CIS 610.003, Fall 2008 First Midterm Prof. J. Calvin

Print Name (last name first): ____

Do not open this exam until instructed to do so. The exam consists of 5 pages, numbered 1 through 5. Before starting to work, make sure that you have all 5 pages. There are four problems, each counting 20 points. Write all answers on the exam.

During this exam it is prohibited to:

(1) exchange information with any other person in any way, including by talking or exchanging papers or books;
(2) use any electronic aid, including calculators;
(3) use any books or notes;
(4) leave the exam room before you complete and turn in your exam.

I have read and understand all of the instructions above. On my honor, I pledge that I have not violated the provisions of the NJIT Academic Honor Code.

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Signature and Date

Stirling's approximation: $n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n (1 + O(1/n))$.
Master Theorem: The solution $T(n)$ to $T(n) = aT(n/b) + f(n)$ can be bounded as follows:

(1) If $f(n) = O\left(n^{\log_b a - \epsilon}\right)$ for some $\epsilon > 0$, then $T(n) = \Theta\left(n^{\log_b a}\right)$.
(2) If $f(n) = \Theta\left(n^{\log_b a}\lg(n)^k\right)$ for some $k \geq 0$, then $T(n) = \Theta\left(n^{\log_b a}\lg(n)^{k+1}\right)$.
(3) If $f(n) = \Omega\left(n^{\log_b a + \epsilon}\right)$ for some $\epsilon > 0$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) = \Theta\left(f(n)\right)$.

1. For each of the following pairs of functions, replace the ? with either a Θ, $\mathcal{O}$, or Ω, as appropriate (answering Θ wherever it applies). You must justify your answers to receive credit.

a) $n^5 =? \qquad (3/2)^n$

b) $\lg\lg(n) =? \qquad \lg(n)/n$

c) $2^{\sqrt{n}} \lg n =? \qquad \sqrt{n}$

d) $(n+\sqrt{n})^5 =? \qquad n^5$

2. Find tight bounds for the solution of the following recurrences.

a. $T(n) = 3T(n/2) + \sqrt{n}$

b. $T(n) = T(n-1) + \lg(n)$