

A4 Solutions

$$G(s) = \frac{500}{s(s+1)(s+10)} = \frac{500}{s^2 + 11s + 10}$$

$$G(j\omega) = \frac{500}{-j\omega^2 + j10\omega - 11\omega^2} = \frac{500(-11\omega^2)}{121\omega^4 + (10\omega - \omega^3)^2} + j \frac{500(\omega^3 - 10\omega)}{121\omega^4 + (10\omega - \omega^3)^2}$$

$$\text{Im } G = 0 \quad \omega = 1\sqrt{10} \text{ rad/s}$$

$$\text{Re } G(j\sqrt{10}) = \frac{-50}{11}$$

From table $DF(a) = \frac{2}{\pi} \left[\sin^{-1}\left(\frac{10}{a}\right) + \left(\frac{10}{a}\right) \cos \sin^{-1}\left(\frac{10}{a}\right) \right]$

$$= \frac{2}{\pi} \left[\sin^{-1}\left(\frac{10}{a}\right) + \frac{10}{a} \sqrt{1 - \left(\frac{10}{a}\right)^2} \right]$$

$$-\frac{1}{DF(a)} = -\frac{\pi}{2} \frac{1}{\sin^{-1}\left(\frac{10}{a}\right) + \frac{10}{a} \sqrt{1 - \left(\frac{10}{a}\right)^2}}$$

$$a = 0 \quad -\frac{1}{DF(a)} = 0$$

$$a = +\infty \quad -\frac{1}{DF(a)} = -\infty$$

$$-\frac{1}{DF(a)} = \frac{-50}{11} \quad \omega \approx 1.2$$

frequency $\sqrt{10}$ rad/s $\approx 0.5 \text{ Hz}$

