

$$\dot{r} = r(C + 2r^2 - r^4)$$

$$\dot{\theta} = 2\pi$$

From Lecture 4

$$r^2 = x^2 + y^2$$

$$r\dot{r} = x\dot{x} + y\dot{y}$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$$\dot{\theta} = \frac{1}{1 + \left(\frac{y}{x}\right)^2} \frac{d}{dt} \frac{y}{x}$$

$$= \frac{x\dot{y} - y\dot{x}}{x^2 + y^2} \leftarrow r^2$$

$$\dot{r} = r(C + 2r^2 - r^4)$$

$$r\dot{r} = r^2(C + 2r^2 - r^4)$$

$$x\dot{x} + y\dot{y} = r^2(C + 2r^2 - r^4)$$

$$\dot{\theta} = 2\pi$$

$$2\pi r^2 = x\dot{y} - y\dot{x}$$

$$x\dot{x} + \frac{y}{x}(2\pi r^2 + y\dot{x}) = r^2(C + 2r^2 - r^4) \quad \therefore y\dot{y} = \frac{y}{x}(2\pi r^2 + y\dot{x})$$

$$\dot{x} = -2\pi y + x(C + 2(x^2 + y^2) - (x^2 + y^2)^2)$$

Sim

$$\dot{y} = 2\pi x + y(C + 2(x^2 + y^2) - (x^2 + y^2)^2)$$

For $C = -0.5$

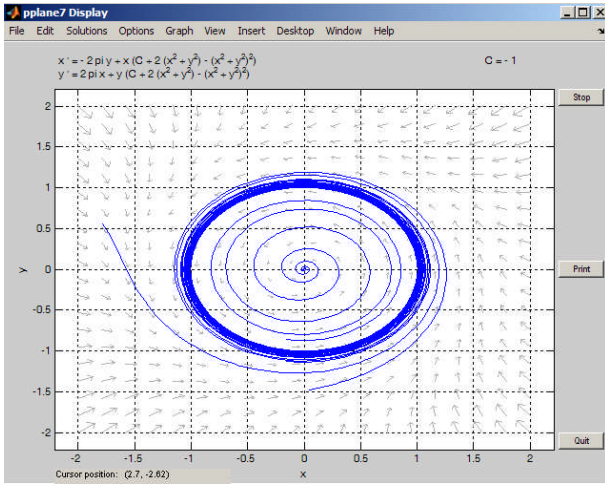
at about $r(0) = 0.545$, $\theta(0)$ any value
 \Rightarrow separatrix focus and limit cycle

$$r(0) < 0.545 \Rightarrow \text{decay to } 0$$

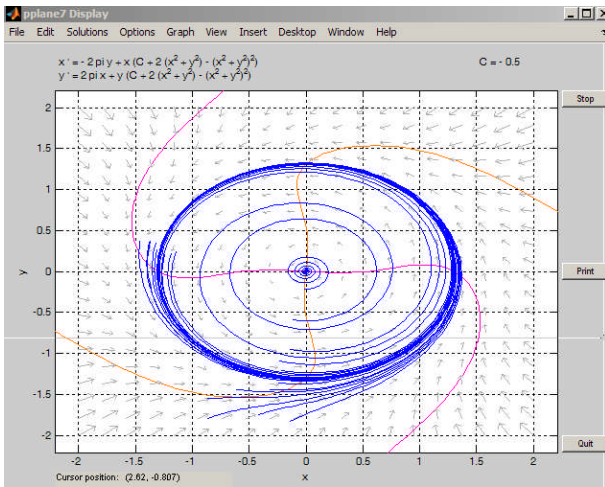
$$r(0) > 0.545 \Rightarrow \text{converge to limit cycle}$$

Similarly $r(0)$ boundary ≈ 1 $C = -1$

C=-1



C=-0.5



C=0

