Math 244 fall 2011 Final Exam Solution

#1

zero here $\downarrow 0$ no zero in here

$\boxed{5}\boxed{4}\boxed{3}\boxed{1} + \boxed{4}\boxed{4}\boxed{3}\boxed{2}$

$60 + 96 = 156$

#2

D: Queen has disease of hemophilia

R: Prince has hemoplia

$P(D) = .5$

$P(R/D) = .5 \Rightarrow P(R'/D) = .5$

$P(R \mid D') = 0$

$$P(D \mid R_1' \cap R_2' \cap R_3') = \frac{P(R_1' \cap R_2' \cap R_3' \mid D)P(D)}{P(R_1' \cap R_2' \cap R_3')}$$

$$= \frac{(1/8)(\frac{1}{2})}{1/16 + P(R_1' \cap R_2' \cap R_3' \mid D')P(D')} = \frac{1/16}{1/16 + \frac{1}{2}}$$

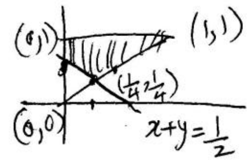
$$= .1111 = \frac{\frac{1}{16}}{9/16} = \frac{1}{9}$$

3.

$$P(\text{system works})$$
$$= 1 - P(\text{system fails})$$
$$= 1 - P(AB \text{ fails})\,P(CDF \text{ fails})$$
$$= 1 - [1-(.7)(.8)][1-(.9)^2(.8)]$$
$$= 1 - [1-.56][1-(.81)(.8)]$$
$$= 1 - (.44)(1-.648]$$
$$= 1 - (.44)(.352) = 1 - .15488$$
$$= .84512$$

$$P(A \text{ fails} / \text{system work}) = \frac{P(A \text{ fails} \cap \text{sytem works})}{P(\text{system works})}$$
$$= \frac{P(A \text{ fails} \cap C \text{ works} \cap D \text{ works} \cap E \text{ works})}{.84512}$$
$$= \frac{(.3)(.648)}{.84512} = \frac{.1944}{.84512} = .2300265505$$

4.



$$1 - P\left(X+Y<\frac{1}{2}\right)$$

$$= 1 - \int_0^{1/4}\int_x^{1/2-x} \frac{1}{y}\,dy\,dx$$

$$= 1 - \int_0^{1/4} \ln y \Big|_x^{1/2-x}\,dx$$

$$= 1 - \int_0^{1/4} \ln\left(\tfrac{1}{2}-x\right) - \ln x \; dx$$

$$= 1 - \int_0^{1/4} \ln\left(\tfrac{1}{2}-x\right)dx + \int_0^{1/4}\ln x\,dx$$

$$= 1 - \left[x\ln\left(\tfrac{1}{2}-x\right)\Big|_0^{1/4} + \int_0^{1/4}\frac{x}{1/2-x}\,dx\right] + x\ln x\Big|_0^{1/4} - \int_0^{1/4}dx$$

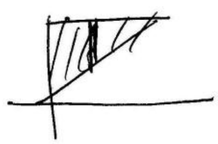
$$= 1 - \left[\cancel{\tfrac{1}{4}\ln\tfrac{1}{4}} - \cancel{\tfrac{1}{4}}\tfrac{1}{2}\int_0^{1/4}\frac{1}{1/2-x}\,dx\right] + \cancel{\tfrac{1}{4}\ln\tfrac{1}{4}} - \cancel{\tfrac{1}{4}} = 1 - \frac{1}{2}\left[\frac{\ln(1/2-x)}{-1}\right]_0^{1/4}$$

$$\text{Ans} = 1 - \frac{1}{2}\left[\ln\frac{1}{2} - \ln\frac{1}{4}\right]$$

$$= 1 + \frac{1}{2}\ln\frac{1}{2} = .65342641$$

5 (a) X & Y are dependent because the density sits on a triangle.

(b) $E(Y-X) = \int_0^1 \int_x^1 2(y-x)\, dy\, dx$



$$= \int_0^1 y^2 - 2xy \Big|_x^1 \, dx$$

$$= \int_0^1 1 - 2x - x^2 + 2x^2 \, dx$$

$$= \int_0^1 x^2 - 2x + 1 \, dx$$

$$= \int_0^1 (x-1)^2 \, dx$$

$$= \frac{(x-1)^3}{3} \Big|_0^1 = 1/3 \,//$$

#6

(a) $\binom{5}{3} \cdot (.7)^4 (.3)^2$

$10\,(.49)^2(.09)$

$= .21609 \,//$

(b) $(.7)(.3)^3$

$.7(.027)$

$= .0189 \,//$

Ans 333.33 liters

6. Suppose that the probability that any given person will believe a tale about the transgressions of a famous actress is 0.7. What is the probability that :

a. The sixth person to hear this tale is the fourth one to believe it?

b. The fourth person to hear this tale is the first one to believe it?

(Please see 5.59, p. 165) (14 points)

7. A random variable X has the discrete uniform distribution

$$f(x;k)=\begin{cases} 1/k, & x=1,2,\cdots k, \\ 0, & elsewhere. \end{cases}$$

Show that the moment-generating function of X is

$$M_X(t)=\frac{e^t\left(1-e^{kt}\right)}{k\left(1-e^t\right)}.$$ (10 points) (5.139, page 275)

$$M_X(t)=\sum_{i=1}^{k}\frac{1}{k}e^{it}=\frac{1}{k}\sum_{i=1}^{k}e^{it}$$

finite sum of geometric series

$$=\frac{1}{k}e^t\frac{\left(1-(e^t)^k\right)}{1-e^t}$$

= desired result.

8. A lawyer commutes daily form his suburban home to his midtown office. The average time for his oneway trip is 24 minute, with a standard deviation of 3 minutes. Assume the distribution of trip times to be normally distributed.

a. What is the probability that a trip will take at least ½ hour?
b. Find the length of time above which we find the slowest 10% of the trips.
c. Find the probability that exactly one of the next three trips will take at least ½ hour.

(8 points each) (6.15, page 187)

a) $P(X > 30)$

$$P\left(Z > \frac{30-24}{3}\right) = P(Z > 2) = .0228$$

(b) $P(X > x) = .10$

$$P\left(Z > \frac{x-24}{3}\right) = .10$$

$$\frac{x-24}{3} = 1.28$$

$$x - 24 = 3.84$$

$$x = 27.84 \text{ min}$$

$$P(Y=1) = \binom{3}{1} (.0228)^1 (1-.0228)^2$$

$$= 3\,(.0228)(0.9772)^2$$

$$= .065316517$$