CS 610, Spring 2009, Prof. Calvin
Homework #1 Solutions

R-1.6

$$1/n = O\left(2^{100}\right) = O\left(\log\log n\right) = O\left(\sqrt{\log n}\right) = O\left(\log^2 n\right) = O\left(n^{0.01}\right) = O\left(\lceil\sqrt{n}\rceil\right)$$
$$= \Theta\left(3n^{0.5}\right) = O\left(2^{\log n}\right) = \Theta\left(5n\right) = O\left(n\log_4 n\right) = \Theta\left(6n\log n\right) = O\left(\lfloor 2n\log^2 n\rfloor\right)$$
$$= O\left(4n^{3/2}\right) = O\left(4^{\log n}\right) = O\left(n^2\log n\right) = O\left(n^3\right) = O\left(2^n\right) = O\left(4^n\right) = O\left(2^{2^n}\right).$$

R-1.15 Suppose that $f(n) \le c_1 g(n)$ for all $n \ge n_1$ and $d(n) \le c_2 h(n)$ for all $n \ge n_2$. Then

$$f(n) + d(n) \le (c_1 + c_2)\left(g(n) + h(n)\right)$$

for $n \ge \max(n_1, n_2)$.

C-1.1 For $j = 1$ to $j = \lfloor n/3 \rfloor$, the $3j$th job will take time $\Theta(3j)$ and the other $n - \lfloor n/3 \rfloor$ jobs will take time $\Theta(1)$. Therefore the total time is

$$\sum_{j=1}^{\lfloor n/3\rfloor} \Theta(3j) + \left(n - \lfloor n/3\rfloor\right)\Theta(1) = \Theta\left(3\frac{\lfloor n/3\rfloor(\lfloor n/3\rfloor + 1)}{2} + n - \lfloor n/3\rfloor\right) = \Theta\left(n^2\right).$$

C-1.4 For the base case, we have $T(1) = 1$ so the claimed solution is valid for $n = 1$. Assume that

$$T(k) = \frac{k(k+1)}{2}$$

for all $k < n$, for some $n > 1$. Then

$$\begin{aligned} T(n) &= T(n-1) + n \quad \text{by the defining recurrence} \\ &= \frac{(n-1)n}{2} + n \quad \text{by the induction assumtion} \\ &= \frac{n(n+1)}{2}. \end{aligned}$$