wave curve), Riemann problems involving curved detonations can be solved. We demonstrate the method using the Forest Fire [11] rate law with HOM equation of state [12] for the explosive PBX-9404.

CURVED DETONATION JUMP CONDITIONS

We start with the Euler equations for conservation of mass, momentum and energy and an equation for reaction progress in curved geometry.

$$
\begin{aligned}
(\rho A)_t \;+\; (\rho A u)_x &= 0 \\
(\rho A u)_t \;+\; \left(\rho A\left[u^2 + PV\right]\right)_x &= PA_x \\
(\rho A \mathcal{E})_t \;+\; (\rho A u\left[\mathcal{E} + PV\right])_x &= -PA_t \\
(\rho A \lambda)_t \;+\; (\rho A u \lambda)_x &= \rho A \mathcal{R}
\end{aligned}
\tag{1}
$$

where ρ, u, E and λ are the fluid density, particle velocity, specific internal energy and mass fraction of the reaction products ($\lambda = 0$ unburnt, $\lambda = 1$ completely burnt), $V = 1/\rho$ is the specific volume, $\mathcal{E} = u^2/2 + E$ is the total specific energy, $P(V, E, \lambda)$ is the pressure, $\mathcal{R}(V, E, \lambda)$ is the specific reaction rate, and A is the cross-sectional area.

If the flow in the reaction zone is quasi-steady, the PDEs can be reduced to the following ODEs.

$$
\begin{aligned}
[\rho \mathrm{v}]_x &= \rho u \kappa \\
\left[\rho \mathrm{v}^2 + P\right]_x &= \rho \mathrm{v} u \kappa \\
\left[E + PV + \frac{\mathrm{v}^2}{2}\right]_x &= 0 \\
-\mathrm{v}\lambda_x &= \mathcal{R}
\end{aligned}
\tag{2}
$$

where $\mathrm{v} = D - u$, $\kappa = A/A_x$ is the curvature and x is the distance relative to the front. In this form, the influence of the curvature of the wave on the standard planar jump conditions is apparent. Integrating through the reaction zone yields the following set of jump conditions.

$$
\begin{aligned}
\Delta[\rho \mathrm{v}] &= \kappa w \langle \rho u \rangle \\
\Delta\left[(\rho \mathrm{v})^2 V + P\right] &= \kappa w \langle \rho \mathrm{v} u \rangle \\
\Delta\left[E + PV + \frac{\mathrm{v}^2}{2}\right] &= 0
\end{aligned}
\tag{3}
$$

where $\Delta[f] = f(x_a) - f(x_b)$ is the change of variable f across the detonation reaction zone. The subscripts a and b denote ahead of the detonation front and behind the reaction zone, respectively. The reaction zone width is $w = x_a - x_b$ and

$$
\langle f \rangle \;=\; \frac{1}{w}\int_{x_1}^{x_0} dx\, f \tag{4}
$$

is the average value of f in the reaction zone.

PARAMETRIZING THE SOURCE TERMS

In order to solve the jump conditions (3) for the Hugoniot curve, we must find explicit approximations for the reaction zone quantities $w\langle \rho u \rangle$ and $w\langle \rho \mathrm{v} u \rangle$. Thus, the ODEs in equations (2) must be solved. First the ODEs are put into the equivalent form

$$
-[c^2 - (D-u)^2]\frac{d}{dx}\begin{pmatrix} V \\ D-u \\ \lambda \\ P \end{pmatrix} =
\begin{pmatrix} [c^2\sigma\mathcal{R} - \mathrm{v}^2 u\kappa]V/\mathrm{v} \\ c^2(\sigma\mathcal{R} - u\kappa) \\ (c^2 - \mathrm{v}^2)\mathcal{R}/\mathrm{v} \\ -\rho\ \mathrm{v}\ \mathrm{v}_x \end{pmatrix} \tag{5}
$$

Here, $c^2 = \partial_\rho P|_{S,\lambda}$ is the square of the frozen sound speed, and $\sigma = (\rho c^2)^{-1}\,\partial_\lambda P|_{V,E}$ is the thermicity.

The ZND model for detonations is employed. The lead front shocks the material up to a high pressure (von Neumann spike). Planar jump conditions without reaction are used, and then the ODEs (5) are solved through the reaction zone. The state at the end of the reaction zone depends on both the detonation velocity and the front curvature, κ. (The HOM equation of state [12] for PBX-9404 with the Forest Fire reaction rate parameters [11] was used for the computations discussed below. The the Forest Fire rate is derived from distance of run to detonation experiments (Pop plots). It is computed for values of pressure below the Chapman-Jouguet (CJ) point and the rate blows up above CJ. A spline was fit to the log of the rate and extrapolated to pressures above CJ.)

The source terms in equations (3) are linear in κ. This was verified by performing computations with constant detonation velocity while varying κ. In Fig. 1, the other portion of the source terms are plotted. The values of these terms for a given detonation speed were computed for several values of κ (corresponding to both converging and diverging detonations) and averaged. The range of values for D was from slightly below CJ ($D = 0.886$ cm/μsec) up to about 8% above CJ. Detonation velocities below CJ apply only to diverging waves. The values decay exponentially over the region studied. A good fit is obtained using the functions

$$\begin{aligned} w\langle\rho u\rangle &\approx 0.000852 e^{-22.15(D-0.9123)} \\ w\langle\rho u \mathrm{v}\rangle &\approx 0.000492 e^{-22.4(D-0.9123)} \end{aligned} \quad (6)$$

SOLVING THE JUMP CONDITIONS

In this section, we describe how to find the state behind a curved detonation wave from the jump conditions (3) using the source term fits (6). Thus, we need to solve

$$\begin{aligned} \Delta[\rho \mathrm{v}] &= \kappa F_1 \\ \Delta\left[(\rho \mathrm{v})^2 V + P\right] &= \kappa F_2 \\ \Delta\left[E + PV + \frac{\mathrm{v}^2}{2}\right] &= 0 \end{aligned} \quad (7)$$

where F_1 and F_2 are the right hand sides of (6).

A range for the density behind the reaction zone for our PBX-9404 example is 2-3 gm/cm^3. Given a detonation speed and curvature, we iterate on ρ_I (I for iterate) as follows. $V_I = 1/\rho_I$. The first and second of equations (7) give

$$\begin{aligned} v_I &= (\rho_a v_a - \kappa F_1) V_I \\ P_I &= \rho_a \mathrm{v}_a^2 + P_a - \rho_I v_I^2 - \kappa F_2 \end{aligned} \quad (8)$$

and we compute

$$E(\rho_a, P_a, 0) + P_a V_a + \frac{\mathrm{v_a}^2}{2} - E(\rho_I, P_I, 1) - P_I V_I - \frac{\mathrm{v_I}^2}{2} \quad (9)$$

We iterate using the secant method until the expression (9) is zero.

A comparison of the results of solving the full ODEs and the jump conditions is presented in Fig. 2. The dots represent points on the planar Hugoniot curve; the circles represent Hugoniot points for $\kappa = 3$ computed using the full ODEs; the triangles represent Hugoniot points for $\kappa = 3$ computed using the jump conditions. The lowest dot corresponds to the lowest circle and triangle. The second lowest dot corresponds to the second lowest circle and triangle, etc. The $\kappa = 3$ and planar Hugoniots are almost indistinguishable. However, for a given detonation speed, the Hugoniot values are quite different (especially near CJ) and the jump conditions capture this difference accurately. Solving the ODEs for the HOM equation of state takes over 20 CPU seconds for a single Hugoniot point. The jump conditions can compute 100 Hugoniot points in 1 CPU second.

DISCUSSION AND CONCLUSION

We have shown how one can efficiently find the Hugoniot curve for curve detonations once the source terms are parametrized. The secant method solution can solve for a state behind the reaction zone more than 1000 times as fast as solving using the ODEs. Thus, in a computation in which Riemann problems are to be solved at each time step (such as in a Front Tracking framework), the preprocessing involved in order to find the jump correction terms (6) is well worth the effort.

Although we have used a realistic equation of state and rate law, the rate law is probably too high in the ZND reaction zone. The form of the rate law may have given rise to the exponential fit for the source terms. This may not be the case in general. In the future, we plan to use a model for burning on the way up to CJ pressure. The theory is also valid for such models.

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