Math 244 Exam II, Fall **Name:**_________________

November 02, 2011 **Student #:** _________________

Instructor: Dhar

Must show all work to receive full credit!!!

I pledge that I have not violated the NJIT code of honor__________________________

1. From a basket of fruit containing 3 oranges, 2 apple, and 3 bananas a random sample of 4 fruits are selected. If X is the number of oranges and Y is the number of apples in the sample, find:
   (a) The joint probability distribution of X and Y, with all probability values.
   (b) $P[X+Y \le 1]$. (Please see 3.39, page 105)
   (20 points)

(a) $P(X=x, Y=y) = \dfrac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}$, $x = 0,1,2,3$, $y = 0,1,2$.

$x \backslash y$	0	1	2	
0	0	2/70	3/70	5/70
1	3/70	18/70	9/70	30/70
2	9/70	18/70	3/70	30/70
3	3/70	2/70	0	5/70
	15/70	40/70	15/70	1

(b) $P(X+Y \le 1) = f(1,0) + f(0,1) = \frac{5}{70}$
$= 1/14 = .071428571$

2. If a dealer's profit, in units of \$3000.0, on a new automobile can be looked upon as a random variable X having density function

$$f(x) = \begin{cases} 3(1-x)^2, 0 \le x \le 1, \\ 0, \quad otherwise, \end{cases}$$

Find (a) the expected profit per automobile and (b) the standard deviation of profit in dollars
(15 points) (Please see 4.12, page 117)

$$EX = \int_0^1 3x(1-x)^2\,dx = 3\int_0^1 x^{2-1}(1-x)^{3-1}\,dx = 3\,\frac{\Gamma(2)\cdot\Gamma(3)}{\Gamma(5)} = \frac{3(2)}{4!} = \frac{1}{4}$$

$= .25$, i.e. $3000 \times \frac{1}{4} = \underline{\$750.00}$

$$E(X^2) = \int_0^1 3x^2(1-x)^2\,dx = 3\int_0^1 x^{3-1}(1-x)^{3-1}\,dx = 3\,\Gamma(3)\Gamma(3)/\Gamma(6) = \frac{3(4)}{5!} = \frac{1}{10}$$

$$Var(X) = \frac{1}{10} - \frac{1}{16} = \frac{6}{160} = \frac{3}{80} \qquad \sigma = .1936, \text{ i.e. } \$580.95$$

3. Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as

f(x, y)		x			
		1	2	3	
	1	0.05	0.05	0.10	.20
	3	0.05	0.10	0.35	.50
y	5	0	0.20	0.10	.30
		0.10	.35	.55	1

Determine the correlation coefficient between X and Y. (Please see problem 3.49, page 106 and 4.51 page 127) (20 pts)

$$EX = .1 + .7 + (.55)3$$
$$= .8 + 1.65 = 2.45$$
$$EX^2 = .1 + 1.4 + (1.65)3 = 1.50$$

$$\begin{array}{r} 4.95 \\ \hline 6.45 \end{array}$$

$$V(X) = 6.45 - (2.45)^2 = .4475$$

$$EY = .2 + 1.5 + 1.5 = 3.2$$
$$EY^2 = .2 + 4.5 + 7.5 = 12.2$$
$$V(Y) = 12.2 - (3.2)^2 = 12.2 - 10.24 = 1.96$$

$$\begin{aligned} EXY = &\ .05 + .1 + .30 \\ &+ .15 + .6 + 3.15 \\ &\ 0 + 2.0 + 1.50 \\ \hline &\ .20 + 2.70 + 4.95 = 7.85 \end{aligned}$$

$$\rho = \frac{\underbrace{7.85 - (3.2)(2.45)}_{7.84}}{\sqrt[2]{(.4475)(1.96)}} = \frac{.01}{.8771} = .01068$$

4. The waiting, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution function given by $F(x)$.

$$F(x)=\begin{cases} 0, & x<0, \\ 4x, & 0\le x<\frac{1}{4}, \\ 1, & \frac{1}{4}\le x. \end{cases}$$

(a) What the probability the waiting time is exactly 15 minutes?

$$P\left(X=\frac{15}{60}\right) = P\left(X=\frac{1}{4}\right) = 0 = \text{Jump at } \frac{1}{4} \text{ in } F$$

(b) Find the probability the waiting time is at least ten minutes.

(Please see problem 3.14, page 92) (15 pts)

$$P\left(X\ge\frac{10}{60}\right) = P\left(X>\frac{1}{6}\right) = 1-F\left(\frac{1}{6}\right) = 1-\frac{4}{6} = 1-\frac{2}{3} = \frac{1}{3}$$

5. Two electronic components of a missile system work in harmony for the success of the total system. Let X and Y denote the life in hours of the two components. The joint density of X and Y is $f(x,y)=\begin{cases} ye^{-y(1+x)}, & x\ge 0, y\ge 0, \\ 0, & \textit{otherwise.}\end{cases}$

a. Give marginal density of both random variables.
b. Are X and Y independent? Explain.
c. What is the probability the lives of both components will exceed 1 hour.

(Please see problems number 3.63, page 107). (20 pts)

$$(a)\ g(x) = \int_0^\infty y e^{-y(1+x)} dy = y \frac{e^{-y(1+x)}}{-(1+x)}\Bigg|_{y=0}^{y=\infty} + \int_0^\infty \frac{e^{-y(1+x)}}{(1+x)} dy$$

$$= \frac{1}{1+x}\int_0^\infty e^{-y(1+x)} dy = \begin{cases} \frac{1}{(1+x)^2}, & 0<x<1, \\ 0, & \text{otherwise.}\end{cases}$$

$$(a)\ h(y) = \int_0^\infty y e^{-y}\cdot e^{-yx} dx = \cancel{y} e^{-y} \frac{e^{-yx}}{-\cancel{y}}\Bigg|_{x=0}^{x=\infty}$$

$$= \begin{cases} e^{-y} & 0<y<\infty, \\ 0, \text{ elsewhere,} \end{cases}$$

(b) **X and Y are dependent because the exponential term in the joint density f(x,y) can not be factored.**

(c) $P(X>1, Y>1)$

$$= \int_1^\infty \int_1^\infty y e^{-y(1+x)} dx\, dy = \int_1^\infty \cancel{y} e^{-y} \frac{e^{-yx}}{-\cancel{y}}\Bigg|_{x=1}^{x=\infty} dy$$

$$= \int_1^\infty e^{-2y} dy = \frac{e^{-2y}}{-2}\Bigg|_1^\infty = \boxed{e^{-2}/2} = 0.067667642$$

6. The joint density for the random variables (X, Y), where X is the unit temperature change and Y is the proportion of spectrum shift that a certain atomic particle produces is $f(x,y)=\begin{cases}8xy, 0<x<y<1,\\ 0, \quad otherwise.\end{cases}$

Find the probability that the spectrum shifts less than half of the total observations, given that the temperature is increased by 0.25 units (Please see example 3.19, page 100). (10 pts)

$$P\left(Y<\frac{1}{2} \,\middle|\, X=\frac{1}{4}\right) = \int_{1/4}^{1/2} f\left(y/X=\frac{1}{4}\right) dy$$

$$f(x\overset{1/4}{=}, y) = \begin{cases} 2y, & \frac{1}{4}<y<1 \\ 0, & \text{elsewhere} \end{cases}$$

$$g(x) = 8x\int_x^1 y\,dy = 4x(1-x^2), 0<x<1$$

$$g\left(\frac{1}{4}\right) = 1-\frac{1}{16} = \frac{15}{16}$$

$$f\left(y/X=\frac{1}{4}\right) = \begin{cases} \frac{32}{15}y, & \frac{1}{4}<y<1 \\ 0 & \text{elsewhere.} \end{cases}$$

$$\int_{1/4}^{1/2} \frac{32}{15}y\,dy = \frac{\overset{16}{\cancel{32}}}{15}\frac{y^2}{\cancel{2}}\Big|_{1/4}^{1/2}$$

$$= \frac{16}{15}\left(\frac{1}{4}-\frac{1}{16}\right)$$

$$= \frac{3}{15} = \frac{1}{5} = 0.2 \;//$$

check

$$\frac{16}{15}y^2\Big|_{1/4}^{1}$$

$$\frac{16}{15}\left(1-\frac{1}{16}\right)$$

$$= 1 \checkmark$$

END