## MATH 333: Probability \& Statistics. Final Examination

(Fall 2004)

December 15, 2004 (A) NJIT

## Name:

SSN:
Section \#
Instructors: A. Jain, K. Johnson, H. Khan, K. Rappaport, S. Roychaudhury
$\rightarrow$ Must show all work to receive full credit.

I pledge my honor that I have abided by the Honor System. $\qquad$ (Signature)

|  | Score |
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| $\# 1$ |  |
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1. Suppose that the population of the gripping strength of industrial workers has a mean of 110 psi and a standard deviation of 10 psi. A random sample of 81 workers is taken. What is the probability that the sample mean will be:
a. Between 109 and 112 ? ( 4 points)
b. Greater than 111? (4 points)
c. Less than or equal to 108 . (4 points)

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2. At a certain gas station, $40 \%$ of the customers buy regular gas, $35 \%$ buy plus gas, and $25 \%$ buy premium gas. Of those customers buying regular gas, $30 \%$ fill their tanks. Of those buying plus gas, $60 \%$ fill their tanks. Of those buying premium gas, $50 \%$ fill their tanks. ( 5 points each)
a. What is the probability that the next customer will buy plus gas and fill the tank?
b. What is the probability that the next customer fills the tank?
c. If the next customer fills the tank, what is the probability that regular gas is bought?

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3. In a hypotheses-testing situation, where the alternate hypothesis is one-sided, suppose $\mathrm{H}_{0}$ was "not rejected" at $\alpha=0.02$. Answer the following questions as "yes" or "no" or "cannot tell". Explain briefly the reason for your answer. (No credit will be given, if you do not justify your answer)
a. Would $\mathrm{H}_{0}$ also be "not rejected" at $\alpha=0.01$ ? ( 2 points)
b. Would $\mathrm{H}_{0}$ also be "not rejected" at $\alpha=0.05$ ? ( 2 points)
c. Would the P -value be smaller than 0.02 ? ( 2 points)

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4. Based on past records, the population mean of the time needed to treat a disease by a standard therapy is 15 days. It is claimed that a new therapy can reduce the mean treatment time. To test this claim, the new therapy is tried on 70 patients that are randomly chosen from a large pool of such patients.
a. Formulate your hypotheses and explain why you chose these hypotheses. (5 points)
b. Suppose the data from 70 patients reveals that the sample mean and standard deviation are found to be 14.6 days and 3.0 days, respectively. Perform the hypothesis test using a $2.5 \%$ level of significance. ( 5 points)
c. Calculate the P -value and interpret the meaning of this P -value. (7 points)

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5. Based on past studies, it is considered reasonable to assume that the amount of meat consumed by an adult in U.S.A. in a year follows a normal distribution. The population mean is 218.4 pounds and the variance is 25 pounds.
a. Find the probability that an adult selected at random consumes more than 235 pounds per year. (7 points)
b. Suppose a random sample of 10,000 adults is selected. Compute the probability that exactly one adult will consume more than 235 pounds per year. ( 8 points)

## Math 333: December 15, 2004 (A)

6. Lifetimes of fans used in diesel engines follow an exponential distribution with the mean $=$ 25,000 hours. What is the probability that a randomly selected fan will last:
a. At least 20,000 hours? (3 points)
b. At most 30,000 hours? (3 points)
c. Between 20,000 hours and 30,000 hours? ( 3 points)
d. If a fan has worked for 20,000 hours, what is the probability that it will work for another 20,000 hours? (3 points)

## Math 333: December 15, 2004 (A)

7. Based on past studies, shear stress of glass-fibers follows a normal distribution. A random sample of 8 glass-fibers was chosen, which yielded the sample mean of 30.5 and the sample standard deviation is 3.0. Derive a $99 \%$ confidence interval for
a. The population mean. (4 points)
b. The population variance. (4 points)
c. Derive a $95 \%$ confidence interval for the population standard deviation. (4 points)

## Math 333: December 15, 2004 (A)

8. An urn contains 10 balls: 4 red and 6 green. One ball is selected at random from the urn and its color is recorded. The selected ball is then returned to the urn along with three new balls of the same color, which yields 13 balls in the urn. A second ball is selected at random from the urn that now contains 13 balls. What is the probability that:
a. The first ball selected is green? (2 points)
b. The second ball selected is red, given that the first ball selected is green? (3 points)
c. The second ball selected is green? ( 6 points)
