Math 111 Pre-test(Show all work. No calculators. This will **NOT** count toward your grade.

1. Find the area of the region bounded by $y = x+3$, the x-axis, x=3 and x=5.

2. Find the equation of the line with slope 2 that intersects the x-axis where x=3.

3. The parabola $y = x^2$ has slope 6 at the point (3, 9). Thus, the tangent line to the curve at this point has slope 6. Find all points where the line perpendicular to this tangent line intersects the parabola.

4. Let $y = f(x) = \dfrac{1}{x+6}$. Find $f(1), f(-5)$, $f(2+h)$, and $\dfrac{f(2+h)-f(2)}{h}$ (simplify)

5. Let $y = f(x) = x^2+1$. Find $f(0), f(2)$, $f(3+h)$, and $\dfrac{f(3+h)-f(3)}{h}$ (simplify).

6. Consider $\dfrac{\sqrt{9+h}-3}{h}$. Rationalize the numerator.

7. Evaluate $f(3), f(1), f(-1)$ and $f(-3)$ if $f(x) = \dfrac{\sqrt{3x^2-2}}{x-2}$.

8. If $f(x) = x^2 - 1$ on $x < 3$
   $g(x) = 2ax$ on $x > 3$
   and f and g meet at x=3, find a.

9. Find the inverse of: $f(x) = \dfrac{x+2}{3x+5}$

10. Find the remainder when $x^3 - 6x^2 + 2x - 4$ is divided by x^2+2

11. Sketch the set of graphs by building up from the first and using scaling and translations: (label vertex & intercepts)
    $y = x^2$; $\quad y = 4-(x-3)^2$;
    $y = (x-3)^2$; $\quad y = \frac{1}{4}\left(4-(x-3)^2\right)$;
    $y = -(x-3)^2$; $\quad y = \frac{1}{4}\left(4-[2(x-3)]^2\right)$;

12. A circular piece of paper with a 4 inch radius has a sector with arclength x cut from it. (See below). The edges of the remaining figure are joined to make a cone. Find the circumference of the base of the cone, the radius of the cone and the height of the cone all in terms of x.

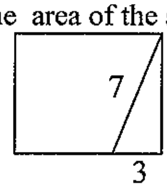
13. Find the roots of $f(x) = \dfrac{x^3-4x^2+3x}{(x-2)(x+4)^2}$ and all values of x where the function lies above the x-axis.

14. Find all values of x satisfying $|x-3| < 4$ both algebraically and graphically and explain what the region means in terms of distance.

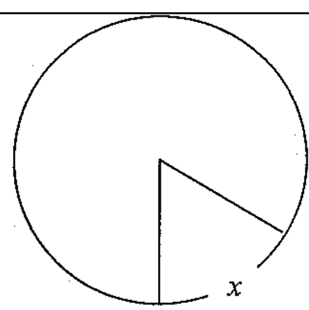
15. Sketch $y = -\sin(2x)$ on $-2\pi \le x \le 2\pi$

16. Solve for x: $2\ln(x-3) - \ln(x+15) = \ln 3$

17. Find the area of the square:

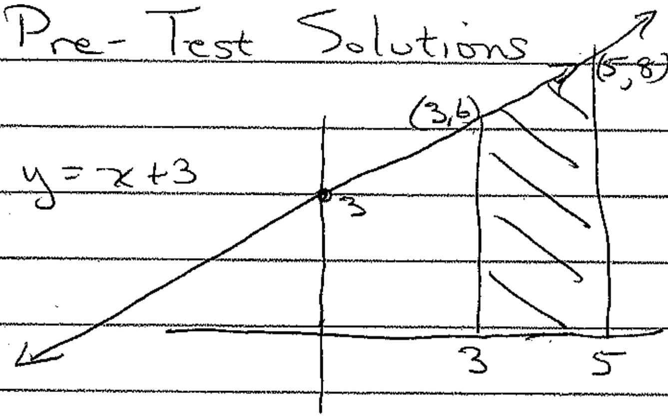


18. An open top box is constructed from a rectangular piece of cardboard with dimensions 14 in X 22 in by cutting equal squares with side x from the corners and folding the sides up. Express the volume of the resulting box in terms of x.



N

Pre-Test Solutions



1. $y = x + 3$

Area = (Trapezoid) $\frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}(5-3)(8+6)$

$= \boxed{14 \text{ sq. units}}$

(or use rectangle & triangle

$(6 \times 2) + (\frac{1}{2})(2)(2) = 12 + 2 = 14$

2. Slope $= 2$; x intercept $(3, 0)$

$y - y_1 = m(x - x_1) \Rightarrow y - 0 = 2(x-3)$

$\boxed{y = 2x - 6}$

3. Tangent line has slope $= 6$ & passes through $(3, 9)$

$\Rightarrow y - 9 = 6(x-3) \Rightarrow$ Eq of perpendicular is:

Intersections of $y = x^2$ &

$y - 9 = -\frac{1}{6}(x-3)$

$y = 9 - \frac{1}{6}x + \frac{1}{2}$

$y = \frac{19}{2} - \frac{1}{6}x$

$\Rightarrow x^2 = \frac{19}{2} - \frac{1}{6}x$

$x^2 + \frac{1}{6}x - \frac{19}{2} = 0$ (Note one answer must be $x = 3$)

$(x-3)(x + \frac{19}{6}) = 0$

$\Rightarrow x = 3, \ x = -\frac{19}{6}$ (when $x = \frac{-19}{6}$, $y = x^2 = \left(\frac{-19}{6}\right)^2$

So the intersections are at $\boxed{(3, 9) \ \& \ \left(-\frac{19}{6}, \frac{361}{36}\right)}$

4. $f(x) = \frac{1}{x+6}$

$\Rightarrow f(1) = \frac{1}{1+6} = \boxed{\frac{1}{7}}$; $f(-5) = \frac{1}{-5+6} = \frac{1}{1} = \boxed{1}$

$f(2+h) = \boxed{\frac{1}{8+h}}$; $\frac{f(2+h) - f(2)}{h} = \frac{\frac{1}{8+h} - \frac{1}{8}}{h} = \frac{8 - (8+h)}{h(8+h)8}$

$= \frac{-h}{h(8)(8+h)} = \boxed{\frac{-1}{8(8+h)}}$

5. $y = x^2+1 \Rightarrow f(0) = 0+1 = \boxed{1}$; $f(2) = 2^2+1 = \boxed{5}$

$f(3+h) = (3+h)^2+1 = 9+6h+h^2+1 = \boxed{10+6h+h^2}$

$$\frac{f(3+h)-f(3)}{h} = \frac{\cancel{10}+6h+h^2-\cancel{(3^2+1)}}{h}$$

$$= \frac{(6+h)\cancel{h}}{\cancel{h}} = \boxed{6+h}$$

6. $$\frac{\sqrt{9+h}-3}{h} = \frac{(\sqrt{9+h}-3)(\sqrt{9+h}+3)}{h(\sqrt{9+h}+3)} = \frac{9+h-9}{h(\sqrt{9+h}+3)}$$

$$= \frac{h}{h(\sqrt{9+h}+3)} = \boxed{\frac{1}{\sqrt{9+h}+3}}$$

7. $f(x) = \dfrac{\sqrt{3x^2-2}}{x-2} \Rightarrow f(3) = \dfrac{\sqrt{27-2}}{3-2} = \boxed{5}$; $f(1) = \dfrac{\sqrt{3-2}}{1-2} = \dfrac{1}{-1} = \boxed{-1}$

$f(-1) = \dfrac{\sqrt{3-2}}{-1-2} = \boxed{-\frac{1}{3}}$; $f(-3) = \dfrac{\sqrt{27-2}}{-3-2} = \dfrac{5}{-5} = \boxed{-1}$

8. $f(x) \to 3^2-1 = 8$ as $x \to 3 \Rightarrow 6a = 8 \Rightarrow a = 8/6 = \boxed{4/3}$

$g(x) \to 2a3 = 6a$

9. $f(x) = \dfrac{x+2}{3x+5}$ or $y = \dfrac{x+2}{3x+5} \Rightarrow$ inverse satisfies $x = \dfrac{y+2}{3y+5}$

$\Rightarrow 3xy+5x = y+2 \Rightarrow (3xy-y) = 2-5x$

$\Rightarrow y(3x-1) = 2-5x \Rightarrow \boxed{y = \dfrac{2-5x}{3x-1}}$ (Note $f(f^{-1}(x)) = f^{-1}(f(x)) = x$)

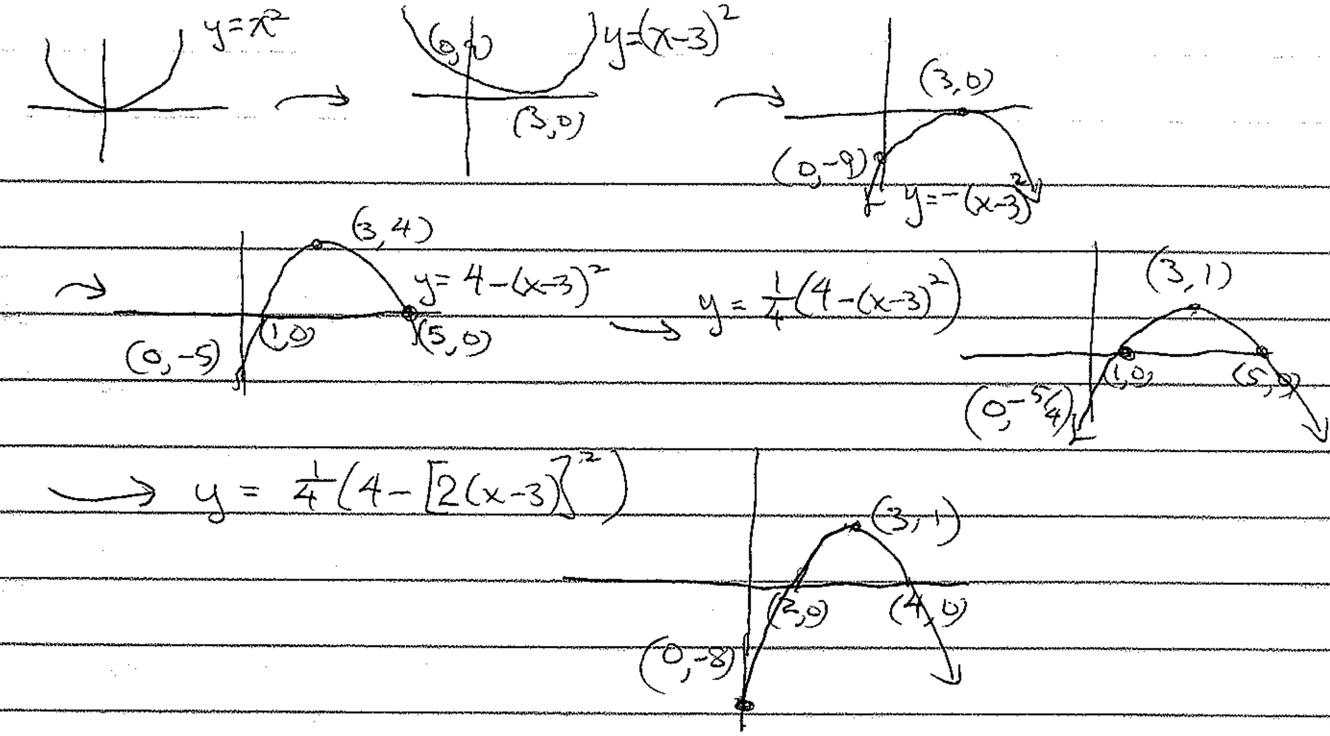
10.

$$\begin{array}{r|l} & x-6 \\ \hline x^2+2 & x^3-6x^2+2x-4 \\ & \underline{x^3 \qquad\quad +2x \qquad} \\ & -6x^2 \qquad -4 \\ & \underline{-6x^2 \qquad -12} \\ & 8 \end{array}$$

So the remainder is $\underline{\underline{8}}$

or $\dfrac{x^3-6x^2+2x-4}{x^2+2} = x-6+\dfrac{8}{x^2+2}$

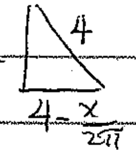
11.



12. Base of cone has circumference $2\pi r - x = 8\pi - x$

Radius of base $r = \frac{\text{Circumference}}{2\pi} = \frac{8\pi - x}{2\pi} = 4 - \frac{x}{2\pi}$

Height of cone



$h = \sqrt{4^2 - (4-\frac{x}{2\pi})^2}$

13. ~~Root~~ $\frac{x^3-4x^2+3x}{(x-2)(x+4)^2} = 0$ if $x^3-4x^2+3x=0 \Rightarrow (x-3)(x-1)x=0$

$\Rightarrow \boxed{x=0,1,3 \text{ Roots}}$

$$\begin{array}{ccccccccc} ++++ & & +++ & --- & ++ & --- & & +++ \\ & -4 & & 0 & 1 & 2 & 3 & \end{array}$$

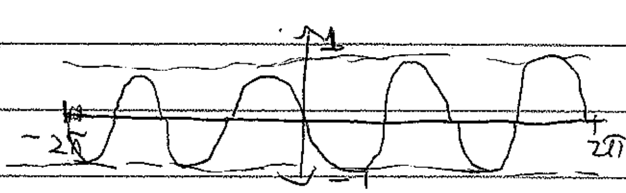
So $f(x) > 0$ for $(-\infty,-4) \cup (-4,0) \cup (1,2) \cup (3,\infty)$

14. $|x-3| < 4$ means all points within 4 units of $x=3$ (not including endpoints) $\Rightarrow -1 < x < 7$

or $x-3 < 4$ AND $x-3 > -4$

$x < 7$ AND $x > -1 \Rightarrow -1 < x < 7$ ✓

15. $y = -\sin(2x)$ on $-2\pi \le x \le 2\pi$



16. $2\ln(x-3) - \ln(x+15) = \ln 3$

$\Rightarrow \ln\left[\frac{(x-3)^2}{(x+15)}\right] = \ln 3 \Rightarrow \frac{(x-3)^2}{x+15} = 3$

$\Rightarrow x^2 - 6x + 9 = 3x + 45 \Rightarrow x^2 - 9x - 36 = 0$

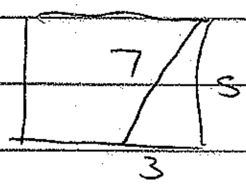
$\Rightarrow (x-12)(x+3) = 0 \Rightarrow x = 12$ or $x = -3$

Check: If $x = 12$ $\quad 2\ln(12-3) - \ln(12+15) \stackrel{?}{=} \ln 3$

$2\ln 9 - \ln 27 \stackrel{?}{=} \ln 3$

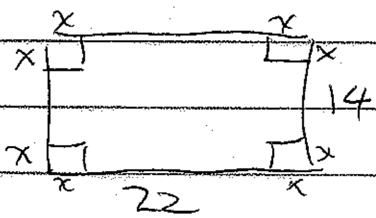
$\ln \frac{81}{27} = \ln 3 = \ln 3$ ✓

If $x = -3$ $\ln(-3-3)$ does not exist. So only $\boxed{x = 12}$ works

17. 

$s^2 = \text{Area}$

$3^2 + s^2 = 7^2 \Rightarrow s^2 = 49 - 9 = \boxed{40 = \text{Area}}$

18. 

Sides will be $(14-2x)$; $(22-2x)$ & x

$\boxed{\text{Volume} = (14-2x)(22-2x)\,x}$

cubic units.