AS Solution

$\dot{r} = r(C + 2r^2 - r^4)$

$\dot{\Theta} = 2\pi$

From Lecture 4 . $r^2 = x^2 + y^2$ $\theta = \tan^{-1}\frac{y}{x}$

$r\dot{r} = x\dot{x} + y\dot{y}$

$\dot{\theta} = \frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{d}{dt}\frac{y}{x}$

$= \frac{x\dot{y} - y\dot{x}}{x^2+y^2} \leftarrow r^2$

$\dot{r} = r(C + 2r^2 - r^4)$

$r\dot{r} = r^2(C + 2r^2 - r^4)$

$x\dot{x} + y\dot{y} = r^2(C + 2r^2 - r^4)$

$\dot{\theta} = 2\pi$

$2\pi r^2 = x\dot{y} - y\dot{x}$

$x\dot{x} + \frac{y}{x}(2\pi r^2 + y\dot{x}) = r^2(C + 2r^2 - r^4)$ $\therefore y\dot{y} = \frac{y}{x}(2\pi r^2 + y\dot{x})$

$\dot{x} = -2\pi y + x(C + 2(x^2+y^2) - (x^2+y^2)^2)$

Sim

$\dot{y} = 2\pi x + y(C + 2(x^2+y^2) - (x^2+y^2)^2)$

For $C = -0.5$

at about $r(0) = 0.545$. $\Theta(0)$ any value

separates focus and limit cycle

$r(0) < 0.545 \Rightarrow$ decay into 0

$r(0) > 0.545 \Rightarrow$ converge to limit cycle.

Similarly $r(0)$ boundary $= 1$ $C = -1$