

FORMULA SHEET

$$\bar{x} = \frac{x_1 + \dots + x_n}{n} = \frac{\sum_{i=1}^n x_i}{n}, s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} = \frac{\sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2 / n}{n-1}. \quad \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

$$f(x) \geq 0, \int_a^b f(x) dx = 1. P(a < X < b) = \int_a^b f(x) dx. V(X) = EX^2 - \mu^2,$$

$$\mu = E(X) = \int xf(x) dx \text{ and } V(X) = \int (x - \mu)^2 f(x) dx. EX^2 = \int x^2 f(x) dx.$$

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du$$

$$f(x_i) = P(X = x_i). \mu = E(X) = \sum_{i=1}^n x_i f(x_i) \text{ and } \sigma^2 = V(X) =$$

$$\sum_{i=1}^n (x_i - \mu)^2 f(x_i) = \sum_{i=1}^n x_i^2 f(x_i) - \mu^2, F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$$

Random variable X is uniformly distributed over (a, b) then: $f(x) = \frac{1}{b-a}$, $a < x < b$, $f(x) = 0$,

elsewhere, and $E(X) = \frac{a+b}{2}$, $V(X) = \frac{(b-a)^2}{12}$.

Normal random variable: $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ $E(X) = \mu$, $V(X) = \sigma^2$

$$\Phi(z) = P(Z \leq z). Z = \frac{X - \mu}{\sigma}, V(Z) = 1.$$

Binomial distribution

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, \dots, n, \mu = E(X) = np \text{ and } \sigma^2 = V(X) = np(1-p).$$

Poisson distribution

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, \dots \quad E(X) = \lambda \text{ and } V(X) = \lambda.$$

Exponential distribution

$$f(x) = \lambda e^{-\lambda x}, \text{ for } 0 \leq x < \infty, E(X) = \frac{1}{\lambda} \text{ and } V(X) = \frac{1}{\lambda^2}. P(X > x) = e^{-\lambda x}, \text{ for } 0 < x < \infty.$$

The random variables X_1, \dots, X_n are independent if

$$P(X_1 \in E_1, X_2 \in E_2, \dots, X_n \in E_n) = P(X_1 \in E_1) \cdot P(X_2 \in E_2) \cdots P(X_n \in E_n) \text{ for any sets } E_1, \dots, E_n.$$

Let $Y = c_0 + c_1 X_1 + c_2 X_2 + \dots + c_n X_n$, then $E(Y) = c_0 + c_1 E(X_1) + c_2 E(X_2) + \dots + c_n E(X_n) = c_0 + c_1 \mu_1 + c_2 \mu_2 + \dots + c_n \mu_n$. If X_1, \dots, X_n are mutually independent then

$$V(Y) = c_1^2 \sigma_1^2 + c_2^2 \sigma_2^2 + \dots + c_n^2 \sigma_n^2$$

$$\rho_{X_1 X_2} = \frac{E(X_1 X_2) - \mu_1 \mu_2}{\sqrt{\sigma_1^2 \sigma_2^2}} = \frac{Cov(X_1, X_2)}{\sqrt{\sigma_1^2 \sigma_2^2}}.$$

Let X_1, \dots, X_n be random variables with mean $E(X_i) = \mu_i$ and variances

$$V(X_i) = \sigma_i^2, i = 1, 2, \dots, n, \text{ and covariance } Cov(X_i, X_j), i, j = 1, 2, \dots, n, i < j.$$

$E(Y) = c_0 + c_1 \mu_1 + c_2 \mu_2 + \dots + c_n \mu_n$ and the variance of Y is

$$V(Y) = c_1^2 \sigma_1^2 + c_2^2 \sigma_2^2 + \dots + c_n^2 \sigma_n^2 + 2 \sum \sum c_i c_j Cov(X_i, X_j)$$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}, \text{ SE of } \bar{X} = \frac{\sigma}{\sqrt{n}},$$

100(1-a) % Confidence interval on the mean, μ , σ^2 variance known.

$$\bar{x} - \frac{z_{\alpha/2} \sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + \frac{z_{\alpha/2} \sigma}{\sqrt{n}}. \quad n = \left(\frac{z_{\alpha/2} \sigma}{E} \right)^2.$$

100(1-a) % Confidence interval on the mean of a normal distribution, μ , σ^2 variance unknown

$$\bar{x} - \frac{t_{\alpha/2, n-1} s}{\sqrt{n}} \leq \mu \leq \bar{x} + \frac{t_{\alpha/2, n-1} s}{\sqrt{n}}. \quad T = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}, v = n-1.$$

100(1-a) % Confidence interval on the variance of a normal distribution, σ^2 is

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}. \quad \chi^2 = \frac{(n-1)s^2}{\sigma^2}.$$