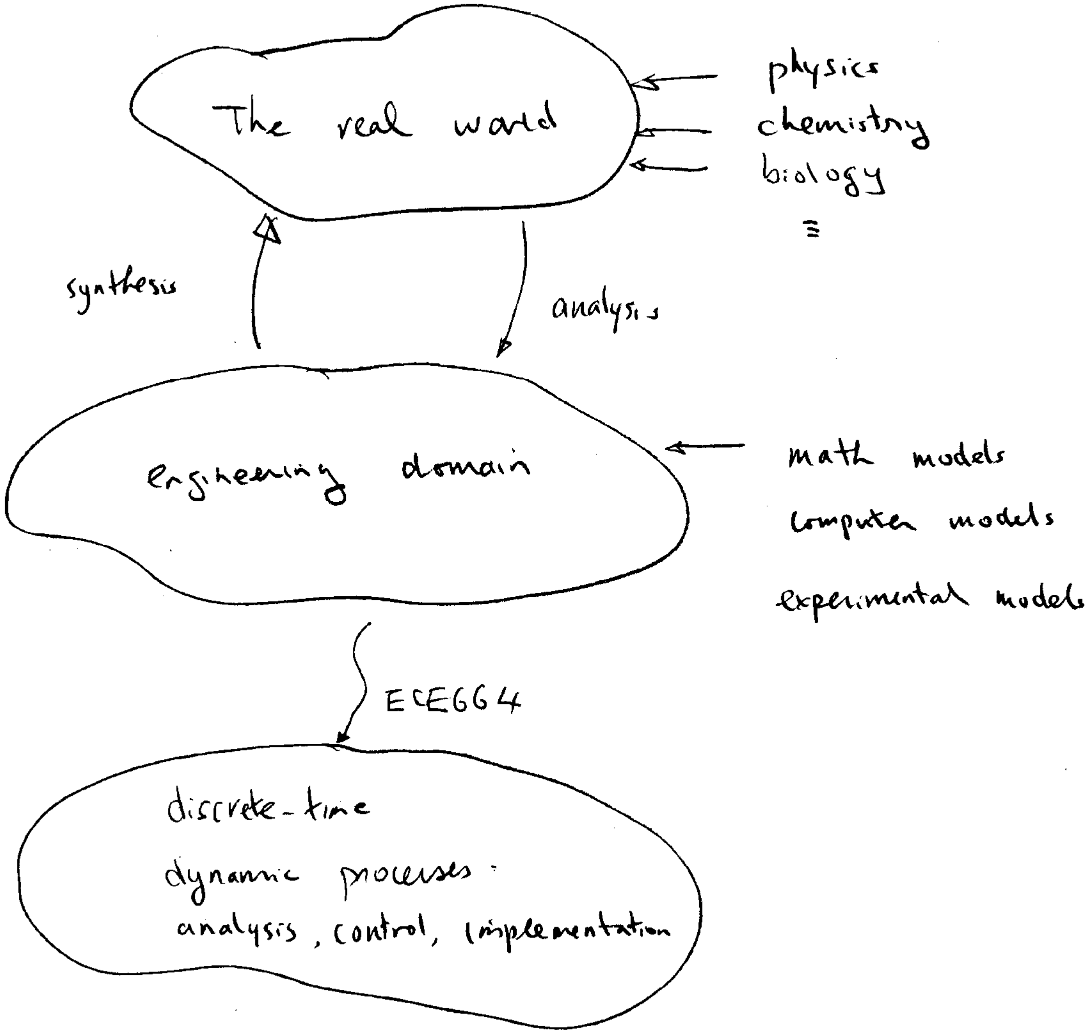
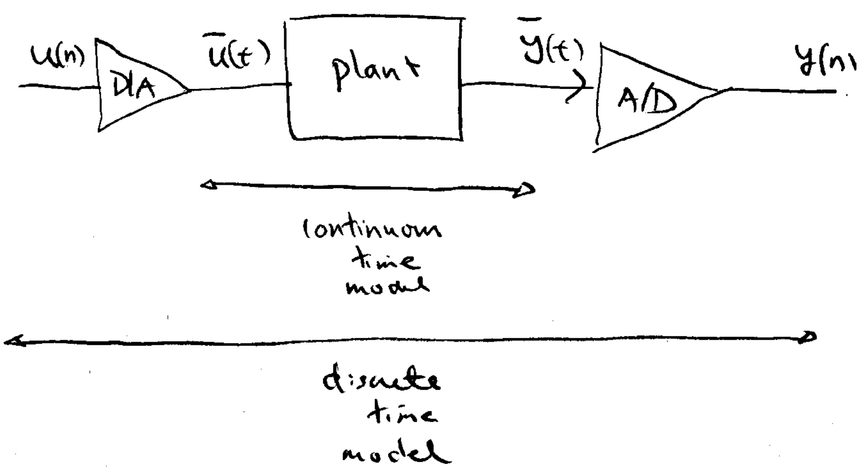
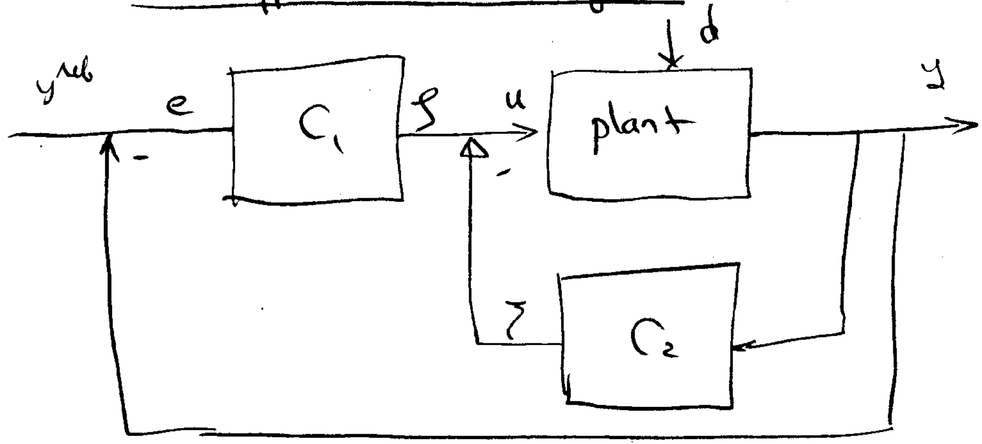
ECE664 Summary & Review



Real-time process:



A Typical Control system:



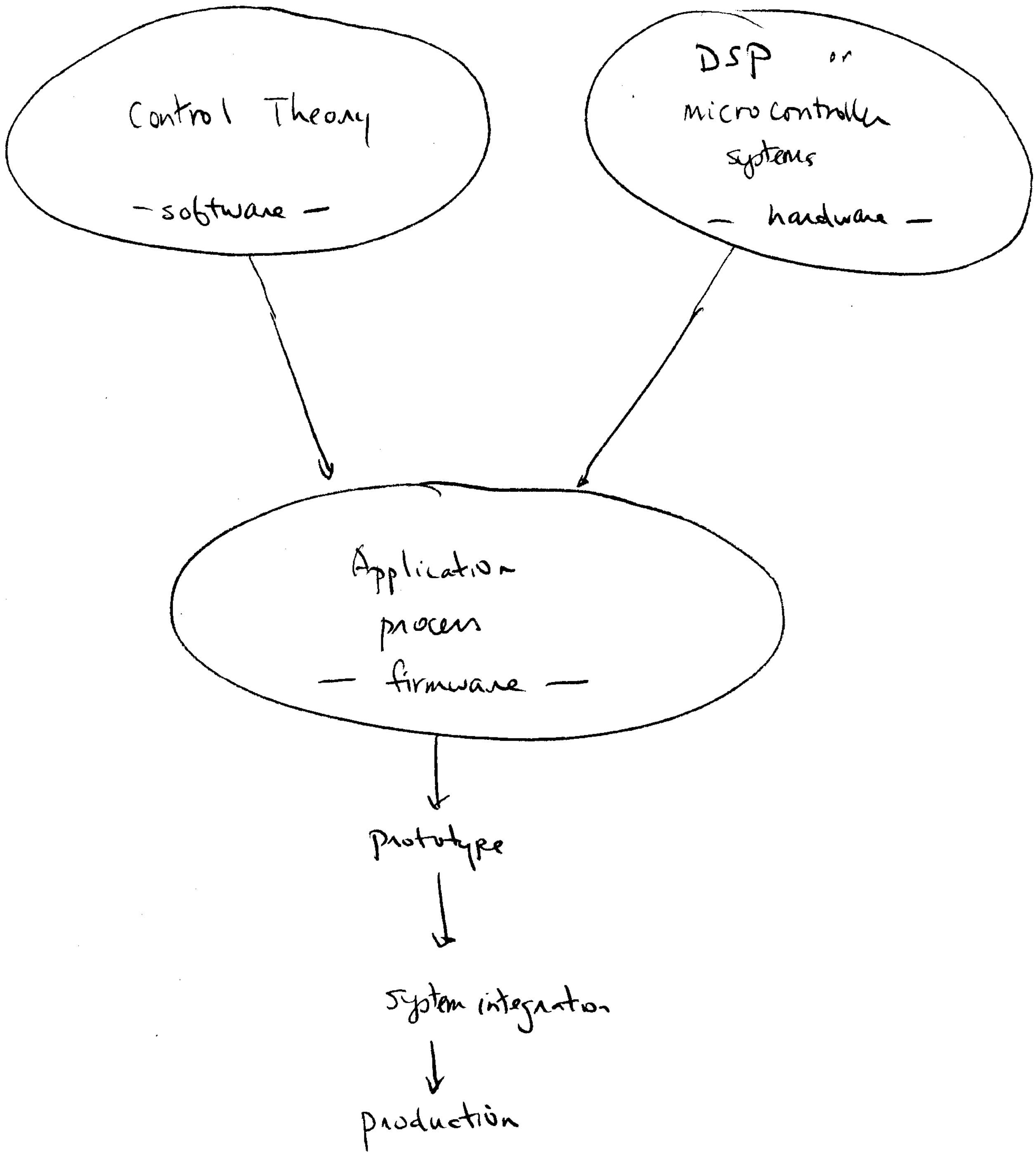
Design C_1, C_2 so that one or more of the following conditions are met:

a) closed loop stability
b) reduce steady state error ($|e| \to 0$)
c) disturbance rejection ($|e| \to 0$ despite presence of d)
d) good transient characteristics
e) robust wrt plant parameter variations

Why discrete time

a) Can implement complex algorithms (& get better results)
b) Performance is consistent and less susceptible to environment
c) Low cost (in large quantity)
d) Flexible
⋮

The Elements



The gateway : sampling, reconstruction } Poisson's theorem

Shannon's theorems

- sampling = 2 × w_c
- reconstruction =

$$\bar{y}(t) = \sum_{n=-\infty}^{\infty} y(nT_s)\ \text{sinc}\left(\frac{t-nT_s}{T_s}\right)$$

zero order hold approx

$$\bar{Y}(s) = G_{ho}(s)\, Y^*(s)$$

$$Y^*(s) = \sum_{n=0}^{\infty} y(nT_s)\, e^{-snT_s} \qquad G_{ho} = \frac{1-e^{-sT_s}}{s}$$

$\downarrow$ on letting $z = e^{sT_s}$

$$Y(z) = \sum_{n=0}^{\infty} y(n)\, z^{-n}$$

$y(n)$ to mean $y(nT_s)$

$$\begin{aligned}\dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u \\ \bar{y} &= \bar{C}\bar{x} + \bar{D}\bar{u}\end{aligned} \quad \xrightarrow[T_s]{} \quad \begin{aligned} x(n+1) &= A\,x(n) + B\,u(n) \\ y(n) &= C\,x(n) + D\,u(n)\end{aligned}$$

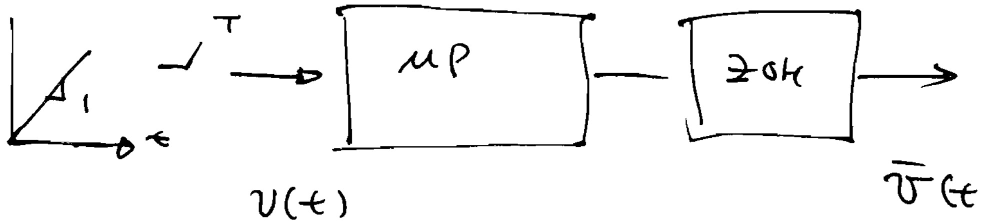
$$A = e^{\bar{A}T_s}, \quad B = \int_0^{T_s} e^{\bar{A}\tau}\bar{B}\, d\tau$$

$$\updownarrow \qquad\qquad \updownarrow$$

$$\bar{G}(s) = \bar{D} + \bar{C}(sI-\bar{A})^{-1}\bar{B} \quad \xrightarrow[T_s]{} \quad G(z) = C(zI-A)^{-1}B + D$$

$$G(z) = \mathcal{Z}\left[\frac{\bar{G}(s)}{s}\right](1-z^{-1})$$

Example: ramp input with ZOH (The DeBella problem)



$v(t) = t$ $\qquad \bar{v}(t) = nT \qquad n = 0, 1, 2, \ldots$

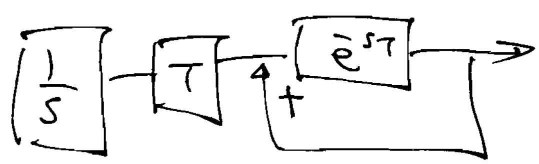
Frequency domain

$$\mathcal{Z}(t) = \frac{Tz}{(z-1)^2}$$ ← from table, for signals

but $z = e^{sT}$

∴ overall: $$\underbrace{\frac{Te^{sT}}{(1-e^{-sT})^2}}_{\text{ramp}} \; \underbrace{\frac{1-e^{-sT}}{s}}_{\text{ZOH}} = \frac{Te^{-sT}}{s(1-e^{-sT})}$$

simulation realization:



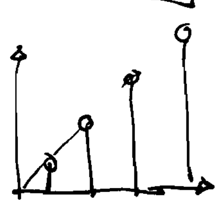
zohramp. mdl

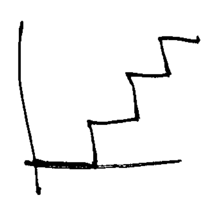
unit step

$$\Downarrow \quad \frac{e^{-sT}}{1-e^{-sT}}$$

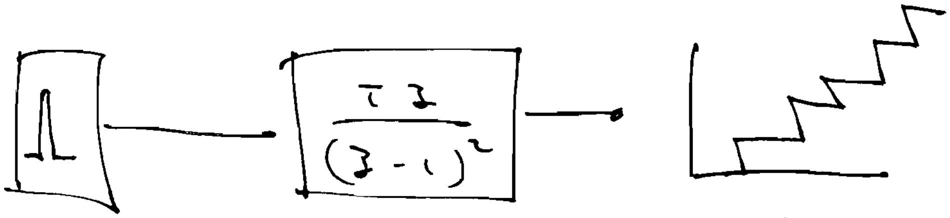
Discrete time approach

$$\boxed{\frac{Tz}{(z-1)^2}} \longrightarrow \boxed{\frac{1-e^{-sT}}{s}}$$





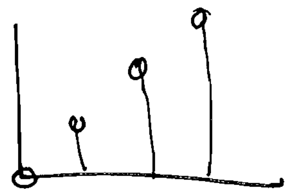
Matlab / Simulink



pulse

= unit sample

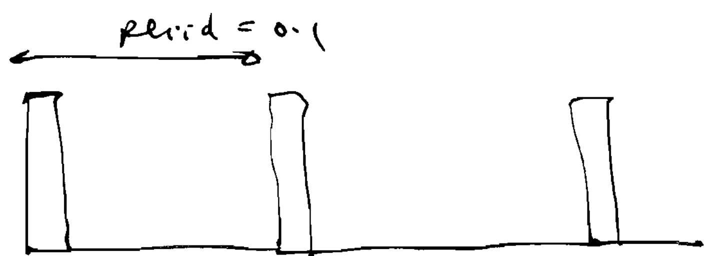
This is not a correct interpretation of the discrete time signal, which should



instead.

zohramp.mdl : $T = 0.1$ s

impulse train :



width $= 10^{-4}$

height $= 10^{4}$

area = 1

Basic Properties

- Model conversion / Coordinate transformation
- Frequency response
- Stability / speed of response
- Time / Frequency Domain Solutions

Intermediate Properties

- System with delay
- Controllability / Observability
- Pole placement / observer design
- Selection of sampling rate
- Implementation Issue

Control Properties

- Parameter optimized : eg PID
- State — observer based
  industrial regulator
  LQR
- Feedforward / deadbeat / Smith predictor / Shaping

$$\dot{x} = \underbrace{\begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}}_{A_c} x + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{B_c} u$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x$$

continuous time eigenvalues : $\pm j\sqrt{2}$ undamped

(I) Convert to discrete time with $T_s = 0.1\,s$

$$A = e^{A_c T_s}$$

$$= \begin{pmatrix} s & -1 \\ 2 & s \end{pmatrix}^{-1} \Bigg|_{t=0.1} = \begin{pmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ \sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{pmatrix}_{t=0.1}$$

$$= \begin{pmatrix} 0.9900 & 0.0997 \\ -0.1993 & 0.9900 \end{pmatrix}$$

$$B = \int_0^{T_s} e^{At}\,dt\, B_c = \begin{pmatrix} 0.00499 \\ 0.09967 \end{pmatrix}$$

(II) Pole placement

$$\mathrm{eig}(A) = 0.99 \pm j\,0.14$$

$\boxed{U\text{-do}}$

Desired : $\frac{1}{2}, \frac{1}{2} \rightarrow$ continuous time : -6.93

controllability $M_c = [B \;\; AB] = \begin{bmatrix} 0.0050 & 0.0149 \\ 0.0997 & 0.0977 \end{bmatrix}$

rank $M_c = 2$
controllable

$$M_c^{-1} = \begin{bmatrix} -98.1668 & 14.9499 \\ 100.168 & -5.0167 \end{bmatrix} \leftarrow e_2'$$

$$P = \begin{pmatrix} 100.1668 & -5.0167 \\ 100.1688 & +5.0167 \end{pmatrix}$$

Old characteristics poly: $z^2 - 1.98z + 1$

New " " : $z^2 - z + 0.25$

$\therefore K = [+0.25 - 1 \quad -1 + 1.98] \, P$

$= [23.038 \quad 8.6789]$

Check $\operatorname{eig}(A - BK) = 0.5041,\ 0.4959 \sim 0.5,\ 0.5$ ok

$\boxed{\text{U-do}}$ Find K so that $\operatorname{eig}(A-BK) = 0, 0$

ans: $[98.17 \quad 14.95]$

deadbeat response

(III) Industrial Regulator

$$\left.\begin{aligned} x(n+1) &= A\,x(n) + B\,u(n) \\ y(n) &= C\,x(n) \\ e(n) &= y^{ref} - y(n) \\ \eta(n+1) &= \eta(n) + e(n) \end{aligned}\right\} \begin{pmatrix} x(n+1) \\ \eta(n+1) \end{pmatrix} = \mathcal{A} \begin{bmatrix} x(n) \\ \eta(n) \end{bmatrix} + \mathcal{B}\,u(n)$$

$$\mathcal{A} = \begin{pmatrix} A & 0 \\ -C & 1 \end{pmatrix} \qquad \mathcal{B} = \begin{pmatrix} B \\ 0 \end{pmatrix}$$

eigenvalue : $0.99 \pm j0.14$, 1
to shift to 0.5, 0.5, 0.5

$$M_c = [B \quad AB \quad A^2B] = \begin{bmatrix} 0.005 & 0.0149 & 0.0245 \\ 0.0997 & 0.0977 & 0.0937 \\ 0 & -0.005 & -0.0199 \end{bmatrix}$$

rank = 3 OK

u-do

$$\begin{array}{ll} M_c^{-1} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ -50.08 & 2.508 & -100.17 \end{bmatrix} & \leftarrow e_3' \end{array}$$

$$P = \begin{bmatrix} e_3' \\ e_3'A \\ e_3'A^2 \end{bmatrix} = \begin{bmatrix} -50.08 & 2.508 & -100.17 \\ 50.08 & -2.508 & -100.17 \\ 150.25 & 2.508 & -100.17 \end{bmatrix}$$

$\boxed{\text{U-do}}$ $PAP^{-1} = ?$

Old CP: $z^3 - 2.98z^2 + 2.98z - 1$
New CP = $z^3 - 1.5z^2 + 0.75z - 0.125$

$\therefore K = [-0.125+1 \quad 0.75-2.98 \quad -1.5+2.98]\, P$
$= [66.86 \quad 11.50 \quad -12.52]$

Check eig$(A - BK)$ = $0.51 \pm j0.02$, 0.48 ~ 0.5, 0.5, 0.5

$\therefore u(n) = -66.86\, x_1(n) - 11.5\, x_2(n) + 12.52\, x_3(n)$

$\boxed{\text{U-do}}$ place all poles to 0, 0, 0

ans $K = [\,248.42 \quad 17.46 \quad -100.17\,]$

(IV) Observer design

$$\hat{x}(n+1) = A\hat{x}(n) + Bu(n) + L[y(x) - C\hat{x}(n)]$$

Design L so that (A-LC) has eigenvalues twice as fast as closed loop plant (0.5, 0.5)

$0.5 \longrightarrow -6.93$

twice as fast $: -13.86 \xrightarrow{-/-} \lambda = \frac{1}{4}$

$\boxed{\text{U-do}}$ thrice as fast

$\lambda = \frac{1}{8}$ not $\frac{1}{6}$

Using duality

let $B_{dual} = C' = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$A_{dual} = A' = \begin{pmatrix} 0.99 & -0.1993 \\ 0.0997 & 0.9900 \end{pmatrix}$$

Find K_{dual} so that $(A_{dual} - B_{dual} K_{dual})$ has eigenvalues at 0.25, 0.25

U-do

$K_{dual} = [\,1.4800 \quad 5.2952\,]$

then $L = K_{dual}' = \begin{bmatrix} 1.48 \\ 5.30 \end{bmatrix}$

check eig$(A - LC) = 0.25, 0.25$ checks

Undo Find L to shift observer eigenvalues to $0, 0$

ans $L = \begin{bmatrix} 1.9800 \\ 9.6347 \end{bmatrix}$

(V) Combined control and estimation

(A) State feedback only

$$x(n+1) = A\,x(n) + Bu(n)$$
$$y(n) = Cx(n)$$
$$\hat{x}(n+1) = A\,\hat{x}(n) + Bu(n) + L\,(y(n) - C\hat{x}(n))$$
$$u(n) = -K\hat{x}(n)$$

or

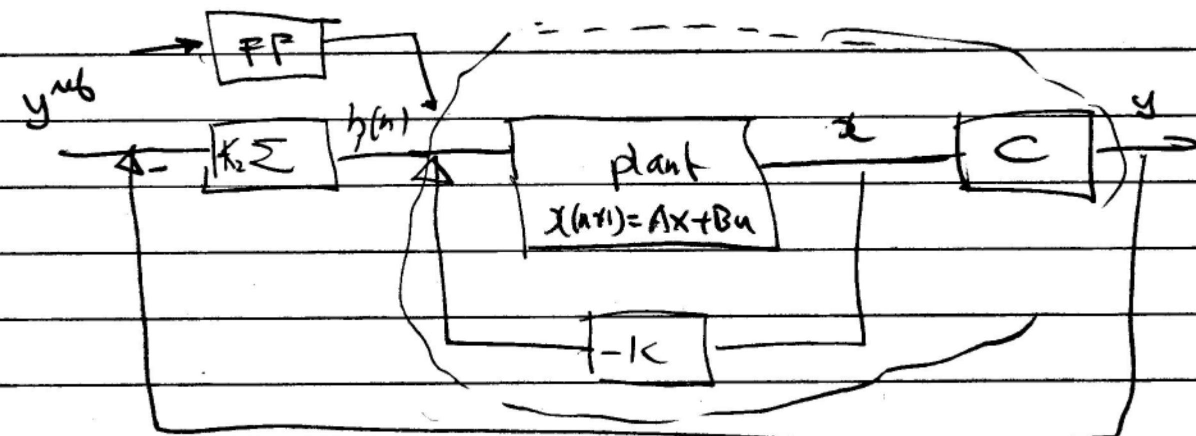
$$\left[\begin{aligned} \hat{x}(n+1) &= (A - BK - LC)\hat{x} + Ly(n) \\ u(n) &= -K\,\hat{x}(n) \end{aligned}\right] \text{ controller}$$

(B) Industrial regulator

add
$$\left.\begin{aligned} \eta(n+1) &= \eta(n) + e(n) \\ e(n) &= y^{ref} - y(n) \\ \hat{x}(n+1) &= (A - BK_1 - LC)\,\hat{x}(n) - BK_2\,\eta(n) + Ly(n) \\ u(n) &= -K_1\,\hat{x}(n) - K_2\,\eta(n) \end{aligned}\right\} \text{ controller}$$

where $K = [K_1 \mid K_2]$

(VI) Feedforward control



$\eta(n)$ sees
$$x(n+1) = (A-BK)\,x(n) + B\,v(n)$$
$$y(n) = C\,x(n)$$
$$v(n) = -K_2\eta(n)$$

$\therefore$ DC gain $\left. C\,(zI - A + BK)^{-1} B \right|_{z=1}$

$= 0.0399$

$\therefore$ set $-\boxed{FF}-$ to $\frac{1}{0.0399} = 25.04$

to help $\eta(n)$ to reach the "right" steady state:

$y(\infty) = 1$

$\Rightarrow \eta(\infty) = 25.04$! (U-do)

(XII) Input shaping

continuous time system $\frac{1}{s^2+2}$

must put into $\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$

$$\therefore \quad G(s) = \frac{1}{2}\left(\frac{2}{s^2+2}\right)$$

$\zeta = 0$ $\quad\therefore\quad M_p = 1$

$\omega_0 = \sqrt{2}$ $\quad \Delta T = \frac{\pi}{\sqrt{2}}$

Shaper $\quad A_1 = 2 \cdot \frac{1}{2} = 1 = A_2 \,, \quad \Delta T = \frac{\pi}{\sqrt{2}}$