ECE436/ECE698 Ex2 solutions

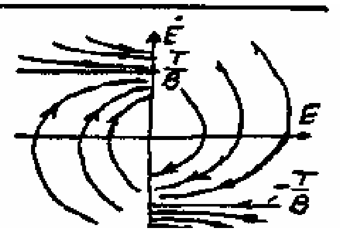
1.

$J\ddot\theta_o + B\dot\theta_o = T\,\mathrm{sgn}\,E$; $E = \theta_i - \theta_o$

$\theta_i = A$, $\dot\theta_o = -\dot E$, $\ddot\theta_o = -\ddot E \rightarrow$

$J\ddot E + B\dot E = -T\,\mathrm{sgn}E \rightarrow \frac{d\dot E}{dE} = \frac{-B\dot E - T\,\mathrm{sgn}E}{J\dot E}$

zero slopes when $\dot E = -(T/B)\,\mathrm{sgn}E$



2.

$J\ddot\theta_o + B\dot\theta_o = T\,\mathrm{sgn}\,E$; $E = \theta_i - \theta_o$

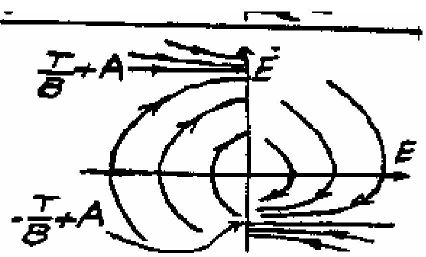
$\theta_i = At$, $\dot E = A - \dot\theta_o$, $\ddot E = -\ddot\theta_o \rightarrow$

$-J\ddot E + B(A - \dot E) = T\,\mathrm{sgn}E \rightarrow$

$\frac{d\dot E}{dE} = \frac{-B\dot E + BA - T\,\mathrm{sgn}E}{J\dot E}$

$E<0$: $d\dot E/dE = 0$ for $\dot E = \frac{T}{B} + A$

$E>0$: " " $\dot E = -\frac{T}{B} + A$



3.

(a) $J\ddot\theta_o + B\dot\theta_o = +f(E)$; $\dot\theta_o = -\dot E$, (b)

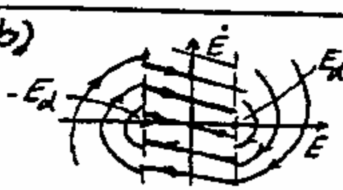
$\ddot\theta_o = -\ddot E \rightarrow J\ddot E + B\dot E + f(E) = 0$

$\rightarrow \frac{d\dot E}{dE} = \frac{-B\dot E - f(E)}{J\dot E}$

$-E_d < E < E_d$: $f(E) = 0$: $\frac{d\dot E}{dE} = -\frac{B}{J}$:
straight lines with slope $-B/J$.

$E > E_d$: $f(E) = K(E - E_d)$: Isocline for $d\dot E/dE = m$:
$\dot E = \frac{-KE + KE_d}{mJ + B}$: straight lines intersecting $\dot E = 0$ axis at $E = E_d$ (focus).

$E < -E_d$: $f(E) = K(E + E_d)$: Focus at $E = -E_d$



4.

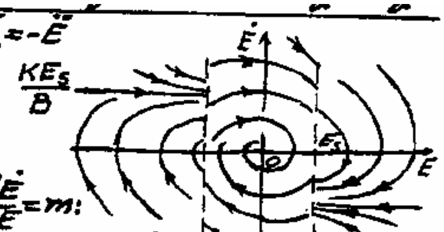
$J\ddot\theta_o + B\dot\theta_o = f(E)$; $\dot\theta_o = -\dot E$, $\ddot\theta_o = -\ddot E$

$\rightarrow J\ddot E + B\dot E + f(E) = 0$

$\rightarrow \frac{d\dot E}{dE} = \frac{-B\dot E - f(E)}{J\dot E}$

$-E_s < E < E_s$: $f(E) = KE$:
Isocline for slope $\frac{d\dot E}{dE} = m$:
$\dot E = \frac{-KE}{mJ + B}$: Isoclines are straight lines through 0, assumed to be a focus.

$E > E_s$: $d\dot E/dE = (-B\dot E - KE_s)/(J\dot E)$: Depends only on $\dot E$, so isoclines are horizontal. Slopes are 0 for $\dot E = -KE_s/B$. analogous for $E < -E_s$. These are velocity limits.



5) a) $\dot{r} = -r(r^2 - 1)$

$\dot{\theta} = -1$

$r < 1$, $\dot{r} > 0$ state tends towards $|r| = 1$

$r > 1$, $\dot{r} < 0$ " " away from $|r| = 1$

$\therefore$ stable limit cycle

note : $r = \dfrac{1}{(1 + c_0 e^{-2t})^{1/2}}$, $\theta = \theta_0 - t$

$c_0 = \dfrac{1}{r_0^2} - 1$

as $t \to \infty$ $r \to 1$

b) unstable limit cycle

6) $\dfrac{\partial f_1}{\partial x_1} + \dfrac{\partial f_2}{\partial x_2} = x_1^2 + x_2^2 > 0$

no limit cycle in the $x_1 - x_2$ plane

7) linearized model characteristics equation

$\lambda^2 - \mu\lambda + 1 = 0$

$\mu > 2$, λ_1, λ_2 distinct real $\Rightarrow$ unstable node

$\mu = 2$, $\lambda_1 = \lambda_2 = 1$

$0 < \mu < 2$, complex $\Rightarrow$ unstable spiral

$\left.\begin{array}{l} x_e = 0 \\ \text{Index} = 1 \end{array}\right\}$ possible to have limit cycle