{P_1P_2P_3}{(1-P_1)(1-P_2)(1-P_3)}}{1 + \frac{(1-P_1)P_2P_3}{P_1(1-P_2)(1-P_3)}} = \log \frac{1+x}{1+x\frac{(1-P_1)^2}{P_1^2}}, \\ \varepsilon_2 &= \log \frac{1 + \frac{P_1P_2P_3}{(1-P_1)(1-P_2)(1-P_3)}}{1 + \frac{(1-P_2)P_1P_3}{P_2(1-P_1)(1-P_3)}} = \log \frac{1+x}{1+x\frac{(1-P_2)^2}{P_2^2}}, \\ \varepsilon_3 &= \log \frac{1 + \frac{P_1P_2P_3}{(1-P_1)(1-P_2)(1-P_3)}}{1 + \frac{(1-P_3)P_1P_2}{P_3(1-P_1)(1-P_2)}} = \log \frac{1+x}{1+x\frac{(1-P_3)^2}{P_3^2}}, \end{aligned} \tag{4}$$

where $x = \frac{P_1P_2P_3}{(1-P1)(1-P2)(1-P3)}$. The following propositions leading to the derivation of the minimum error probability show the relationship between the optimal and estimated weights.

Proposition 1

If $P_1 < 0.5$, $P_2 < 0.5$, and $P_3 < 0.5$, then $a_1 > a_2 > a_3 \Longrightarrow b_1 > b_2 > b_3$.
The proof is given in Appendix A.

Proposition 2

If $P_1 < 0.5$, $P_2 < 0.5$, and $P_3 < 0.5$, then $b_1 < b_2 + b_3, b_2 < b_1 + b_3$, and $b_3 < b_1 + b_2$.
The proof is given in Appendix B.

Proposition 3

The minimum error probability using the adaptive fusion algorithm is

$$P_m = P_1P_2 + P_1P_3 + P_2P_3 - 2P_1P_2P_3.$$

[1]Implicit assumption is made that full interleaving is used.

Proof:

From Proposition 2, $b_2 + b_3 > b_1$, $b_1 + b_3 > b_2$, and $b_1 + b_2 > b_3$, implies that the vectors which make $\lambda > 0$ are [1,1,1], [1, 1 −1], [1, −1, 1], and [−1, 1,1]. Thus, the error probability when the adaptive fusion scheme is used and u_i, $i = 1, 2, 3$ is independent will be :

$$\begin{aligned} & P(\lambda > 0|H_0) = \\ & P(u_1 = +1, u_2 = +1, u_3 = +1|H_0) + \\ & P(u_1 = -1, u_2 = +1, u_3 = +1|H_0) + \\ & P(u_1 = +1, u_2 = -1, u_3 = +1|H_0) + \\ & P(u_1 = +1, u_2 = +1, u_3 = -1|H_0) \\ = \; & P(u_1 = +1|H_0)P(u_2 = +1|H_0)P(u_3 = +1|H_0) + \\ & P(u_1 = -1|H_0)P(u_2 = +1|H_0)P(u_3 = +1|H_0) + \\ & P(u_1 = +1|H_0)P(u_2 = -1|H_0)P(u_3 = +1|H_0) + \\ & P(u_1 = +1|H_0)P(u_2 = +1|H_0)P(u_3 = -1|H_0) \\ = \; & P_1P_2 + P_1P_3 + P_2P_3 - 2P_1P_2P_3 \\ = \; & P_m. \end{aligned} \tag{5}$$

3. A REALIZATION OF THE OPTIMAL FUSION SCHEME FOR MACRO DIVERSITY

In current macroscopic diversity systems, when selection diversity is used to form the final decision about the information transmitted, the achievable bit error rate (BER) of the final decision, P_f, is

$$P_f = \min\{P_1, P_2, P_3\} \tag{6}$$

where P_i, $i = 1, 2, 3$ as defined previously, is the BER of the ith base station for the same information bit. When the optimal fusion scheme (Eq. 1) is used for macroscopic diversity combination, it has been proved [3] that

$$P_f = \min\{P_1, P_2, P_3, P_m\}, \tag{7}$$

where $P_m = P_1P_2+P_1P_3+P_2P_3-2P_1P_2P_3$ as in Proposition 3. It has been shown that the fusion scheme has better performance than selection diversity when only shadowing is considered [5]. Whenever P_m is less than $\min\{P_1, P_2, P_3\}$, especially when differences between P_1, P_2 and P_3 are small, P_f in Eq. (7) is less than the P_f in Eq. (6). The drawback of the fusion method is its higher complexity compared to the selection diversity. All of the u_1, u_2 and u_3 have to be transmitted to a switching or fusion center where the optimal combination based on P_1, P_2 and P_3 is performed according to Eq. (1). In this paper, based on the analysis resulting in Eq. (7), we propose a simplified realization of the optimal fusion scheme that has lower complexity. In this realization, the final detection of a transmitted information bit, u_f, will be an element selected from the binary data set $D = \{u_1, u_2, u_3, \text{Maj}(u_1, u_2, u_3)\}$ with the smallest error probability, where Maj(.) stands for the majority operator defined by

$$\text{Maj}(u_1, u_2, u_3) = \begin{cases} +1 & \text{if } u_1 + u_2 + u_3 > 0, \\ -1 & \text{if } u_1 + u_2 + u_3 < 0. \end{cases}$$

When u_i's are mutually independent with respective BERs P_i for $i = 1, 2, 3$, the BER for this majority operator is :

$$\begin{aligned} & P(\text{Maj}(u_1, u_2, u_3) = +1 | H_0) \\ = \; & P(u_1 = +1, u_2 = +1, u_3 = +1 | H_0) + \\ & P(u_1 = -1, u_2 = +1, u_3 = +1 | H_0) + \\ & P(u_1 = +1, u_2 = -1, u_3 = +1 | H_0) + \\ & P(u_1 = +1, u_2 = +1, u_3 = -1 | H_0) \\ = \; & P_1P_2 + P_1P_3 + P_2P_3 - 2P_1P_2P_3 \\ = \; & P_m. \end{aligned} \quad (8)$$

The above equation implies that the majority operator yields a BER P_m. Therefore, the above realization implements the optimal fusion rule. The realization is much easier than the direct realization according to Eq. (1), because only selection and the majority operator are required. The majority operator for macroscopic diversity has been proposed in [8]. Another advantage of this realization is that the entire $U = \{u_1, u_2, u_3\}$ does not always have to be sent to the switching center. Only when $P_m < \min\{P_1, P_2, P_3\}$, all three elements of U are required at the switching center for the majority operation. Otherwise, only the element u_i with the smallest BER is transmitted to the switching center.

4. PERFORMANCE COMPARISON

In [5], we compared the performance of using the fusion scheme with the selection diversity when only shadowing distortion is considered. Here, both the shadowing and fading effects are taken into consideration. The flat Rayleigh fading and shadowing effect modeled by log-normal distribution are assumed. The error probability is considered as the performance index. In addition, as in a practical system, we assume that the maximal ratio combiner is used for the microscopic diversity to combat the fading distortion. According to [9], the instantaneous received power at the output of the L-branch micro combiner is a chi-square distributed random variable with 2L degrees of freedom. The conditional error probability for the fixed local mean received power will be the instantaneous bit error probability averaged over fading channel statistics, which can be written as follows

$$P_i = \left[\frac{1}{2}(1-\mu_i)\right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[\frac{1}{2}(1+\mu_i)\right]^k, \quad (9)$$

where, by definition,

$$\mu_i = \sqrt{\frac{\gamma_i}{1+\gamma_i}},$$

and γ_i is the averaged signal to noise ratio (SNR) over fading statistics for the ith base station (local mean of SNR). When the shadowing effect is considered, the local mean of the received power is a log-normal distributed random variable. When the power spectrum density of thermal noise and interference are assumed to be a constant, the local mean of the SNR is also log-normal distributed. Thus the area-mean BER, when *no macroscopic diversity* is employed, equals to

$$P_f = \int P(\gamma) f(\gamma) d\gamma, \quad (10)$$

where $f(\gamma)$ is the probability density function of γ, the local mean of the SNR, at a base station. According to the previous discussion, $f(\gamma_1)$ is a log-normal function with a mean (determined by the distance between the mobile user and the base station, and the propagation environment), and a variance (determined by the power control scheme). When the macroscopic selection diversity is used, the area-mean BER is

$$P_f = \int\int\int \min(P_1, P_2, P_3) f(\gamma_1, \gamma_2, \gamma_3) d\gamma_1 d\gamma_2 d\gamma_3, \quad (11)$$

where $f(\gamma_1, \gamma_2, \gamma_3)$ is the joint probability density function. When the fusion based macroscopic diversity is implemented for three base stations, the area-mean BER is

$$P_f = \int\int\int \min(P_1, P_2, P_3, P_m) f(\gamma_1, \gamma_2, \gamma_3) d\gamma_1 d\gamma_2 d\gamma_3. \quad (12)$$

Since base stations are far away from each other, the random variables γ_1, γ_2 and γ_3 can be regarded as independent variables. When a mobile user is equidistant from the three base stations, the local mean SNR have the same statistical parameters, and thus

$$f(\gamma_1, \gamma_2, \gamma_3) = f(\gamma_1) f(\gamma_2) f(\gamma_3).$$

Figure 2 shows the curve of the area-mean BER versus SNR for the nonmacro diversity, selection macro diversity, and fusion based macro diversity obtained, by numerically calculating Eqs (10)-(12). The numerical results are derived for L=3, and the standard deviation of the local-mean SNR of 1.5 dB. From this Figure, it is shown that significant improvement can be achieved by the fusion based macro diversity even in the presence of both fading and shadowing.

5. CONCLUSION

The performance of the adaptive fusion algorithm for macroscopic diversity has been analyzed. The minimum error probability that can be achieved by the adaptive fusion method equals to that by the majority rule. A less complex realization of the optimal fusion scheme is also proposed. The realization is equivalent to the combination of the conventional macro selection diversity and a majority operator, and is demonstrated to outperform the conventional macroscopic selection diversity when both fading and shadowing are involved.

Appendix A: Proof of Proposition 1

$a_1 > a_2 > a_3$ implies $P_3 > P_2 > P_1$. From the condition

$$1 > 2P_3,$$

we have

$$P_2 - P_1 > 2P_3(P_2 - P_1).$$

Subtracting $P_1P_2P_3$ from both sides and rearranging the terms, we have

$$P_2(1-P_3) + (1-P_2)P_1P_3 > P_1(1-P_3) + (1-P_1)P_2P_3.$$

Dividing both sides by $(1-P_3) > 0$,

$$P_2 + \frac{(1-P_2)P_1P_3}{1-P_3} > P_1 + \frac{(1-P_1)P_2P_3}{1-P_3}.$$

Subtracting P_1P_2 from both sides and factoring out $(1-P_1)P_2$ from the left side and $(1-P_2)P_1$ from the right side, the above inequality becomes

$$(1-P_1)P_2[1 + \frac{(1-P_2)P_1P_3}{P_2(1-P_1)(1-P_3)}] > P_1(1-P_2)[1 + \frac{(1-P_1)P_2P_3}{P_1(1-P_2)(1-P_3)}]. \quad (13)$$

Since

$$P_1P_2[1 + \frac{(1-P_2)P_1P_3}{P_2(1-P_1)(1-P_3)}][1 + \frac{(1-P_1)P_2P_3}{P_1(1-P_2)(1-P_3)}] > 0, \quad (14)$$

dividing Eq. (13) by Eq. (14),

$$\frac{1-P_1}{P_1\left[1 + \frac{(1-P_1)P_2P_3}{P_1(1-P_2)(1-P_3)}\right]} > \frac{1-P_2}{P_2\left[1 + \frac{(1-P_2)P_1P_3}{P_2(1-P_1)(1-P_3)}\right]}.$$

Multiplying both sides by $1+x > 0$ and taking the logarithm on both sides,

$$\log\frac{1-P_1}{P_1} + \log\frac{1+x}{1+x\frac{(1-P_1)^2}{P_1^2}} > \log\frac{1-P_2}{P_2} + \log\frac{1+x}{1+x\frac{(1-P_2)^2}{P_2^2}}.$$

According to Eqs. (2)-(4), the above inequality implies that

$$b_1 = a_1 + \varepsilon_1 > b_2 = a_2 + \varepsilon_2.$$

$b_2 > b_3$ can be similarly proved. Therefore, when $P_1 < 0.5$, $P_2 < 0.5$, and $P_3 < 0.5$, then $a_1 > a_2 > a_3 \Longrightarrow b_1 > b_2 > b_3$.

Appendix B: Proof of Proposition 2

According to the condition

$$P_3 < \frac{1}{2},$$

we have

$$P_3^2 < (1-P_3)^2.$$

Multiplying both sides by $1-2P_2$,

$$P_3^2(1-2P_2) < (1-P_3)^2(1-2P_2),$$

and

$$P_3^2[(1-P_2)^2 - P_2^2] < (1-P_3)^2[(1-P_2)^2 - P_2^2].$$

Multiplying both sides by $P_1(1-P_1)$ and rearranging the terms,

$$(1-P_2)^2(1-P_1)P_1P_3^2 + (1-P_1)(1-P_3)^2P_1P_2^2 < P_1(1-P_1)(1-P_2)^2(1-P_3)^2 + (1-P_1)P_1P_2^2P_3^2.$$

Dividing both sides of the above inequality by $(1-P_1)(1-P_2)(1-P_3)$, and after further manipulation,

$$\left[(1-P_1)P_2 + \frac{(1-P_2)P_1P_3}{1-P_3}\right]\left[P_3 + x\frac{(1-P_3)^2}{P_3^2}\right] < [P_1(1-P_2)(1-P_3) + (1-P_1)P_2P_3][1+x].$$

Factoring out $(1-P_1)P_2P_3$ from the left side, and $P_1(1-P_2)(1-P_3)$ from the right side,

$$(1-P_1)P_2P_3[1 + x\frac{(1-P_2)^2}{P_2^2}][1 + x\frac{(1-P_3)^2}{P_3^2}] < P_1(1-P_2)(1-P_3)[1 + x\frac{(1-P_1)^2}{P_1^2}][1+x].$$

Dividing both sides of the above inequality by the following factor,

$$P_1P_2P_3[1 + x\frac{(1-P_1)^2}{P_1^2}][1 + x\frac{(1-P_2)^2}{P_2^2}][1 + x\frac{(1-P_3)^2}{P_3^2}],$$

we get

$$\frac{(1-P_1)}{P_1[1 + x\frac{(1-P_1)^2}{P_1^2}]} < \frac{(1-P_2)(1-P_3)[1+x]}{P_2P_3[1 + x\frac{(1-P_2)^2}{P_2^2}][1 + x\frac{(1-P_3)^2}{P_3^2}]}.$$

Multiplying both sides by $1+x$ and taking the logarithm operation,

$$\log\left(\frac{1-P_1}{P_1}\cdot\frac{1+x}{1+x\frac{(1-P_1)^2}{P_1^2}}\right) < \log\left(\frac{1-P_2}{P_2}\cdot\frac{1+x}{1+x\frac{(1-P_2)^2}{P_2^2}}\cdot\frac{1-P_3}{P_3}\cdot\frac{1+x}{1+\frac{(1-P_3)^2}{P_3^2}}\right).$$

Again according to Eqs. (2)-(4), the above inequality implies that

$$b_1 < b_2 + b_3.$$

Similarly, we can prove that $b_2 < b_1 + b_3$, and $b_3 < b_1 + b_2$.

1. REFERENCES

[1] W. C. Jakes, *Microwave Mobile Communication.* New York, NY: J. Wiley & Sons, 1974, reprinted by IEEE Press, 1994.

[2] D. G. Brennan, "Linear diversity combining techniques," *Proc. of IRE*, vol. 47, pp. 1075–1102, June 1959.

[3] J. Chen, N. Ansari, and Z. Siveski, "Distributed detection for cellular CDMA," *IEE Electronics Letters*, vol. 32, pp. 169–171, Feb. 1996.

[4] J. Chen, N. Ansari, and Z. Siveski, "Improving multiuser detection performance by data fusion," *Proc. of GLOBECOM'96* , (London, England), pp. 1794–1798, Nov. 1996.

[5] J. Chen, N. Ansari, and Z. Siveski, "Adaptive data fusion for handoffs in cellular CDMA," Tech. Rep. CCSPR-96-NA-1, Dept. of ECE, NJIT, Feb. 1996.

[6] Z. Chair and P. K. Varshney, "Optimal data fusion in multiple sensor detection systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 22, pp. 98–101, Jan. 1986.

[7] N. Ansari, E. Hou, B. Zhu, and J. Chen, "An adaptive fusion model for distributed detection systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 32, pp. 524–531, Apr. 1996.

[8] L. F. Chang and J. C. I. Chuang, "Diversity selection using coding in a portable radio communications channel with frequency-selective fading," *IEEE JSAC*, vol. 7, pp. 89–98, Jan. 1989.

[9] J. G. Proakis, *Digital Communications.* New York, NY: McGraw-Hill, 1995.

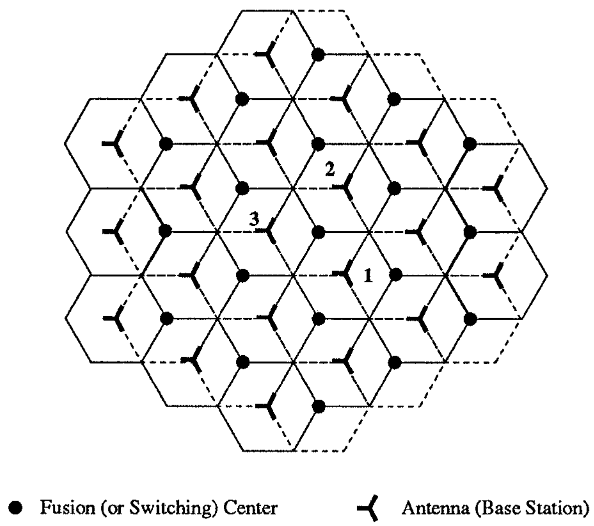

Figure 1: A fusion macroscopic diversity scheme.

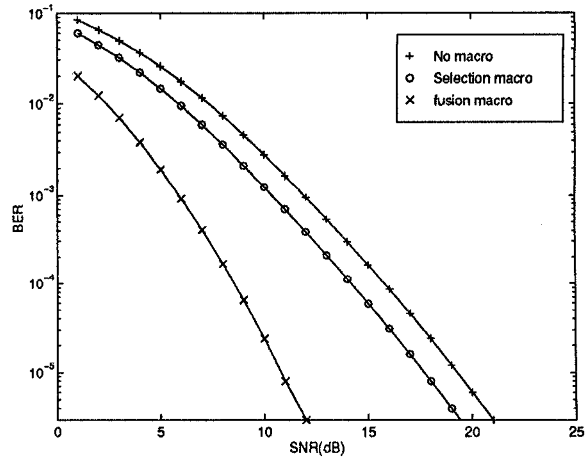

Figure 2: Performance of different macro diversity schemes.