Math 244-Final Exam

Name:________________

December 16, 2011 **Student #:** ______________

Instructor: Dhar

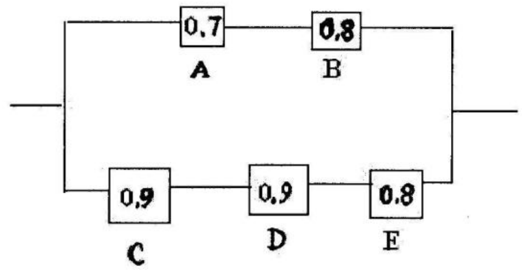
Must show all work to receive full credit!!!

I pledge that I have not violated the NJIT code of honor____________________________

1. How many even four digit numbers can be formed from the digits 0, 1, 2, 5, 6, and 9 if each digit can be used only once? Show all work. (10 points)
   (Example 2.17, page 46)

2. There is a 50-50 chance that the queen carries the gene of hemophilia. If she is a carrier, then each prince has a 50-50 chance of having hemophilia independently. If the queen is not a carrier, the prince will not have the disease. Suppose the queen has had three princes without the disease. What is the probability the queen is a carrier? Show all work. (10 points)
   (2.127, page 79)

3. A circuit system is given in the figure below. Assume that the components (A, B, C, D, and E) fail independently.
   a. What is the probability that the entire system works?
   b. Given that the system works, what is the probability that the component A is not working? (10 points) (2.93, page 71)



4. Let X denote the diameter of an armored electric cable and Y denote the diameter of the ceramic mould that makes the cable. Both X and Y are scaled so that they range between 0 and 1. Suppose that X and Y have the joint density

$$f(x,y)=\begin{cases}\dfrac{1}{y}, & 0<x<y<1,\\ 0, & \text{elsewhere.}\end{cases}$$

Find the probability P(X + Y > ½). (10 points) (3.45, page 105)

5. The amount of kerosene, in thousands of liters, in a tank at the beginning of any day is a random amount Y from which a random amount X is sold during the day. Suppose that the tank is not resupplied during the day so that $x \le y$, and assume that the joint density function of these variables is $f(x,y)=\begin{cases}2, & 0\le x\le y<1,\\ 0, & \text{elsewhere.}\end{cases}$

a. Determine if X and Y are independent.
b. Find the average amount of kerosene left in the tank at the end of the day in liters. (Please see 3.47, p. 105 and 4.81, p. 130) (12 points)

6. Suppose that the probability that any given person will believe a tale about the transgressions of a famous actress is 0.7. What is the probability that :

a. The sixth person to hear this tale is the fourth one to believe it?

b. The fourth person to hear this tale is the first one to believe it?

(14 points) (Please see 5.59, p. 165)

7. A random variable X has the discrete uniform distribution

$$f(x;k)=\begin{cases} 1/k, & x=1,2,\cdots k, \\ 0, & elsewhere. \end{cases}$$

Show that the moment-generating function of X is

$$M_X(t)=\frac{e^t\left(1-e^{kt}\right)}{k\left(1-e^t\right)}.$$

(10 points) (7.17, page 224)

8. A lawyer commutes daily form his suburban home to his midtown office. The average time for his one-way trip is 24 minute, with a standard deviation of 3 minutes. Assume the distribution of trip times to be normally distributed.

a. What is the probability that a trip will take at least ½ hour?
b. Find the length of time above which we find the slowest 10% of the trips.
c. Find the probability that exactly one of the next three trips will take at least ½ hour.

(24 points) (6.15, page 186-7)