**MATH 333**                   **FINAL EXAM**                          **December 15, 1999**

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I pledge my honor that I have abided by the Honor System.  ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯
Show your work for full credit.

1. The following data, recorded in days, represent the length of time to recovery for patients randomly treated with one of two medications to clear up severe infections:

| Medication I | Medication II |
|---|---|
| $n_1 = 14$ | $n_2 = 14$ |
| $\bar{x}_1 = 17$ | $\bar{x}_2 = 19$ |
| $s_1^2 = 1.5$ | $s_2^2 = 1.8$ |

   (a) (10 pts) Find a 99% confidence interval for the difference in mean recovery times for the two medications.

   (b) (4 pts) What assumptions about the underlying populations are necessary for your confidence interval to be reliable?

2. (10 pts) If an electrical connector on a laptop computer is kept dry, the probability that it fails during the warranty period is 1%. If the connector is ever wet, the probability of failure during the warranty period increases to 5%. Suppose 90% of the connectors are kept dry and the other 10% get wet. Compute the proportion of connectors that fail during the warranty period.

3. A telephone company is trying to decide whether some new lines in a large community should be installed underground. Because a small surcharge will be added to telephone bills to pay for the extra installation costs, the company has decided to survey customers and proceed only if the survey strongly indicates that more than 60% of all customers favor underground installation. If 118 of 160 customers surveyed favor underground installation despite the surcharge, what should the company do?

   (a) (5 pts) State the relevant null and alternative hypotheses.

   (b) (5 pts) Compute the $p$-value for the test of part (a).

(c) (5 pts) State your conclusion concerning the test of part (a) in the words of the problem, assuming $\alpha = 0.05$.

4. Suppose a particular state allows individuals filing tax returns to itemize deductions only if the total of all itemized deductions is at least $5000. Let $X$ (in thousands of dollars) be the total of itemized deductions on a randomly chosen form. Assume that $X$ has a probability density function ($pdf$)

$$f(x) = \frac{k}{x^3}, x \geq 5, \quad f(x) = 0 \text{ otherwise.}$$

(a) (5 pts) Find $k$ such that $f$ is a valid $pdf$.

(b) (10 pts) What is the expected total deduction on a randomly chosen form that contains itemized deductions? If you did not answer part (a), assume $k = 100$.

(c) (5 pts) Find the deduction amount which is exceeded for only 10% of the individuals. If you did not answer part (a), assume $k = 100$.

5. The dean of students claims that the average time spent studying for a statistics final at a certain technological institution is 5 hours, with standard deviation 2 hours.

(a) ( 10 pts) Suppose we survey 100 randomly selected students about their study times. Assuming the dean's claim is true, what is the probability that the sample average study time for these 100 students is more than 5.5 hours?

(b) (5 pts) Suppose we do not trust the dean's claim that the average study time is 5 hours and wish to estimate for ourselves the true average study time. Assuming that the standard deviation in study times is 2 hours, how many students would we need to survey in order to estimate the true mean study time to within 10 minutes with at least 95% confidence?

6. On Christmas decorations, lights are usually connected in parallel, so that bulbs will still light up even if some fail. If they are connected in series, one failure of a light bulb causes the entire string to fail. Assume each light bulb fails with probability 0.01, independently of the others, and we have a string of Christmas lights containing 20 light bulbs.

   (a) (7 pts) What is the probability of the whole string failing to light if the bulbs are connected in parallel?

   (b) (7 pts) What is the probability of the whole string failing to light if the bulbs are connected in series?

7. The boxplots below describe the temperatures in Celsius at which a certain chemical reaction started under two conditions, A and B. Twenty experiments were conducted under Condition A, while 40 were conducted under Condition B.

   (a) (7 pts) In total, how many times did the reactions require a temperature of 475 Celsius or above to start?

   (b) (5 pts) Which condition has the larger variability in reaction temperatures?
   A ☐                    B ☐                    They are approximately the same ☐