## MATH 333A: Probability \& Statistics. Examination \#2 <br> (Fall 2006)

November 15, 2006 NJIT

| Name: | SSN: | Section \# |
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$\rightarrow$ Must show all steps for each problem to receive full credit.

| Score |  |
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| $\# 1$ |  |
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I pledge my honor that I have abided by the Honor System. $\qquad$
(Signature)

1. The number of patients arriving for emergency treatment in a one-hour period in a hospital can be modeled by a Poisson process with a mean of 6. (7 pts each)
a. What is the probability that exactly 5 arrivals occur in a one-hour period?
b. What is the probability that at least 3 arrivals occur in a one half-hour period?
c. Let $Y$ be the number of people arriving at the emergency room in a 90-minute period. Find the mean and standard deviation of Y .

## Math 333: November 15, 2006 (A)

2. A sample of 20 parts is taken every hour from a metal punching process. Typically, $1 \%$ of the parts require rework. Let $X$ denote the number of parts in the sample of 20 in one hour that require rework. Assume that the parts are produced and sampled independently. (7 pts each)
a. If the percentage of parts that require rework remains at $1 \%$, what is the probability that X exceeds its mean by more than 1.5 standard deviations?
b. If the rework percentage increases to $4 \%$, what is the probability that $X$ exceeds 1 ?
c. If the rework percentage increases to $4 \%$, what is the probability that $X$ exceeds 1 in at least one of the next 5 hourly samples of 20 parts each?

## Math 333: November 15, 2006 (A)

3. The time it takes for a cell to divide (a process, biologists call mitosis) can be considered to be Normally distributed. ( 9 pts each)
a. If $14 \%$ of all cells complete mitosis within 54.6 minutes, while $10 \%$ of all cells need over 66.5 minutes to divide, find the mean and the standard deviation of the time taken by a cell to undergo mitosis.
b. By what time do $99 \%$ of all cells complete mitosis ?

## Math 333: November 15, 2006 (A)

4. At a local branch of a certain bank with a single drive-in service window, the amount of time a teller spends in servicing a customer is exponentially distributed with a mean of 5 minutes. ( 7 pts each)
a. What is the probability that at least 3 customers are served at the drive-in window within 15 minutes?
b. If there is one car ahead of you at the drive-in service window at the time you arrive, what is the probability that you will be still waiting for service after the next 4 minutes?
c. What is the median time spent by a teller in servicing a customer?

## Math 333: November 15, 2006 (A)

5. A filling station gets its gasoline supply once a week. Its weekly sales volume (in thousand-gallon units) is distributed with the following probability density function:

$$
f(x)=5(1-x)^{4}, \quad \text { if } \quad 0 \leq x \leq 1 .
$$

a. What should the capacity of the filling station's underground tank be, so that the chance of the filling station running out of gas in a given week is only $1 \%$ ? ( 7 pts )
b. If gasoline is sold at $\$ 2.05 /$ gallon, what is the average weekly revenue of the filling station? ( 5 pts)
c. What is the cumulative distribution function of the weekly sales volume? (7 pts)

