Formulas: $F(x) = P(X \le x)$. If X is discrete random variable taking values $x_1, x_2, \ldots$. then $x_{n-1} \le t < x_n$, $F(t) = \sum_{i=1}^{n-1} P(X = x_i)$. $E(X) = \sum_{x \in A} xp(x)$. $E[g(X)] = \sum_{x \in A} g(x)p(x)$.

$E[\alpha_1 g_1(X) + \alpha_2 g_2(X) + \cdots + \alpha_n g_n(X)] = \alpha_1 E[g_1(X)] + \alpha_2 E[g_2(X)] + \cdots + \alpha_n E[g_n(X)]$.

Var(X) = E[(X-μ)^2]. [EX]$^2 \le$ E[X^2]. $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Binomial $p(x) = \binom{n}{x} p^x (1-p)^{n-x}$, x = 0, 1, …, n, $p(x) = 0$, elsewhere. E(X) = np, Var(X) = $np(1-p)$.

Poisson $P\{N(t) = n\} = e^{-\lambda t} \frac{(\lambda t)^n}{n!}$, $n = 0, 1, 2, \ldots$, $p(x) = 0$, elsewhere.
$E[N(t)] = \mathrm{Var}[N(t)] = \lambda t$.

Geometric $p(x) = (1-p)^{x-1} p, 0 < p < 1, x = 1, 2, 3, \ldots$, $p(x) = 0$, elsewhere. E(X) = $\frac{1}{p}$, Var(X) = $\frac{1-p}{p^2}$.

Negative Binomial $p(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, 0 < p < 1, x = r, r+1, r+2, \ldots$, $p(x) = 0$, elsewhere. E(X) = $\frac{r}{p}$, Var(X) = $\frac{r(1-p)}{p^2}$. $P(X \in A) = \int_A f(x)\, dx$. If f the density function is continuous then $F'(x) = f(x)$.

Let X is a continuous random variable with density function f_X and the set of possible values A. For the invertible function h: $A \to$ R , let $Y = h(X)$ be a random variable with the set of possible values $B = h(A) = \{ h(a)$: $a \in A\}$. Suppose the inverse of $y = h(x)$ ($y = h(x)$ can be uniquely solved for x) is the function, $x = h^{-1}(y)$, which is differentiable for all y in B. Then f_Y, the density function of Y is given by

$$f_Y(y) = f[h^{-1}(y)] \left| \left(h^{-1}\right)'(y) \right|, \ y \in B.$$

$\mu = E(X) = \int_{-\infty}^{\infty} xf(x)\, dx$. $E(X) = \int_0^{\infty} [1 - F(t)]\, dt - \int_0^{\infty} F(-t)\, dt$. If X is non-negative then $E(X) = \int_0^{\infty} [1 - F(t)]\, dt$ and $E(X^r) = r\int_0^{\infty} t^{r-1} [1 - F(t)]\, dt$. $E[h(X)] = \int_{-\infty}^{\infty} h(x) f(x)\, dx$.

Gamma Density $f(x) = \begin{cases} \dfrac{\lambda e^{-\lambda x} (\lambda x)^{r-1}}{\Gamma(r)}, & if \quad x > 0 \\ 0, & if \quad x \le 0 \end{cases}$, $E[X] = \frac{r}{\lambda}$,

$Var(X) = \frac{r}{\lambda^2}$. $\Gamma(r) = \int_0^\infty x^{r-1}e^{-x}dx$, $\Gamma(r) = (r-1)\Gamma(r-1)$, $\Gamma(n+1) = n!$.

$P(X = x, Y = y) = p(x, y)$. $P\left[(X,Y) \in A\right] = \sum\sum_{A} p(x, y)$. Marginal Distributions:
$p_X(x) = \sum_{y \in B} p(x, y)$ and $p_Y(y) = \sum_{x \in A} p(x, y)$, $E[h(X,Y)] = \sum_{x\in A}\sum_{y\in B} h(x, y)p(x, y)$,

$P\left[(X,Y) \in R\right] = \iint_R f(x, y)dxdy$. $f_X(x) = \int_{-\infty}^{\infty} f(x, y)\,dy$ and $f_Y(y) = \int_{-\infty}^{\infty} f(x, y)\,dx$.

The joint probability distribution function is given by $F(t,u) = P(X \le t, Y \le u)$. If he partial derivatives of F exit, then $f(x, y) = \frac{\partial^2}{\partial x \partial y}F(x, y)$.

$E[h(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x, y) f(x, y)\,dxdy$. If X and Y are independent random variables then $E[g(X)h(Y)] = E[g(X)]E[h(Y)]$. $p_{X|Y}(x|y) = \frac{p(x, y)}{p_Y(y)}$.

$E(X|Y = y) = \sum_{x\in A} x p_{X|Y}(x|y)$. $E(h(X)|Y = y) = \sum_{x\in A} h(x)p_{X|Y}(x|y)$. $f_{X|Y}(x|y) = \frac{f(x, y)}{f_Y(y)}$.

$E(X|Y = y) = \int_{-\infty}^{\infty} x f_{X|Y}(x|y)dx$. $E(h(X)|Y = y) = \int_{-\infty}^{\infty} h(x) f_{X|Y}(x|y)dx$.

$F_{X|X>s}(t) = P(X \le t | X > s)$. $f_{X|X>s}(t) = F'_{X|X>s}(t)$.

Let X and Y be continuous random variables with joint density $f(x,y)$. Let h_1 and h_2 be real-valued functions of two variables, $U = h_1(X,Y)$ and $V = h_2(X,Y)$. Suppose that $u = h_1(x, y)$ and $v = h_2(x, y)$ define a one-to-one transformation of a set R in the xy-plane onto a set Q in the uv-plane. That is for $(u, v) \in Q$ the system of two equations in two unknowns $u = h_1(x, y)$ *and* $v = h_2(x, y)$ has a unique solution $x = w_1(u,v)$, $y = w_2(u,v)$ for all x and y in terms of u and v; and the functions w_1 and w_2 have continuous partial derivatives, and the Jacobian of the transformation $x = w_1(u,v)$, $y = w_2(u,v)$ is nonzero on Q:

$$J = \begin{vmatrix} \frac{\partial w_1}{\partial u} & \frac{\partial w_1}{\partial v} \\ \frac{\partial w_2}{\partial u} & \frac{\partial w_2}{\partial v} \end{vmatrix} = \frac{\partial w_1}{\partial u}\frac{\partial w_2}{\partial v} - \frac{\partial w_1}{\partial v}\frac{\partial w_2}{\partial u} \neq 0.$$

Then the random variables U and V are jointly continuous with the joint density function $g(u, v)$ given by $g(u,v) = \begin{cases} f\left(w_1(u,v), w_2(u,v)\right)\left|J\right|, (u,v) \in Q \\ 0, \quad elsewhere. \end{cases}$

Convolution Theorem: $g(t)=\int_{-\infty}^{\infty} f_1(x) f_2(t-x)dx,\;\; G(t)=\int_{-\infty}^{\infty} f_1(x)F_2(t-x)dx,$ and $p(z)=\sum_x p_1(x)p_2(z-x).$

$M_X(t)=E(e^{tX}).\quad M_X(t)=\sum_{x\in A} e^{tx}p(x).\quad M_X(t)=\int_{-\infty}^{\infty} e^{tx}f(x)dx.\; E\left(X^n\right)=M_X^{(n)}(0).$

$M_{X_1,X_2}(t_1,t_2)=E(e^{t_1X_1+t_2X_2}).$

$p(x_1,x_2,...,x_n)=P(X=x_1,X_2=x_2,...,X_n=x_n).\; \sum_{x_i\in A_i,1\le i\le n} p(x_1,x_2,...,x_n)=1.$

$p_{X,Y}(x,y)=\sum_z p(x,y,z).\; F(t_1,t_2,...,t_n)=P(X_1\le t_1,X_2\le t_2,...,X_n\le t_n).$

$E[h(X_1,X_2,...,X_n)]=\sum_{x_n\in A_n}...\sum_{x_1\in A_1} h(x_1,x_2,...,x_n)p(x_1,x_2,...,x_n).$ Let R be any region in $\mathbf{R^n}=$ $\mathbf{R\times R\times ...\times R},$ $P\left((x_1,x_2,...,x_n)\in R\right)=\int_R ...\int f(x_1,x_2,...,x_n)dx_1,dx_2,...,dx_n.$

$\int_{-\infty}^{\infty}....\int_{-\infty}^{\infty} f(x_1,x_2,...,x_n)dx_1,dx_2,...,dx_n=1.\;\; f(x_1,x_2,...,x_n)=\dfrac{\partial^n F(x_1,x_2,...,x_n)}{\partial x_1,\partial x_2,...,\partial x_n}.$

$E[h(X_1,X_2,...,X_n)]=\int_{-\infty}^{\infty}....\int_{-\infty}^{\infty} h(x_1,x_2,...,x_n)f(x_1,x_2,...,x_n)dx_1,dx_2,...,dx_n.$ Let $X_{(k)}$ represent the k^{th} order statistics then its distribution can be described by $F_k(x)=\sum_{i=k}^{n}\binom{n}{i}[F(x)]^i[1-F(x)]^{n-i}\;\; f_k(x)=\dfrac{n!}{(k-1)!(n-k)!}f(x)[F(x)]^{k-1}[1-F(x)]^{n-k}.$

Let $X_{(i)}$ represent the i^{th} order statistics and $X_{(j)}$ represent the j^{th} order statistics ($i<j$)then their joint density function can be described by

$f_{ij}(x,y)=\dfrac{n!}{(i-1)!(j-i-1)!(n-j)!}f(x)f(y)[F(x)]^{i-1}[F(y)-F(x)]^{j-i-1}[1-F(y)]^{n-j}, x<y,$

0, otherwise. The joint denisty of all the n order statistics { $X_{(1)},\; X_{(2)},\; ...,\; X_{(n)}$ } is given by $f_{12...n}(x_1,\ldots,x_n)=n!\,f(x_1)f(x_2)\ldots f(x_n),\quad x_1<x_2\ldots<x_n,$ 0, otherwise.

Multinomial Distributions:

$p(x_1,\ldots,x_r)=P(X_1=x_1,X_2=x_2,\ldots,X_r=x_r)=\dfrac{n!}{x_1!,x_2!\ldots,x_r!}[p_1]^{x_1}[p_2]^{x_2}...[p_r]^{x_r}.$

Let N be a discrete random variable with set of possible values $\{1,2,\ldots\}$. Then $\sum_{i=1}^{\infty} p(N\ge i).$ Cauchy-Schwarz Inequality:

$\left|E(XY)\right|\le\sqrt{E\left(X^2\right)E\left(Y^2\right)}.\; cov(X,Y)=E\left[\left(X-E(X)\right)\left(Y-E(Y)\right)\right].$

$\text{Var}\left(aX+bY\right)=a^2\text{Var}(X)+b^2\text{Var}(Y)+2ab\text{Cov}(X,Y).\;\text{Cov}(X,Y)=E[XY]-\mu_X\mu_Y.$

$\mathrm{Cov}(aX+b,cY+d)=ac\mathrm{Cov}(X,Y)$. $\mathrm{Cov}\left(\sum_{i=1}^{n}a_iX_i,\sum_{j=1}^{m}b_jY_j\right)=\sum_{i=1}^{n}\sum_{j=1}^{m}a_ib_j\mathrm{Cov}(X_i,Y_j)$.

$\mathrm{Var}\left(\sum_{i=1}^{n}a_iX_i\right)=\sum_{i=1}^{n}a_i^2\mathrm{Var}(X_i)+2\sum_{i<j}\sum a_ia_j\mathrm{Cov}(X_i,X_j)$. $\rho=\dfrac{\mathrm{Cov}(X,Y)}{\sigma_X\sigma_Y}$.

$E\left[E(X|Y)\right]=E(X)$. $E\left[f(Y)\,X|Y\right]=f(Y)E(X|Y)$. Let $X_1,X_2,\ldots$ be independent and identically distributed random variables (r.v.s) with finite mean $E(X)$ and Var(X) independent of N, which is an integer-valued r.v. with $E(N) < \infty$ and Var(N). Then

$E\left(\sum_{i=1}^{N}X_i\right)=E(N)E(X)$ and $\mathrm{Var}\left(\sum_{i=1}^{N}X_i\right)=E(N)\mathrm{Var}(X)+\left[E(X)\right]^2\mathrm{Var}(N)$.

$P(B)=\sum_{x\in A}P(B|X=x)P(X=x)$. $P(B)=\int_{-\infty}^{\infty}P(B|X=x)f(x)\,dx$.

$\mathrm{Var}\left(X|Y\right)=E\left[\left(X-E(X|Y)\right)^2\Big|Y\right]$. $\mathrm{Var}\left(X|Y\right)=E\left[X^2|Y\right]-\left(E(X|Y)\right)^2$.

$\mathrm{Var}(X)=E\left[\mathrm{Var}\left(X|Y\right)\right]+\mathrm{Var}\left(E(X|Y)\right)$.

Normal Density: $f(x)=\dfrac{1}{\sigma_X\sqrt{2\pi}}\exp\left[-\dfrac{(x-\mu_X)^2}{2\sigma_X^2}\right]$. $Z=\dfrac{X-\mu_X}{\sigma_X}$.

Bivariate Normal Density:

$p_{ij}^{(n+m)}=\sum_{i=0}^{\infty}p_{ik}^{(n)}p_{kj}^{(m)}$. $f(x,y)=\dfrac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}\exp\left[-\dfrac{1}{2(1-\rho^2)}Q(x,y)\right]$.

$Q(x,y)=\left(\dfrac{x-\mu_X}{\sigma_X}\right)^2-2\rho\left(\dfrac{x-\mu_X}{\sigma_X}\right)\left(\dfrac{y-\mu_Y}{\sigma_Y}\right)+\left(\dfrac{y-\mu_Y}{\sigma_Y}\right)^2$.

$M_{X_1,X_2}(t_1,t_2)=e^{\mu_1t_1+\mu_2t_2+\frac{1}{2}\left(\sigma_1^2t_1^2+\sigma_2^2t_2^2+2\rho t_1t_2\sigma_1\sigma_2\right)}$.

$f_{Y|X}(y|x)=\dfrac{1}{\sigma_Y\sqrt{2\pi}\sqrt{1-\rho^2}}\exp\left[-\dfrac{[y-\mu_Y-\rho(\sigma_Y/\sigma_X)(x-\mu_X)]^2}{2\sigma_Y^2(1-\rho^2)}\right]$.

Markov Chain:

$P\left(X_{n+1}=j|X_n=i,X_{n-1}=i_{n-1},\ldots,X_1=i_1,X_0=i_0\right)=P\left(X_{n+1}=j|X_n=i\right)=p_{i,j}$.

Chapman Kolmogorov Equations: $p_{ij}^{(n+m)}=\sum_{k=0}^{\infty}p_{ik}^{(n)}p_{kj}^{(m)}$. $P^{(n+m)}=P^{(n)}P^{(m)}$.

The n[th] **time** X_n distribution is given by $P(X_n=j)=\sum_{i=0}^{\infty}p_{X_0}(i)p_{ij}^{(n)}$.

Note that $\sum_{n=1}^{\infty}p_{ii}^{(n)}<\infty$ **if and only if state 'i' is transient.**

Note that $\sum_{n=1}^{\infty}p_{ii}^{(n)}=\infty$ **then state 'i' is recurrent.**