REVISION
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Optimal Patterns for Suturing Wounds

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Abstract

A mathematical model for computing stresses in sutured human skin wounds is presented. The model uses the incremental law of elasticity and elastic constants valid for *in vivo* orthotropic skin. The model is applied to compute the principal stress and displacements resulting from suturing small elliptical and circular wounds in a large flat sheet of skin, in order to determine the optimal suturing patterns. It is observed that the average stress index for a circular wound sutured toward the center is almost double that of a wound sutured perpendicular to a diameter. Thus, the latter type of suturing pattern is preferable. Similarly, suturing an elliptical wound perpendicular to its major axis produces a lower average stress index than a circular wound of the same area. It is also found that the optimal ratio of semi-major to semi-minor axis of an elliptical wound is near 3 (for abdomenal wounds), i.e., this ratio produces the most uniform stresses along the wound edges, where wound healing is the lowest.

Since high stresses have adverse effects on healing and blood flow, this work, depicting regions of high stresses, may be used along with other biological factors to help predict regions of slower healing in sutured wounds.

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Key words. human skin, orthotropy, wound, suturing, stress

Introduction

Many suturing techniques have been developed for the closure of skin wounds (see Thorek and Linden, 1970). It has been found that it is preferable to suture a wound rather than close it with tape, since sutured wounds in pig skin tend to have higher breaking strength values than those that are taped closed (Ordman and Gilman, 1966). In addition, closure by free skin grafts or non-adjacent skin pedicle grafts is less satisfactory and should be considered only when adjacent tissues are not available (Morris, 1973).

When the skin is stretched during suturing, stresses are produced. Stress is one of the many biological factors that plays an important role in the wound closure and healing process. Skin tension is of particular importance in wound healing. A number of detrimental effects are associated with tissue stresses in sutured skin. For example, maximal tension across a sutured wound is likely to produce a stretched hypertrophic scar at that site (Gibson, 1990). Scars produced when the axes of the wound are placed parallel to Langer's lines are better than those for which the axes cross the lines (Cacou and Muir, 1995). Dehiscence, ischemia or necrosis may be expected in regions of high stresses through compromise of circulation in the subdermal plexus (Brian and Manuel, 1995). Animal studies show that blood flow is inversely proportional to wound closure tension (Larrabee, et al. 1984). Compressive stresses are to be avoided as far as possible, since the skin cannot support negative stresses (Danielson and Natrajan, 1975). In addition, cellular production and release of inflammatory mediators are altered by mechanical stress (Sumpio and Widmann 1990, Sumpio et al. 1987, Nollert et al. 1989) Thus, areas of high stress concentration after wound closure may contribute to an enhanced inflammatory response and retard wound healing. Tensile stresses at the wound edges that are nearly the yield stress (whose magnitude depends upon the site) are likely to be more detrimental. In view of this, it is worthwhile to determine stresses theoretically at each location for a given pattern of suturing, especially when no experimental device, invasive or non-invasive, has been developed to measure such stresses (Gibson, 1990). A preferred suturing pattern should produce low average principal stresses as far as possible.

An accurate stress analysis of the skin would take into account the facts that the skin is orthotropic (Reihsner, et al. 1995 and Tong and Fung 1976), undergoes large deformations (Danielson and Natrajan, 1975) and has a non-linear stress-strain relation (Larrabee and Galt, 1986) which depends on the initial strain state (Reihsner, et al. 1995). Taking into account all these aspects makes the mathematics intractable. Numerical solution may be attempted, but the results might not lead to insights that could be obtained from analytical solution. In order to obtain an exact analytical solution for the stress distribution near a sutured wound, we assume that the wound is small and the skin undergoes small deformations and small strains. Under these assumptions, we apply linear elasticity theory for plane stress problems in an *orthotropic material* (Lekhnitski, 1981) and use elastic parameters in an orthotropic model of skin as have been obtained experimentally (Reihsner, et al. 1995).

As early as 1931, it was known that transverse suturing (i.e., normal to the major axis of the wound) in the repair of an elliptical umbilical hernia wound is preferable to longitudinal suturing (see Fig. 804, Ashhurst 1931). However, no theoretical proof exists for this finding. Similarly, in surgical practice, it is well known that a circular wound should not be sutured towards the center, but again there is no theoretical explanation for this practice. Typically, a circular wound is given an elliptical shape by making incisions along the diameter of the circle. The resulting ellipse is then sutured in a direction normal to its major axis. In order to provide a theoretical explanation for these surgical practices, we perform a mathematical analysis in this paper to compute wound closure stresses in humans, in the *in-vivo* configuration. We are especially interested in the location and magnitude of principal stresses since these stresses affect the biological activity of the skin (Danielson and Natrajan, 1975). We hypothesize that suturing the circular wounds transversely (i.e. in the direction normal to the diameter) will produce lower tensile and compressive stresses than suturing towards the center. We also hypothesize that a sutured elliptical wound would produce lower stresses than a circular wound of the same wound area, and seek to determine the optimal ratio of semi-major to semi-minor axis.

Method

We consider a small elliptical wound with major axis length between 0.01 and 0.08m and minor axis length from 0.01 to 0.02m in the center of a much larger flat sheet of skin of small thickness as shown in Fig. 1. An elliptical wound is chosen since one can approximate a number of shapes (e.g., circle, line segment) by an ellipse. The ellipse has major axis along the x-axis and minor axis along the y-axis. The X-axis and Y-axis represent the principal directions of elasticity (also known as axes of elastic symmetry, depending on the position of collagen fibers) for the orthotropic skin material. ψ is the angle between the y-axis and the Y-axis. It is mentioned here that for an isotropic material, a method for the solution in the case of an elliptical hole subjected to an arbitrary displacement is given in Muskhelishvili (1953).

We first consider an elliptical wound sutured perpendicular to the major axis, i.e., by pulling the boundary of the hole towards the x-axis in a direction parallel to the y-axis, so that the two edges of the wound are sutured together tightly along x-axis (Fig. 1). As a result of suturing, stresses and displacements are produced throughout the plane sheet of skin. However, since the sheet of skin is large compared to the size of the wound, we assume that the stresses die out at the edges of the sheet. We evaluate these stresses and displacements and compute regions of maximum and minimum tension and extension.

Since we wish to consider the human skin in the *in-vivo* configuration, we use the elastic parameters determined experimentally for human skin in the *in-vivo* configuration (Reihsner, et al. 1995) and employ their stress strain relation. This stress-strain relation uses incremental modulii which almost characterize the non linear behavior of the skin. Incremental modulii are obtained by using the incremental law, i.e., a linearized relationship between incremental stress ($d\sigma$) and strain ($d\epsilon$) obtained by subjecting the material to a small perturbation around the equilibrium state. Incremental law is a popular approach to nonlinear elasticity (Fung, 1993). Reihsner, et al. (1995) also used the *in-vivo* configuration as the equilibrium state. They determined the elastic parameters, thickness of the dermis and collagen content at 16 different sites of the human body by taking the dermis samples from five deceased humans. The circular samples of 30mm were taken within 24 hours of death to perform the experiments.

The equation of an ellipse with center at the origin in rectangular coordinates (x,y) is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \tag{1}$$

We assume $a > b$ so that a is the semi-major axis length and b is the semi-minor axis length. (See Fig. 1). We use the stress strain relation of Reihsner, et al. (1995)

$$\begin{bmatrix} \sigma_X \\ \sigma_Y \\ \tau_{XY} \end{bmatrix} = \begin{bmatrix} E_1/(1 - \nu_1\nu_2) & E_1\nu_2/(1 - \nu_1\nu_2) & 0 \\ E_2\nu_1/(1 - \nu_1\nu_2) & E_2/(1 - \nu_1\nu_2) & 0 \\ 0 & 0 & G \end{bmatrix} \begin{bmatrix} \epsilon_X \\ \epsilon_Y \\ \epsilon_{XY} \end{bmatrix} \tag{2}$$

where σ_X and σ_Y are the normal stresses, τ_{XY} is the shear stress and $u(X,Y)$ and $v(X,Y)$ are the displacements in the X and Y directions. E_1, E_2, ν_1, ν_2 and G denote Young's moduli (along principal directions of elasticity as defined for orthotropic material), Poisson's ratios (which are subject to the constraint, $\nu_1 E_2 = \nu_2 E_1$) , and the shear modulus respectively (Halpin and Pagano, 1968). Using the *in vivo* geometry as reference, the strains $\epsilon_{ij},\ i,j = 1,2$ are defined as:

$$\epsilon_{ij} \equiv \frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right) \text{for small strains.} \tag{3}$$

The stress-strain relation (2) is valid if the coordinate axes correspond to the principal directions of elasticity of the orthotropic material. The relationship between (x, y) and (X, Y) coordinates is

$$x = X\cos\psi - Y\sin\psi \qquad y = X\sin\psi + Y\cos\psi \tag{4}$$

For plane stress problems the partial differential equations of equilibrium, in the absence of body forces, are:

$$\begin{aligned} \frac{\partial}{\partial X}\sigma_X + \frac{\partial}{\partial Y}\tau_{XY} &= 0 \\ \frac{\partial}{\partial X}\tau_{XY} + \frac{\partial}{\partial Y}\sigma_Y &= 0. \end{aligned} \tag{5}$$

Following [Lekhnitskii, 1981], σ_X, σ_Y and τ_{XY} are expressed as

$$\sigma_X = \frac{\partial^2 F(X,Y)}{\partial Y^2}, \qquad \sigma_Y = \frac{\partial^2 F(X,Y)}{\partial X^2}, \qquad \tau_{XY} = -\frac{\partial^2 F(X,Y)}{\partial X \partial Y} \tag{6}$$

Then $F(X, Y)$ satisfies the Biharmonic equation:

$$\frac{1}{E_2}\frac{\partial^4 F}{\partial X^4} + \frac{\partial^4 F}{\partial X^2 \partial Y^2}\left(\frac{1}{G} - \frac{2\nu_1}{E_1}\right) + \frac{1}{E_1}\frac{\partial^4 F}{\partial Y^4} = 0 \tag{7}$$

Equation (7) has been obtained from equation (27.4) of (Lekhnitskii, 1981) in which all β_{ij} are replaced by a_{ij} and where $a_{11} = E_1^{-1}$; $a_{22} = E_2^{-1}$; $a_{66} = G^{-1}$; $a_{12} = -\nu_1 E_1^{-1} = -\nu_2 E_2^{-1}$ and all other a_{ij} are zero. Equation (7) is derived using the stress-strain relation (2) , strain equation (3), the differential equations of equilibrium (5) and the compatibility equation of stresses. Therefore, the solution of the plane stress problem reduces to solving the Biharmonic equation (7) subject to prescribed boundary conditions in terms of external displacements.

The solution function $F(X, Y)$ also depends on the complex numbers μ_k which are the roots of the characteristic equation (see Lekhnitskii, 1981):

$$\frac{1}{E_1}\mu^4 + \left(\frac{1}{G} - \frac{2\nu_1}{E_1}\right)\mu^2 + \frac{1}{E_2} = 0. \tag{8}$$

For homogeneous materials, the coefficients of (8) are constants. The roots of (8) are all complex. If these complex roots are distinct, F is given by

$$F = 2\mathrm{Re}\left[F_{11}(Z_1) + F_{12}(Z_2)\right] \tag{9}$$

$$\text{where} \quad Z_1 = X + \mu_1 Y \quad \text{and} \quad Z_2 = X + \mu_2 Y \tag{10}$$

and Re denotes the real part. $F_{11}(Z_1)$ and $F_{12}(Z_2)$ are arbitrary analytic functions of the generalized (complicated) complex variables $(Z_1,\ Z_2)$. (See equations 26.4-5 of Lekhnitskii, 1981). μ_1 and μ_2 are two distinct roots of (8) which are not complex conjugates.

Following [Lekhnitskii, 1981], we define the complex potentials $\Phi_1(Z_1),\ \Phi_2(Z_2)$ as

$$\Phi_1(Z_1) = F'_{11}(Z_1); \qquad \Phi_2(Z_2) = F'_{12}(Z_2) \tag{11}$$

where the primes denote the derivatives of the function with respect to the (complicated) complex variables Z_1, Z_2. The stresses can then be written

$$\begin{aligned}
\sigma_X &= 2\mathrm{Re}\left[\mu_1^2\Phi'_1(Z_1) + \mu_2^2\Phi'_2(Z_2)\right] \\
\sigma_Y &= 2\mathrm{Re}\left[\Phi'_1(Z_1) + \Phi'_2(Z_2)\right] \\
\tau_{XY} &= -2\mathrm{Re}\left[\mu_1\Phi'_1(Z_1) + \mu_2\Phi'_2(Z_2)\right]
\end{aligned} \tag{12}$$

Neglecting rigid body displacements, the displacements u and v are:

$$u = 2\mathrm{Re}\left[p_1\Phi_1(Z_1) + p_2\Phi_2(Z_2)\right] \quad \text{and} \quad v = 2\mathrm{Re}\left[q_1\Phi_1(Z_1) + q_2\Phi_2(Z_2)\right] \tag{13}$$

$$\begin{aligned}
\text{where} \ p_1 &= \frac{1}{E_1}\mu_1^2 - \frac{\nu_1}{E_1} \qquad & p_2 &= \frac{1}{E_1}\mu_2^2 - \frac{\nu_1}{E_1} \\
q_1 &= -\frac{\nu_1\mu_1}{E_1} + \frac{1}{E_2\mu_1} \qquad & q_2 &= -\frac{\nu_1\mu_2}{E_1} + \frac{1}{E_2\mu_2}
\end{aligned} \tag{14}$$

Once the complex potentials $\Phi_1(Z_1)$ and $\Phi_2(Z_2)$ and μ_1 and μ_2 are determined for given elastic constants, the stresses and displacements can be evaluated from (12) and (13). We now find these complex potentials for the elastic parameters of *in vivo* human abdomenal skin given by Reihsner, et al. (1995). However, these elastic parameters are derived from defining the stress as force times thickness divided by collagen mass, and therefore the units of the elastic constants are given as Nm/g. But the traditional definition of stress is force/area. Therefore we neglect the collagen content (73.1 % of the dry weight of abdomen site, see Table 1 of Reihsner, 1995) and use the modified values of the elastic parameters obtained by multiplying their values by 73.1%. Thus, $E_1 = 4.168\mathrm{N/m}^2$; $E_2 = 2.500\mathrm{N/m}^2$; $G = 0.694\mathrm{N/m}^2$; $\nu_1 = 0.498$; and $\nu_2 = 0.299$.

Now, using (8) with these elastic parameters we find that

$$\mu_1 = i\beta_1 = 0.599i, \quad \mu_2 = i\beta_2 = 2.156i, \quad \text{so} \quad \beta_1 = 0.599 \quad \text{and} \quad \beta_2 = 2.156. \tag{15}$$

Since the boundary conditions on its edges are complicated, we map the ellipse (1) written as

$$\begin{aligned} x &= a\cos\theta; & y &= b\sin\theta \\ \text{or } X\cos\psi - Y\sin\psi &= a\cos\theta; & X\sin\psi + Y\cos\psi &= b\sin\theta \end{aligned} \tag{16}$$

onto the boundary of a unit circle $\zeta = e^{i\theta}$ in the ζ-plane ($\zeta = \xi + i\eta$) by the transformations

$$Z_1 = \frac{\tilde{a}_1 - i\mu_1\tilde{b}_1}{2}\zeta_1 + \frac{\tilde{a}_1 + i\mu_1\tilde{b}_1}{2}\zeta_1^{-1} \tag{17}$$

$$Z_2 = \frac{\tilde{a}_2 - i\mu_2\tilde{b}_2}{2}\zeta_2 + \frac{\tilde{a}_2 + i\mu_2\tilde{b}_2}{2}\zeta_2^{-1} \tag{18}$$

where $\tilde{a}_i = a\ \cos\psi - a\mu_i\ \sin\psi$ and $\tilde{b}_i = b\ \cos\psi + (b/\mu_i)\sin\psi$, for $i = 1, 2$. It is noted that for $\psi = 0$, $\tilde{a}_i = a$ and $\tilde{b}_i = b$. Using (15), (17) and (18) these reduce to

$$Z_1 = c_1\zeta_1 + c_2\zeta_1^{-1} \qquad Z_2 = c_3\zeta_2 + c_4\zeta_2^{-1} \tag{19}$$

$$\text{where } c_1 = \frac{\tilde{a}_1 + \tilde{b}_1\beta_1}{2}, \quad c_2 = \frac{\tilde{a}_1 - \tilde{b}_1\beta_1}{2}, \quad c_3 = \frac{\tilde{a}_2 + \tilde{b}_2\beta_2}{2}, \quad c_4 = \frac{\tilde{a}_2 - \tilde{b}_2\beta_2}{2}. \tag{20}$$

When X and Y run along the contour of the ellipse taking the values in (16), Z_1 and Z_2 in (19) take the values $\zeta_1 = \zeta_2 = \zeta = e^{i\theta}$. However, away from the boundary, ζ_1, ζ_2, and ζ are, in general, not equal.

Similarly, using $\zeta_1 = \xi_1 + i\eta_1$, we find that the circle $\xi_1^2 + \eta_1^2 = r_1^2$, of radius r_1 in the ζ-plane maps onto the ellipse

$$\frac{(X\cos\psi - Y\sin\psi)^2}{(c_1 + c_2/r_1^2)^2} + \frac{\beta_1^2(X\sin\psi + Y\cos\psi)^2}{(c_1 - c_2/r_1^2)^2} = r_1^2 \tag{21}$$

For $r_1 = 1$, $\psi = 0$ and $\tilde{a}_1 = a$ this is the ellipse given by (1), as can be seen by using (16) and (20). As $r_1 \to \sqrt{c_2/c_1}$, this ellipse maps into a finite line along the x-axis. This can be seen by noting that the denominator of the second term in (21) tends to zero, forcing the numerator to zero and implying that $Y = y = 0$. The circle of radius $\sqrt{c_2/c_1}$ and its exterior in the ζ_1-plane maps onto the (X,Y)-plane. The relations between (X,Y) and (ξ_1,η_1) coordinates are given by

$$X = \left(c_1 + \frac{c_2}{r_1^2}\right)\xi_1\cos\psi + \left(c_1 - \frac{c_2}{r_1^2}\right)\frac{\eta_1}{\beta_1}\sin\psi; \quad Y = \left(c_1 - \frac{c_2}{r_1^2}\right)\frac{\eta_1}{\beta_1}\cos\psi - \left(c_1 + \frac{c_2}{r_1^2}\right)\xi_1\sin\psi \tag{22}$$

Similarly, corresponding to the value β_2, we have

$$X = \left(c_3 + \frac{c_4}{r_2^2}\right)\xi_2\cos\psi + \left(c_3 - \frac{c_4}{r_2^2}\right)\frac{\eta_2}{\beta_2}\sin\psi; \quad Y = \left(c_3 - \frac{c_4}{r_2^2}\right)\frac{\eta_2}{\beta_2}\cos\psi - \left(c_3 + \frac{c_4}{r_2^2}\right)\xi_2\sin\psi \tag{23}$$

where $r_2^2 = \xi_2^2 + \eta_2^2$.

We now solve for $\Phi_1(Z_1)$ and $\Phi_2(Z_2)$ (and thus the stresses and displacements) for the case where each point on the boundary of the ellipse is sutured perpendicular to the x-axis (see Fig. 1). The displacement of the ellipse boundary in X, Y coordinates is $u + iv = -(X\sin\psi + Y\cos\psi)(\sin\psi + i\cos\psi)$. Since we are considering small displacements, y must be small. Because b is the maximum value of y on the ellipse, b must be small compared to the dimensions of the skin sheet. Thus, at this boundary, (10) and (15) imply

$$u + iv = \left[\frac{(\beta_2 Z_1 - \beta_1 Z_2)}{\beta_1 - \beta_2}(i \sin\psi) + \frac{(Z_2 - Z_1)}{\beta_1 - \beta_2}(\cos\psi) \right] [cos\psi - i\sin\psi] . \tag{24}$$

Since $\zeta = e^{i\theta}$ at the ellipse boundary this can be written, using (20),

$$\begin{aligned} u + iv = \frac{\cos\psi - i\sin\psi}{\beta_1 - \beta_2} \big[e^{i\theta} \{ (c_1\beta_2 - \beta_1 c_3)(i\sin\psi) + (c_3 - c_1)\cos\psi \} \\ + e^{-i\theta} \{ (c_2\beta_2 - c_4\beta_1)(i\sin\psi) + (c_4 - c_2)\cos\psi \} \,] . \end{aligned} \tag{25}$$

Expressing $u + iv$ in terms of complex potentials $\Phi_1(Z_1)$ and $\Phi_2(Z_2)$ we write, from (13),

$$\begin{aligned} u &= p_1\Phi_1(Z_1) + p_2\Phi_2(Z_2) + \overline{p_1}\overline{\Phi_1(Z_1)} + \overline{p_2}\overline{\Phi_2(Z_2)} \\ v &= q_1\Phi_1(Z_1) + q_2\Phi_2(Z_2) + \overline{q_1}\overline{\Phi_1(Z_1)} + \overline{q_2}\overline{\Phi_2(Z_2)} \end{aligned} \tag{26}$$

where the bar denotes complex conjugate. Thus,

$$u+iv = (p_1+iq_1)\Phi_1(Z_1) + (p_2+iq_2)\Phi_2(Z_2) + (\overline{p_1}+i\overline{q_1})\overline{\Phi_1(Z_1)} + (\overline{p_2}+i\overline{q_2})\overline{\Phi_2(Z_2)} \tag{27}$$

General expressions for $\Phi_1(Z_1)$ and $\Phi_2(Z_2)$ in the absence of rigid body displacements can be taken as:

$$\Phi_1(Z_1) \equiv {}^1\Phi(\zeta_1) = K_1 \ln \zeta_1 + \sum_{n=1}^{\infty} A_n \zeta_1^{-n} + \sum_{n=1}^{\infty} D_n \zeta_1^n \tag{28}$$

$$\Phi_2(Z_2) \equiv {}^2\Phi(\zeta_2) = K_2 \ln \zeta_2 + \sum_{n=1}^{\infty} B_n \zeta_2^{-n} + \sum_{n=1}^{\infty} L_n \zeta_2^n \tag{29}$$

where $K_1,\ K_2,\ A_n,\ B_n,\ D_n$ and L_n are arbitrary complex constants to be determined for a specific problem. The terms $\ln\zeta_1$ and $\ln\zeta_2$ are introduced since stresses are expressed as derivatives of $\Phi_1(Z_1)$ and $\Phi_2(Z_2)$ (see (12)).

We take $K_1 = K_2 = L_n = D_n = 0$ so that the displacements and stresses die out for large values of ζ_1 and ζ_2. Furthermore, for displacements to be single-valued, the increment in displacement of a point on the circle when θ increases by 2π must be zero. Therefore, the logarithm term in (28 and 29) must vanish. Thus, $\Phi_1(Z_1)$ and $\Phi_2(Z_2)$ become

$$\Phi_1(Z_1) \;=\; \sum_{n=1}^{\infty} A_n \zeta_1^{-n}; \qquad \Phi_2(Z_2) \;=\; \sum_{n=1}^{\infty} B_n \zeta_2^{-n}. \tag{30}$$

Now equating (25) with (27) and using (30) with $\zeta_1 = \zeta_2 = e^{i\theta}$ to match displacements at the boundary of the ellipse with those expressed in terms of potential functions we obtain

$$\begin{aligned} &\tfrac{\cos\psi - i\sin\psi}{\beta_1-\beta_2} && \left[e^{i\theta}\left\{(c_1\beta_2-\beta_1c_3)i\sin\psi+(c_3-c_1)\cos\psi\right\}+e^{-i\theta}\left\{(c_2\beta_2-c_4\beta_1)i\sin\psi+(c_4-c_2)\cos\psi\right\}\right] \\ &= && \sum_{n=1}^{\infty}\left[A_ne^{-in\theta}(p_1+iq_1)+B_ne^{-in\theta}(p_2+iq_2)+\overline{A_n}e^{in\theta}(\overline{p_1}+i\overline{q_1})+\overline{B_n}e^{in\theta}(\overline{p_2}+i\overline{q_2})\right] \end{aligned} \tag{31}$$

Equating coefficients of $e^{i\theta}$ and $e^{-i\theta}$ yields

$$\begin{aligned} \overline{A_1}(\overline{p_1}\;+\;i\overline{q_1})\;+\;\overline{B_1}(\overline{p_2}\;+\;i\overline{q_2}) &\;=\; A \\ A_1(p_1\;+\;iq_1)\;+\;B_1(p_2\;+\;iq_2) &\;=\; B \end{aligned} \tag{32}$$

where

$$\begin{aligned} A &= \frac{\cos\psi - i\sin\psi}{\beta_1-\beta_2}[(c_1\beta_2-\beta_1c_3)(i\sin\psi)+(c_3-c_1)\cos\psi] \\ B &= \frac{\cos\psi - i\sin\psi}{\beta_1-\beta_2}[(c_2\beta_2-c_4\beta_1)(i\sin\psi)+(c_4-c_2)\cos\psi] \end{aligned}$$

Thus,

$$A_1 \;=\; \frac{-ip_2(\overline{A}-B)+q_2(\overline{A}+B)}{2(p_1q_2-p_2q_1)} \;=\; a_1+ib_1; \quad B_1 \;=\; \frac{ip_1(\overline{A}-B)-q_1(\overline{A}+B)}{2(p_1q_2-p_2q_1)} \;=\; d_1+ie_1. \tag{33}$$

where a_1 is the real part of A_1 and b_1 is the imaginary part of A_1, , etc.

Next, equating coefficients of $e^{\pm in\theta}$ in (31) for $n \geq 2$, we find that $A_n = B_n = 0$, for $n \geq 2$. Hence, we have from (30)

$$\Phi_1(Z_1) = {}^1\Phi(\zeta_1) = A_1\zeta_1^{-1} = \frac{A_1(\xi_1 - i\eta_1)}{\xi_1^2+\eta_1^2} \;\text{ and }\; \Phi_2(Z_2) = {}^2\Phi(\zeta_2) = B_1\zeta_2^{-1} = \frac{B_1(\xi_2 - i\eta_2)}{\xi_2^2+\eta_2^2}. \tag{34}$$

To find the stresses, we have from (34) and (19)

$$\Phi_1'(Z_1) \;=\; \frac{\frac{d}{d\zeta_1}{}^1\Phi(\zeta_1)}{\frac{dZ_1}{d\zeta_1}} \;=\; \frac{A_1}{c_2 \;-\; c_1\zeta_1^2} \;\text{ and }\; \Phi_2'(Z_2) \;=\; \frac{\frac{d}{d\zeta_2}{}^2\Phi(\zeta_2)}{\frac{dZ_2}{d\zeta_2}} \;=\; \frac{B_1}{c_4 \;-\; c_3\zeta_2^2} \tag{35}$$

¿From (35), we find

$$\begin{aligned}
\text{Re } \Phi_1'(Z_1) &= \frac{a_1\left[c_2 - c_1(\xi_1^2 - \eta_1^2)\right] - 2b_1c_1\xi_1\eta_1}{\left[c_2 - c_1(\xi_1^2 - \eta_1^2)\right]^2 + 4c_1^2\xi_1^2\eta_1^2} \\
\text{Im } \Phi_1'(Z_1) &= \frac{2a_1c_1\xi_1\eta_1 + b_1\left[c_2 - c_1(\xi_1^2 - \eta_1^2)\right]}{\left[c_2 - c_1(\xi_1^2 - \eta_1^2)\right]^2 + 4c_1^2\xi_1^2\eta_1^2} \\
\text{Re } \Phi_2'(Z_2) &= \frac{d_1[c_4 - c_3(\xi_2^2 - \eta_2^2)] - 2e_1c_3\xi_1\eta_1}{\left[c_4 - c_3(\xi_2^2 - \eta_2^2)\right]^2 + 4c_3^2\xi_2^2\eta_2^2} \\
\text{Im } \Phi_2'(Z_2) &= \frac{2d_1c_3\xi_2\eta_2 + e_1\left[c_4 - c_3(\xi_2^2 - \eta_2^2)\right]}{\left[c_4 - c_3(\xi_2^2 - \eta_2^2)\right]^2 + 4c_3^2\xi_2^2\eta_2^2}
\end{aligned} \tag{36}$$

The principal stresses, σ_1 and σ_2, at any point (X, Y) in the skin sheet are given by

$$\begin{aligned}
\sigma_1 &= \frac{\sigma_X \;+\; \sigma_Y}{2} \;+\; \frac{1}{2}\sqrt{(\sigma_X \;-\; \sigma_Y)^2 \;+\; 4\tau_{XY}^2} \\
\sigma_2 &= \frac{\sigma_X \;+\; \sigma_Y}{2} \;-\; \frac{1}{2}\sqrt{(\sigma_X \;-\; \sigma_Y)^2 \;+\; 4\tau_{XY}^2}
\end{aligned} \tag{37}$$

where the point (X, Y) corresponds to the point's location before suturing.

If the circular wound were sutured in a direction perpendicular to the x-axis, the analysis for the elliptical wound is applicable. We simply let $a \;=\; b$.

If a circular wound, i.e., $a \;=\; b$, were sutured toward the origin (i.e., toward the center of the wound), then the displacements would be $u \;=\; -X$ and $v \;=\; -Y$ Then using (10),

$$u \;+\; iv \;=\; \frac{1}{\beta_1 \;-\; \beta_2}\left[(Z_1\beta_2 \;-\; Z_2\beta_1) \;+\; (Z_2 \;-\; Z_1)\right]. \tag{38}$$

Using (19) with $\zeta \;=\; e^{i\theta}$, this becomes

$$u + iv = \frac{1}{\beta_1 - \beta_2}\left[e^{i\theta}\left\{(c_1\beta_2 - c_3\beta_1) + (c_3 - c_1))\right\} + e^{-i\theta}\left\{(c_2\beta_2 - c_4\beta_1) + (c_4 - c_2)\right\}\right]. \tag{39}$$

Thus, the left side of (31) can be replaced by the right side of (39) and the right side of (31) remains unchanged. Then A_1 and B_1 are calculated in the same manner as we did earlier, with the result

$$A_1 \;=\; \frac{-ip_2(\overline{A'} - B') + q_2(\overline{A'} + B')}{2(p_1q_2 - p_2q_1)}; \quad B_1 \;=\; \frac{ip_1(\overline{A'} - B') - q_1(\overline{A'} + B')}{2(p_1q_2 - p_2q_1)} \tag{40}$$

with A' and B' given by

$$A' = \frac{1}{\beta_1 - \beta_2}[(c_1\beta_2 - \beta_1 c_3) + (c_3 - c_1)]; \; B' = \frac{1}{\beta_1 - \beta_2}[(c_2\beta_2 - c_4\beta_1) + (c_4 - c_2)]$$

The rest of the analysis is exactly the same as in the elliptical case.

The stresses t^{pr} in (x, y) coordinates can be obtained from the tensorial relation:

$$t^{pr} \;=\; \frac{\partial x^p}{\partial X^q}\frac{\partial x^r}{\partial X^s}\sigma^{qs} \qquad (p, q, r, s \;=\; 1, 2) \tag{41}$$

where σ^{qs} denote stresses in (X, Y) coordinates. For example, using the relation (4) along with (41) we get

$$t^{11} \;=\; t_x \;=\; \sigma_X \cos^2\psi \;-\; 2\tau_{XY}\sin\psi\cos\psi \;+\; \sigma_Y \sin^2\psi. \tag{42}$$

Since the angle ψ for abdomenal skin at a particular site is not known, for illustration we evaluate the stresses and displacements by taking $\psi \;=\; 0$. This amounts to having the major axis of the ellipse coincide with the Langer's lines since the extensions are minimum in this direction (Gibson, 1990). This will be the case when the principal directions of elasticity coincide with the principal axes of the ellipse. Assuming $\psi \;=\; 0$, seems to be appropriate in view of the observation (Larrabee, 1986) that when a specimen of skin is progressively strained, the fibers become oriented in the direction of strain (in this problem, the y-axis). Stresses, however, can be determined by the above analysis for any known angle ψ.

Results

In our model we employ linear elasticity theory valid for orthotropic materials in plane stress problems. Our model is limited to small strains, although the incremental modulii used in our analysis give the nonlinear character to some extent. Since skin can take on large deformations depending upon the size of the wound, a better model incorporating large deformation theory can be the subject of future research. However, so far as comparison of two suturing patterns is concerned (as in this paper), small strain theory is good enough because large deformation theory will produce results which could differ only quantitatively (but not qualitatively) from the results of small strain theory if the shape of the wound is not changed.

We present results only for the first quadrant (for $\psi = 0$), since the results are symmetric about the x and y axes. Furthermore, only the distributions of stresses of physiological interest will be presented, i.e. maximum and minimum principal stresses σ_1 and σ_2, respectively.

We observe from Figure 2 that the principal stress σ_1 along the edge of circular wound sutured toward the center is considerably higher compared to suturing done transversely. A similar situation holds for the magnitude of the stress σ_2. This is also evident from Table I, where the average stress indices (ASI) for different patterns of suturing are shown. The principal average stress index for σ_1 is defined as the integral of the principal stresses over the computational area divided by this area. The principal average stress index for σ_2 is similarly defined. The results of Table I and Fig. 2 confirm our hypothesis that suturing perpendicular to the x-axis is preferable compared to suturing towards the center. This also explains theoretically the usual practice by surgeons of avoiding suturing towards the center, so far as the influence of stress factors are concerned.

Note also from Table I that the ASI for an elliptical wound both for principal tensile and

Table 1: Average Stress Index (ASI) for wounds of equal area ($9.4\times 10^{-4} m^2$) in a .2 $m \times$ .2 m skin sheet

	σ_1	σ_2
Circle sutured toward center	+0.160	-0.163
Circle sutured perpendicular to x-axis	+0.092	-0.085
Ellipse sutured perpendicular to x-axis	+0.067	-0.051

compressive stresses is lower than that for a circular wound of the *same area*, as hypothesized. Hence, from a stress perspective, an elliptical wound is preferable to a circular wound. This also explains the reason why it is a good idea for surgeons to give a circular wound an elliptical shape along the diameter of the circle before suturing. Thus, whenever an excision is to be made it should be done in the form of an ellipse because suturing the resulting wound would produce low stresses.

A complete stress distribution in the skin sheet for the principal stress σ_1 is shown in Fig. 3. From this figure, it is clear that the magnitudes of the stresses are smaller throughout the whole sheet when the circle is sutured perpendicular to the x-axis, than when suturing is done towards the center. In particular, the large tensile stresses do not die out as quickly when suturing is done toward the center. The results for σ_2 are similar, although they are not shown due to space limitations.

In the case of an elliptical wound sutured parallel to the minor axis, we see (Fig. 4a) that the maximum principal stress for $a/b = 5$ occurs at the end of the major axis (although this may be difficult to see from the figure, it is found from the actual numerical data. Thus, the healing will be slowest at this point. Although this is true for $a/b = 5$, it is not true in general, as can be seen in Fig. 5. Further, note that the stress near the end of the major axis decreases quickly and becomes compressive as one moves away from the ellipse boundary. The tensile stresses are also large at the wound edge. The magnitude of the stress σ_2 also is maximal at the end of the major axis (Fig. 4b), and is compressive in nature.

It is interesting to observe from Fig. 5 that whatever the ratio a/b for an elliptical wound, the principal stress σ_1 at the end of the major axis is the same. We also find that the stresses along the edge become more non-uniform as this ratio moves away from approximately 3. Therefore, $a/b \simeq 3$ is the optimal ratio for which the stresses become most uniform along the wound edge, resulting in a more uniform healing process. The results for σ_2 are similar, although they are not shown. Hence, if it is possible to implement this ratio, the healing process may be improved on the basis of stress information, although there are many other biological factors involved in wound healing. We note that this ratio changes with the values of the elastic constants E_1, E_2 and G for different sites of the human body. It is also noted that the location of the maximum principal stress σ_1 shifts from the end of the major axis to the center and eventually increases as a/b becomes less than 3. We observe from Fig. 6 that lines of maximum extension are almost parallel to the minor axis of the ellipse and that the minimum extension lines are almost parallel to the minor axis for $\psi = 0$.

We observe from Fig. 6 that the lines of maximum extension are almost parallel to the minor axis of the ellipse, and those of minimum extension parallel to the minor axis for $\psi = 0$. The minimum extensibility of the skin occurs along Langer's lines (Gibson 1990), hence, if a surgeon has a choice to make an excision in the form of an ellipse, then it is desirable to make the major axis of the ellipse coincide with the Langer's line. With this orientation, the lines of minimum extension (see Fig. 6) will be along the Langer's lines. In that case, better scars will be produced after suturing the wound (Cacou and Muir 1995).

Discussion

Our results are valid for small strain theory which means that displacements are small and computed strains are also small, with small generally defined as being in the range (0.05 to

0.1). Using equations (14), (19), (26), (33) and (35) we find that for this particular problem, displacement gradients and therefore strains are small if the semi-minor axis b of the ellipse is small compared to the dimension of the skin sheet. In that case rotational strain, defined by $\omega_{ij} = (1/2)(\partial u_i/\partial X_j - \partial u_j/\partial X_i)$ is also small. General requirements for the displacement gradients to be small amounts to the condition that the fixed outer boundaries be far away from the wound. Therefore our results are valid if the wound is considered to be in the center of a large flat sheet of skin, as mentioned in the beginning of the methods section.

Some researchers have evaluated wound closure stresses theoretically but no comparison has been made for closing the same wound using different suturing patterns. In order to compare with previous results for isotropic material (since no similar work has been performed for orthotropic material), we have separately computed the principal stresses σ_1 and σ_2 produced upon suturing an elliptical hole in *isotropic* material. This calculation has also been performed by Danielson and Natarjan (1975); our expressions for σ_1 and σ_2 differ from their result by a single term. In particular, the third term contained within the absolute value symbols (see equation (2) in Danielson and Natrajan (1975) is given there as $-\zeta^2(3-2\kappa m^2)$, whereas we find this term to be $-\zeta^2(3-2\kappa m+m^2)$ where $\kappa = (3-\nu)/(1+\nu)$ and $m = (a-b)/(a+b)$. The formula presented in Danielson and Natrajan (1975) gives unbounded stresses at the ends of the major axis (i.e., for $\zeta = \pm 1$) as $m \to 1$. However, upon making the above mentioned correction, the formula gives bounded results. The corrected isotropic formula gives numerical values in complete agreement with our anisotropic results in the limit $E_1 \to E_2 = E$, $\nu_1 \to \nu_2 = \nu$, and $G = E/[2(1+\nu)]$, where E, ν, and G are the elastic parameters for the isotropic case. In comparing the isotropic stress patterns with the orthotropic pattern for $\psi = 0$, we see that the main difference is that the magnitude of the principle stresses for the isotropic case are considerably higher (almost double). However, for $\psi \neq 0$ there are also qualitative differences in the distribution of principal stresses.

Some of the calculations reported here may also be compared, at least qualitatively, with the results of Larrabee and Galt (1986), who used the finite element method and linear elasticity theory to study displacements *in vitro* for rectangular and elliptical wounds. Although these authors do not not suture the wound edges precisely parallel to the minor axis, their computed displacement field is nevertheless qualitatively similar to that shown in our Fig. 6. They further point out that this deformation field is in reasonable agreement with that produced in a clinical wound closure. Larrabee and Galt (1986) considered the skin to be isotropic, did not evaluate stress fields, and did not discuss different suturing patterns.

It is also worth mentioning that Danielson and Natrajan (1975) have employed tension field theory *in vitro* to predict stresses in stretched skin by considering the skin to be a membrane subjected to large deformations. In their work, they employed a uniaxial stress-strain relation which permits the stress to be evaluated *only* in one direction. They also considered the skin to be *isotropic*. A formula for the maximum stress upon closing a skin defect was obtained and, upon adjusting a free parameter, was found to agree with the experimental stress produced by a Z-plasty operation.

Another important biological element involved in wound healing is blood flow. It has been observed (Larrabee, et al. 1984) that blood flow is inversely proportional to the wound closure tension. From this information, it may be inferred that blood flow will be lessened in regions of maximum stresses on the basis of stress information, although there are many other biological factors involved in blood flow due to the complex structure of the skin.

Finally, our analysis can predict the stress and displacement distribution around and away from the wound for different shaped wounds and different patterns of suturing. This information can be helpful to the surgeon to understand in advance, i.e., prior to surgery, the stresses and displacements produced as a result of suturing wounds of different shapes in various patterns.

Acknowledgement

H. R. C. thanks the Department of Physical Medicine at UMDNJ Stratford for support. The work of M. S. was supported in part by NSF Grant DMS-9704746. We also thank the Center for Applied Mathematics and Statistics at N.J.I.T for research facilities.

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Figure Captions

Fig. 1 (a) An elliptical wound with semi-major and minor axes a and b, sutured in the direction parallel to the y-axis. ψ is the angle between the principal directions of elasticity and principal axes of the ellipse.

(b) A circular wound sutured in the direction perpedicular to the x-axis.

(c) A circular wound sutured toward the origin, so that $u = -x, \ v = -y$.

Fig. 2 Principal stresses σ_1 and σ_2 along the x-axis for a circular wound after suturing. Dotted curves represent the stresses for suturing towards the origin, while the solid curves represent the stresses for suturing vertically towards the x-axis. It is clear that σ_1 and σ_2 are higher in magnitude for suturing toward the origin, in agreement with our hypothesis. This also explains the reason why suturing toward the origin is avoided.

Fig. 3 Principal stress σ_1 throughout the first quadrant of the skin sheet with a circular wound. The dimensions of the axes are in meters. The stress is that at the precursor point, i.e., before suturing. Suturing is parallel to the y-axis in Fig. 3a, and toward the center in Fig. 3b. It is clear that σ_1 and σ_2 are higher in magnitude for suturing toward the origin, as hypothesized.

Fig. 4 Principal stresses σ_1 (Fig. 4a) and σ_2 (Fig. 4b) throughout the first quadrant of the skin sheet with an elliptical wound, for $a/b = 5$. The dimensions of the axes are in meters. Horizontal axis is along the semi-major axis and vertical axis along the semi-minor axis. Note that σ_1 and σ_2 are maximal at the end of the major axis.

Fig. 5 Variation of principal stress (σ_1) along the x-axis for $a/b \ = \ 1, \ 2, \ 3, \ 5, \ 8$. It can be seen the stresses are most uniform for $a/b \simeq 3$. Thus the optimal ratio of semi-major to semi-minor axis is approximately 3.

Fig. 6 Magnitude and directions of extensions connecting points of the skin sheet before and after suturing the elliptical wound. Note that maximum extension occurs parallel to the minor axis, and minimum extension parallel to the major axis.