ECE667

A10 Solutions

1) $G(t) = \mathcal{L}^{-1}\left(\dfrac{(s+1)+4}{(s+1)^2+3^2}\right) = e^{-t}\cos 3t + \frac{4}{3}e^{-t}\sin 3t$

same

2) $\omega_d = \omega\sqrt{1-\zeta^2}$. $\zeta\omega = 1 \Rightarrow \zeta = \frac{1}{\sqrt{10}} \Rightarrow \omega_d = 3$

$G^c(s) = \dfrac{1}{2(s+1)}$ $\qquad G^s(s) = \dfrac{-\frac{4}{3}}{2(s+1)}$

$\downarrow \qquad\qquad \downarrow$

$G^c(t) = \frac{1}{2}e^{-t}$ $\qquad G^s(t) = -\frac{4}{3}\cdot\frac{1}{2}e^{-t}$

$\therefore\ G(t) = e^{-t}\cos 3t + \frac{4}{3}\sin 3t$ same as 1)

3) $\dot{x} = \begin{pmatrix} 0 & 1 \\ -10 & -2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$

$y = \begin{pmatrix} 5 & 1 \end{pmatrix} x$

4) $x_1 = \mathcal{L}^{-1}\left[\dfrac{1}{s^2+2s+10}\right] = \frac{1}{3}e^{-t}\sin 3t \rightarrow x_1$

$x_2 = \mathcal{L}^{-1}\left[\dfrac{s}{s^2+2s+10}\right] = e^{-t}\cos 3t - \frac{1}{3}e^{-t}\sin 3t \Rightarrow \underbrace{e^{-t}\cos 3t}_{y^c} = x_1 + x_2$

5) $u = K\, y^c \cos 3t = K(x_1 + x_2)$

$\dot{x} = \begin{pmatrix} 0 & 1 \\ -10+K & -2+K \end{pmatrix} x$

$\therefore\ K = 2$ will do.